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# Statistical analysis of Hoek-Brown shear strength properties for single and multiple intact rock units

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Article

**Keywords**
Rock strength
Hoek-Brown failure criterion
Uniaxial compressive strength test
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Regression analysis

**Abstract**

In the practice of geotechnical mining, the shear strength of intact rock is typically characterized using the Hoek-Brown failure criterion. The presence of veinlets, micro-defects and/or a stockwork texture of tested rock specimens, a common feature of mineralized rock, leads to variability of results from strength testing, resulting in the need to carry out regression analysis of the test results to find a best fit Hoek-Brown failure curve. This paper presents a way of accounting uniaxial compression tests separately from triaxial tests, so to decrease the biasing towards one type of testing in particular. This alternative form of doing the regression analysis may be applicable when the quality of uniaxial compression tests is known to be of similar good quality as triaxial tests, as could be the case in some mining projects. In the proposed methodology, a measure of the quality of the fit of the Hoek-Brown failure curve is obtained by computation and interpretation of a correlation factor. For cases in which several intact rock units are being characterized, a global correlation factor for all geotechnical units considered together is proposed. To further quantify the variability of results obtained from laboratory testing, a probability analysis of the Hoek-Brown properties is also presented.

## 1. Introduction

In geotechnical mining, proper characterization of intact rock strength through laboratory testing is of primary importance, as the strength properties of intact rock, together with empirical rock mass classification systems, allow definition of the strength of the rock mass. Both intact rock and rock mass strength, in turn, allow definition of the mechanical response of excavations and necessary support/reinforcement to recover ore (see Read & Keeney, 2009).

The Hoek-Brown failure criterion is a widely used method for characterizing shear strength of intact rock and rock masses. For intact rock, this non-linear shear strength criterion takes the following form (Hoek & Brown, 1980, 2019):

$$\sigma_1 = \sigma_3 + \sigma_{ci}\sqrt{m_i \frac{\sigma_3}{\sigma_{ci}} + 1} \quad (1)$$

In Equation (1), $\sigma_1$ and $\sigma_3$ are the major and minor principal stresses, respectively; $\sigma_{ci}$ is the uniaxial compressive strength, and $m_i$ is a constant that depends on the type of rock being tested.

Characterization of intact rock properties in terms of the Hoek-Brown failure criterion involves determination of the values of $\sigma_{ci}$ and $m_i$ to use in Equation (1). This is done through lab testing, in particular, uniaxial and triaxial compressive strength tests.

Uniaxial compression strength testing (or UCS-testing), and triaxial compression strength testing (or TX-testing) are common types of tests done on cylindrical specimens of rock to determine intact rock strength — see, for example, Goodman (1989) and Brady & Brown (2006). Detailed specifications and procedures for conducting these tests are provided in suggested methods by the International Society for Rock Mechanics and Rock Engineering Ulusay & Hudson (2007) and Ulusay (2014). In the context of characterizing the strength of intact rock with Equation (1), UCS-tests allow estimation of $\sigma_{ci}$, while TX-tests allow definition of the constant mi. Because several UCS-tests and TX-tests are typically conducted for the same rock unit, representative values of $\sigma_{ci}$ and $m_i$ need to be obtained from statistical interpretation (i.e., regression analysis) of test results. With this regard, it should be mentioned that there are instances in which no TX-tests are conducted, for example, when the importance of the mining excavation is minor and a quick evaluation of excavation response is needed.

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In such cases, it is not uncommon to estimate the parameter $m_i$ in Equation (1) through empirical tables such as those presented in Hoek & Brown (1980 and 1997), and perform the statistical interpretation of UCS-test results only.

Besides UCS-tests and TX-tests, splitting tension tests, or Brazilian tests and point load tests Goodman (1989) and Brady & Brown (2006) are also conducted as part of the characterization of intact rock in geotechnical mining. Although Brazilian test results (in combination with UCS-tests and TX-tests) have been used to characterize the properties $\sigma_{ci}$ and $m_i$ in Equation (1), in this paper, Brazilian tests will not be considered. This agrees with the latest version of the Hoek-Brown failure criterion Hoek & Brown (2019) — see, also, Hoek & Martin (2014) — where the authors indicate that a tension cut-off applies to the Hoek-Brown failure criterion given by Equation (1).

In geotechnical mining it is quite normal to find that intact rock of a same unit shows high variability of strength and therefore variability in the properties $\sigma_{ci}$ and $m_i$ in Equation (1). This is particularly true when dealing with rock that has been mineralized through hydrothermal processes. These deposits tend to produce marked alteration in the rock. Some types of alteration (e.g., silicified) could lead to a rock of higher strength (compared with the non-altered rock of the same unit), while other types (e.g., argillic alteration) result in lower strength — see Flores & Karzulovic (2003), Russo & Hormazabal (2016) and Russo et al. (2020). In general, veinlets, micro-defects and/or a stockwork texture tend to decrease the uniaxial compressive strength of intact rock specimens, compared with the strength of rock without such defects or texture.

An example of alteration of rock within the same rock unit is illustrated in Figure 1. The figure shows specimens of rock prepared for UCS-testing. Figure 1a corresponds to a quartzitic porphyry with chloritic and quartz-sericitic alteration. Figure 1b corresponds to feldspathic porphyry with sericitic alteration. Figure 1c corresponds to feldespathic porphyry with argillic and quartz-sericitic alteration; and Figure 1d corresponds to feldspathic porphyry with chloritic and quartz-sericitic alteration. All samples are characterized by high degree of alterations produced by the hydro- thermal processes that led to mineralization. In particular, the samples in Figure 1b and Figure 1d show veinlets that are weaker than the main matrix of rock containing them. These veinlets decrease the overall strength of the specimen measured in the test. In extreme cases, the degree of alteration could be so high that sampling and cutting of specimens for UCS-testing or TX-testing becomes impossible. In such cases, there is no other option than to sample and test relative sound (non-altered) rock from the core, and to account for the alteration through rock mass classification system — see Hoek et al. (1998), Marinos et al. (2006) and Hoek & Brown (2019).

In general, UCS-testing is easier and more economical to do than TX-testing. Also, UCS-tests can be regarded as TX-tests for which the confining pressure is equal to zero. When reviewing existing geotechnical reports (e.g., to design new excavations in an existing mine) it is quite normal to encounter that a large number of UCS-tests have been done for a rock unit, while a quite limited (if none) number of TX-tests have been conducted. In any case, the number of UCS-test results is almost always larger than the number of TX-test results for any particular fixed value of confining pressure considered.

Although UCS-testing is easier and more economical than TX-testing, quantity does not necessarily imply quality, and in many instances the quality of TX-test results is higher than the quality of UCS-test results. Based on this assumption, Hoek & Brown (2019) recommend doing a regression analysis to compute both parameters $\sigma_{ci}$ and $m_i$ in Equation (1), considering all TX-test results and only the average value of the UCS-test results. In this way, the biasing towards the UCS-test results can be eliminated.

Prior to the latest recommendation in Hoek & Brown (2019), it had been normal practice to apply regression analysis to all laboratory testing results (i.e., to include all UCS-test and TX-test results), in this way giving the same weight to UCS-test and TX-test results, independently of the quality and quantity of the test results.

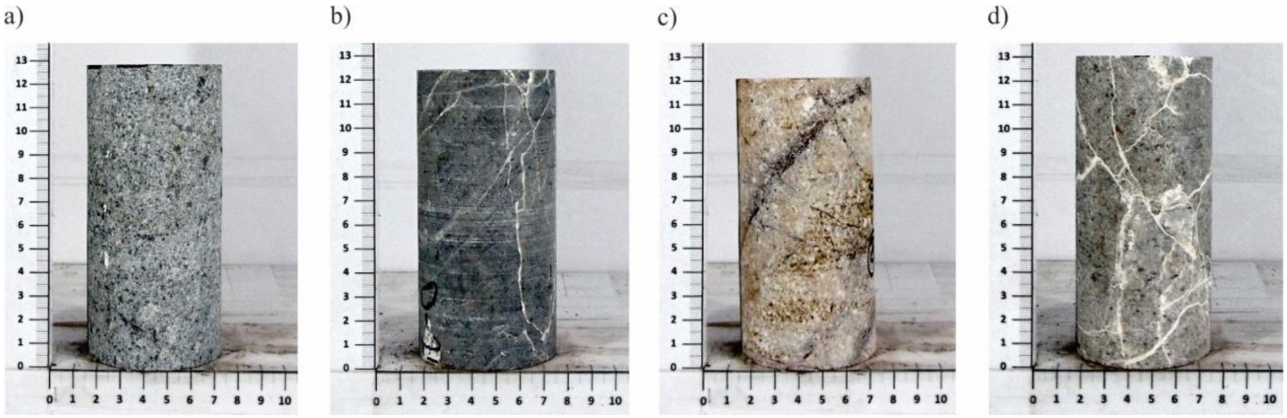



**Figure 1.** Samples of porphyries corresponding to different alterations produced by the hydro-thermal processes that led to mineralization.

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For example, this type of regression analysis is the one that used the software RocLab that accompanied the article Hoek et al. (2002), which was developed and freely distributed to assist the engineer in the estimation of the Hoek-Brown failure criterion parameters; it is also the type of regression analysis currently implemented in the software RocData (Rocscience Inc, 2020). As it will be illustrated in the next section in this paper, this type of analysis (i.e. that equally 'weights' UCS-test and TX-test results), yields estimation of a $\sigma_{ci}$ value for a rock unit that is not necessarily equal to the mean value of the UCS-test results considered by themselves.

One main objective of this paper is to present an alternative way of carrying out the computation of the Hoek-Brown properties $\sigma_{ci}$ and $m_i$ though regression analysis, that accounts for UCS-test and TX-test results separately. This alternative way of computing the parameters is not intended to be a replacement to the recommendation made by Hoek & Brown (2019), but to be considered in situations in which the quantity of UCS-test results is large compared with that of TX-tests, and the quality of UCS-testing is known to be the same as that of TX-testing.

With regard to the regression analysis of UCS-test and TX-test results, various techniques can be applied to find the 'best fit' failure curve. A recent article by Úcar et al. (2024) discusses a rigorous method for adjusting a Hoek-Brown failure curve (and other types of failure curves) to laboratory test results. To simplify the analysis, this paper will illustrate the procedure using the classical least square regression technique.

## 2. Intact rock properties from regression analysis of laboratory testing results

The proposed regression analysis of intact rock properties $\sigma_{ci}$ and $m_i$, introduced in the last section, will be presented and illustrated with an example.

Table 1 lists different values of $\sigma_{ci}$ from uniaxial compressive tests (UCS-tests) for four different geotechnical units — designated as GU-1, GU-2, GU-3 and GU-4. These units correspond to rocks that have been geologically characterized as 1) granodiorite with chlorite alteration; 2) breccia with sericitic alteration; 3) diorite; and 4) andesite. Similarly, Table 2 lists the pairs of values corresponding to confining pressure and axial stress at failure ($\sigma_{3',i}$, $\sigma_{1',i}$) obtained from triaxial tests (TX-tests) carried out for the same units. The results of UCS-tests and TX-tests listed in Table 1 and Table 2 are known to be both of comparable good quality.

As it was discussed in the introduction section, in practical geotechnical mining, TX-test results typically comprise few scattered (and incomplete) series of values of confining pressure, with the number of available TX-tests for a particular fixed value of confining pressure being smaller than the number of UCS-tests for the same unit. This is indeed the case of the test results summarized in Table 1 and Table 2, in which, for example, for GU-1 there are 21 UCS-tests available, whereas there are only 2 TX-tests for the fixed value of confining pressures 15 MPa and 30 MPa, all the other TX-test results being for different values of confining pressure.

With regard to the origin of the data in Table 1 and Table 2, these sets of results are part of a large data set of rock test results carried out throughout a decade or so of geotechnical investigations for the development of a mine. The analysis of this data was commended to the first author as part of a study of future expansion of an open pit mine. The lab reports included details such as photographs of core boxes, location of sampling, and photographs of intact rock specimens before and after the testing. Testing done for the project went beyond the UCS-test and TX-test results summarized in Table 1 and Table 2. As mentioned in the introduction to this paper, the focus here is on discussing the estimation of Hoek-Brown properties $\sigma_{ci}$ and $m_i$ from UCS-testing and TX-testing only, so other testing done will not be discussed.

**Table 1.** Results obtained from uniaxial compression strength tests (UCS-tests) for four different geotechnical units in a mining project.

| | $\sigma_{ci}$ [MPa] | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Test | GU-1 | GU-2 | GU-3 | GU-4 | Test | GU-1 | GU-2 | GU-3 | GU-4 |
| 1 | 114.1 | 51.1 | 141.3 | 99.5 | 14 | 119.1 | 50.4 | 157.9 | 110.1 |
| 2 | 99.8 | 60.9 | 138.0 | 102.9 | 15 | 115.6 | 55.0 | 133.1 | 105.2 |
| 3 | 101.4 | 79.9 | 140.9 | 115.7 | 16 | 108.4 | 51.5 | 148.7 | 102.1 |
| 4 | 122.1 | 75.8 | 134.3 | 95.1 | 17 | 100.9 | 58.9 | 136.9 | 94.5 |
| 5 | 125.7 | 29.8 | 124.8 | 102.0 | 18 | 123.6 | 78.2 | 158.3 | 97.9 |
| 6 | 109.0 | 35.8 | 148.3 | 113.9 | 19 | 127.3 | 63.6 | 145.0 | 95.4 |
| 7 | 140.6 | 64.0 | 145.9 | 111.5 | 20 | 116.9 | 69.0 | 138.5 | 100.7 |
| 8 | 126.7 | 52.3 | 156.1 | 76.4 | 21 | 130.9 | 73.0 | 137.4 | 112.9 |
| 9 | 135.9 | 45.3 | 183.7 | 82.2 | 22 | | 60.6 | 140.2 | 103.8 |
| 10 | 125.3 | 62.3 | 160.1 | 104.1 | 23 | | 65.2 | 156.1 | 107.3 |
| 11 | 105.2 | 73.5 | 162.9 | 95.9 | 24 | | 63.3 | 144.1 | |
| 12 | 131.8 | 65.6 | 160.9 | 90.5 | 25 | | 67.6 | 149.0 | |
| 13 | 106.2 | 61.0 | 161.8 | 103.0 | 26 | | 54.1 | 167.5 | |

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**Table 2.** Results obtained from triaxial compression strength tests (TX-tests) for four different geotechnical units in a mining project.

| | $(\sigma_{3,\,i}, \sigma_{1,\,i})$ [MPa] | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Test | GU-1 | | GU-2 | | GU-3 | | GU-4 | |
| 1 | 5.0 | 145.5 | 2.8 | 92.4 | 5.0 | 188.5 | 2.0 | 113.1 |
| 2 | 7.2 | 150.0 | 4.1 | 75.7 | 5.0 | 182.4 | 2.0 | 102.4 |
| 3 | 8.6 | 153.9 | 5.0 | 70.8 | 6.6 | 207.1 | 4.0 | 158.9 |
| 4 | 11.3 | 193.3 | 5.0 | 70.7 | 10.0 | 260.1 | 4.0 | 148.2 |
| 5 | 15.0 | 217.5 | 5.4 | 111.1 | 12.8 | 262.7 | 4.0 | 115.1 |
| 6 | 15.0 | 170.8 | 6.2 | 68.5 | 14.5 | 252.9 | 4.0 | 150.9 |
| 7 | 17.6 | 187.9 | 7.4 | 80.5 | 15.4 | 267.4 | 5.0 | 136.1 |
| 8 | 19.7 | 215.0 | 8.0 | 75.8 | 18.0 | 327.5 | 5.0 | 143.7 |
| 9 | 25.2 | 277.8 | 9.5 | 114.4 | 19.8 | 276.4 | 5.0 | 186.3 |
| 10 | 25.9 | 253.3 | 11.2 | 111.2 | 20.7 | 333.1 | 6.8 | 179.7 |
| 11 | 26.6 | 254.9 | 10.0 | 97.2 | 21.5 | 301.3 | 10.0 | 194.2 |
| 12 | 30.0 | 311.3 | 10.0 | 127.0 | 23.3 | 361.4 | 10.0 | 194.4 |
| 13 | 30.0 | 299.6 | 15.0 | 144.7 | 25.0 | 372.2 | 14.3 | 225.2 |
| 14 | 32.9 | 251.0 | 15.0 | 138.7 | 25.9 | 362.1 | 14.3 | 236.2 |
| 15 | 36.3 | 279.8 | 20.0 | 125.0 | 25.9 | 357.1 | 17.5 | 251.5 |
| 16 | 40.0 | 369.3 | 20.0 | 135.1 | 26.8 | 386.6 | 18.6 | 221.4 |
| 17 | 50.0 | 393.5 | 20.4 | 176.3 | 28.6 | 444.1 | 20.7 | 239.7 |
| 18 | | | 20.9 | 140.8 | 30.3 | 392.9 | 25.0 | 299.2 |
| 19 | | | 25.0 | 176.8 | 32.1 | 381.9 | 25.0 | 292.1 |
| 20 | | | 25.0 | 184.4 | 33.8 | 445.3 | 28.2 | 298.3 |
| 21 | | | 26.3 | 176.2 | 35.6 | 467.9 | 33.6 | 336.4 |
| 22 | | | | | 38.2 | 411.0 | 40.0 | 411.4 |
| 23 | | | | | 40.0 | 447.2 | 40.0 | 363.4 |
| 24 | | | | | | | 50.0 | 406.7 |
| 25 | | | | | | | 50.0 | 457.2 |

It is also important to mention that both Table 1 and Table 2 contain 'valid' (or reasonably valid) test results. The initial number of test results was larger than the ones listed in the tables, but many test results were discarded by the authors after inspection of photographs of intact rock specimens before and after testing. For example, tests for which failure of the sample took place along pre-existing cracks or veins were discarded; tests for which major principal stresses at failure (particularly for TX-testing) were unreasonably low were also discarded; test results for confining pressures in TX-testing that were larger than the values predicted by Mogi (1966) for the transition from brittle to ductile behavior were discarded (see Mogi, 2007), as it was interpreted that the Hoek-Brown failure criterion applies to brittle behavior of rock mostly.

The analysis of test results in Table 1 and Table 2 starts with the application of some basic descriptive statistics to the UCS-test results.

A representative value of the property $\sigma_{ci}$ for a geotechnical unit is obtained by computing an arithmetic mean ($\sigma_{ci}$) of all UCS-test results for the unit, i.e.:

$$\overline{\sigma}_1 = \frac{1}{n}\sum_{i=1}^{n}\sigma_{1,\,i} \tag{2}$$

The standard deviation ($s.d.\ \sigma_{ci}$) and the coefficient of variation ($c.v.\ \sigma_{ci}$) of the UCS-test results are also computed to quantify the spread of measured values of $\sigma_{ci}$, i.e.:

$$s.d.\sigma_{ci} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\sigma_{cij} - \overline{\sigma}_{ci}\right)^2} \tag{3}$$

and

$$c.v.\sigma_{ci}. = \frac{s.d.\sigma_{ci}}{\overline{\sigma}_{ci}} \tag{4}$$

In Equations (2), (3) and (4), the variables n and $\sigma_{ci,i}$ represent the total number of UCS-tests, and the UCS-test results for the same unit, respectively.

The mean values, the standard deviations and the coefficients of variation of the UCS-test results computed with Equations (2), (3) and (4) are summarized in Table 3 for all different geotechnical units in Table 1.

Computation of the best fit Hoek-Brown failure curve defined by the properties $\sigma_{ci}$ and $m_i$ will be illustrated now for the arbitrarily selected geotechnical unit GU-1.

Figure 2 represents UCS-test results and TX-test results for the unit GU-1 in Table 1 and Table 2, in the interval of confining pressure 0 to 40 MPa. In the principal stress representation shown in the figure, UCS-tests are indicated with square symbols while the TX-tests are indicated with circular symbols.

The Hoek-Brown shear failure curve labelled as 'Traditional' in Figure 2 corresponds to the curve for GU-1 obtained from regression analysis of both UCS-test and TX-test results as given by the software RocData (Rocscience Inc, 2020), using the default Levenberg-Marquardt algorithm. The values of $\sigma_{ci}$ and $m_i$ obtained in this way are indicated in the figure. It is seen that the resulting value $\sigma_{ci}$ 116.05 MPa is different from the arithmetic mean of UCS-test results (Equation 2) for GU-1, that according to Table 3 is $\sigma_{ci}$ 118.41 MPa.

The Hoek-Brown shear failure curve labelled as 'Proposed' in Figure 2 corresponds to the curve for GU-1 obtained from regression analysis of the TX-test results only — i.e., prescribing that the Hoek-Brown failure curve must intersect the vertical axis in the principal stress diagram at the value $\sigma_{ci}$ 118.41 MPa, given by Equation (2). The resulting values of $\sigma_{ci}$ and $m_i$ are indicated in the figure, under the legend 'Proposed'. The resulting value $m_i$ 13.71 computed with the proposed method is different from the one computed earlier on with the traditional method.

The value of the Hoek-Brown parameter $m_i$ under the legend 'Proposed' has been computed by minimizing the sum of the squares of the estimate residuals, i.e., by minimizing the following equation — see, for example, Montgomery & Runger (2015) and Chapra & Canale (2020):

$$s_r\left(m_i\right)=\sum_{i=1}^{n}\left[\sigma_{1,\,i}-\sigma_{3,\,i}-\overline{\sigma}_{ci}\sqrt{m_i\frac{\sigma_{3,i}}{\overline{\sigma}_{ci}}+1}\right]^2 \qquad (5)$$

In Equation (5) and in the equations that follow, $\sigma_{3,\,i}$ and $\sigma_{1,\,i}$ are the pairs of values of confining pressure and axial stress at failure, respectively, for the different TX-tests for the geotechnical unit considered. The minimization process of Equation (5) involves finding the first derivative of the function $S_r\,m_i$ with respect to the variable $m_i$ and equating it to zero, i.e.:

$$s_r\,'\left(m_i\right)=\sum_{i=1}^{n}\frac{\left(\sigma_{3,i}{}^2-\sigma_{3,i}\,\sigma_{1,i}\right)}{\sqrt{m_i\frac{\sigma_{3,i}}{\overline{\sigma}_{ci}}+1}}+\sigma_{ci}\,\sigma_{3,i}=0 \qquad (6)$$

Finding the best fit value of $m_i$ that satisfies Equation (6) can be done using any computational software for engineering or science (e.g., Python, Matlab, Mathcad, to cite some). Alternatively, it can also be done with the software EXCEL (Microsoft Corporation, 2024), using the 'goal seek' function, or using the add-in 'Solver' package — see, for example, Chapra & Canale (2020).

The values of the Hoek-Brown parameters $\sigma_{ci}$ and $m_i$ computed with the 'Traditional' and 'Proposed' methods for all geotechnical units in Table 1 and Table 2, are summarized

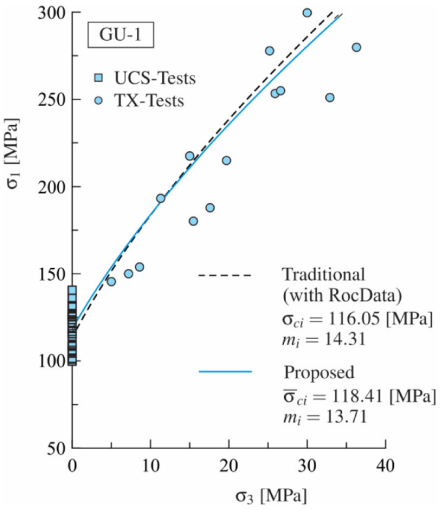



**Figure 2.** Best fit Hoek-Brown shear failure curve for uniaxial and triaxial compressive test results for GU-1 in Table 1 and Table 2, obtained with the traditional method, using the software RocData (Rocscience Inc, 2020), and with the proposed method.

**Table 3.** Summary of statistical analysis of UCS-test results and regression analysis of Hoek-Brown properties, $\sigma_{ci}$ and $m_i$, from TX-test results for the four geotechnical units in Table 1 and Table 2.

| Results | | GU-1 | GU-2 | GU-3 | GU-4 |
|---|---|---|---|---|---|
| $\sigma_{ci}$ [MPa] | | 118.41 | 60.30 | 148.91 | 100.98 |
| *s.d.* $\sigma_{ci}$ [MPa] | | 12.06 | 12.08 | 13.04 | 9.58 |
| *c.v.* $\sigma_{ci}$ [%] | | 10.18 | 20.03 | 8.76 | 9.48 |
| Traditional fitting | $\sigma_{ci}$ [MPa] | 116.05 | 59.33 | 146.17 | 99.08 |
| (with RocData) | $m_i$ | 14.31 | 11.15 | 25.17 | 25.65 |
| Proposed fitting | $\sigma_{ci}$ [MPa] | 118.41 | 60.30 | 148.91 | 100.98 |
| | $m_i$ | 13.71 | 10.72 | 24.18 | 24.78 |
| | $r^2$ | 0.863 | 0.819 | 0.902 | 0.962 |
| | $R^2$ | | | 0.862 | |

in Table 3. For all units, differences between corresponding values of $\sigma_{ci}$ and $m_i$ can be observed.

## 3. Quantifying the quality of the regression analysis

The quality of the best fit values of the Hoek-Brown parameter $m_i$ from TX-tests, as described in the previous section, can be quantified by estimating the coefficient of determination $r^2$. This coefficient is computed with the following equation — see, for example, Montgomery & Runger (2015) and Chapra & Canale (2020):

$$r^2 = \frac{S_t - S_r}{S_t} \tag{7}$$

where $S_r$ is the sum of the squares of the estimate residuals given by Equation (5), and $S_t$ is the sum of the squares of the data residuals, which is computed as:

$$S_t = \sum_{i=1}^{n} \left[ \sigma_{1,\,i} - \overline{\sigma}_1 \right]^2 \tag{8}$$

In Equation (8), $\sigma_1$ is the arithmetic mean of the values of axial stress at failure from the TX-tests for the geotechnical unit being analyzed, i.e.:

$$\overline{\sigma}_1 = \frac{1}{n} \sum_{i=1}^{n} \sigma_{1,\,i} \tag{9}$$

The value of the coefficient of determination $r^2$ computed with Equation (7) typically ranges between 0 and 1 — i.e., if a negative value results from the application of the equations above, the coefficient should be interpreted to be zero.

A value $r^2 = 1$ implies that the fit is perfect; in such case all TX-test results fall on the fitted curve, and Equation (8) is equal to zero. A value $r^2 = 0$ (or a negative value) implies that the fit represents no improvement over considering any other strength curve constructed with any other value of $m_i$. Considering the scarce nature of TX-tests results in typical mining situations, resulting values $r^2 > 0.75$ should be interpreted as acceptable, again, with quality of the fit increasing as $r^2$ becomes closer to one.

To illustrate the variability of the coefficient of determination $r^2$, Figure 3 shows strength curves obtained with the proposed regression analysis presented in the previous section, for the four different geotechnical units listed in Table 1, Table 2 and Table 3. For each of the geotechnical units, the diagrams indicate the resulting values of the $\sigma_{ci}$ and $m_i$ parameters, and the coefficient $r^2$ obtained from application of Equation (7). Note that the second-to-last row in Table 3 also includes the values of $r^2$ for each of the geotechnical units.

In the introduction to this paper, it was already mentioned about the typical variability of UCS- test and TX-test results in altered rock, e.g., mineralized rock. In Table 1, Table 2 and Table 3, the geotechnical units GU-1 and GU-2 correspond to altered mineralized rock (with GU-2 being significantly more altered than GU-1). It can be seen from Table 3 that the computed values of *c.v.* $\sigma_{ci}$ for units GU-1 and GU-2 are larger than the corresponding values for units GU-3 and GU-4. Also, it can be seen that the computed values of $r^2$ for units GU-1 and GU-2 are smaller than for the other units. In all cases, nevertheless, the values of $r^2$ result larger than 0.75 suggesting an acceptable fitting of the Hoek-Brown failure curves.

In addition to computing coefficients of determination ($r^2$) for each individual unit, it is also possible to compute a 'global' coefficient of determination for all units considered together. This can be done by transforming the Hoek-Brown failure criterion according to the rule discussed in Carranza-Torres & Hormazabal (2020) — see also Carranza-Torres (2023).

Referring to the Hoek-Brown failure criterion for intact rock (Equation 1), the transformation involves dividing the values of principal stresses $\sigma_3$ and $\sigma_1$ by the term ($m_i \times \sigma_{ci}$), and adding the term $1/m_i^2$, i.e.:

$$S_3 = \frac{\sigma_3}{m_i\, \sigma_{ci}} + \frac{1}{m_i^{\,2}} \tag{10}$$

and

$$S_1 = \frac{\sigma_1}{m_i\, \sigma_{ci}} + \frac{1}{m_i^{\,2}} \tag{11}$$

With this stress transformation, a dimensionless form of the Hoek-Brown failure criterion for intact rock can be obtained as follows:

$$S_1 = S_3 + \sqrt{S_3} \tag{12}$$

The effect of applying the transformation in the Equations (10) through (12) is illustrated in the diagrams in Figure 4. Figure 4a represents the UCS-test and TX-test results, together with their corresponding best fit shear failure curves for all four geotechnical units, as represented separately in Figure 3. Clearly, there is a marked variability of results (i.e., points representing tests) as these correspond to different types of rock. Figure 4b represents the same results, this time when the UCS-test and TX-test results in Figure 4a have been transformed using the corresponding values of $\sigma_{ci}$ and $m_i$ for each unit (i.e., the values for the proposed fitting listed in Table 3). It is seen that all UCS-test and TX-test results tend to align closer to a unique (transformed) Hoek-Brown failure curve given by Equation (12). Because in the transformed representation of stresses $S_3$ and $S_1$ all Hoek-Brown failure curves 'merge' into a single one, it is possible now to compute a 'global'

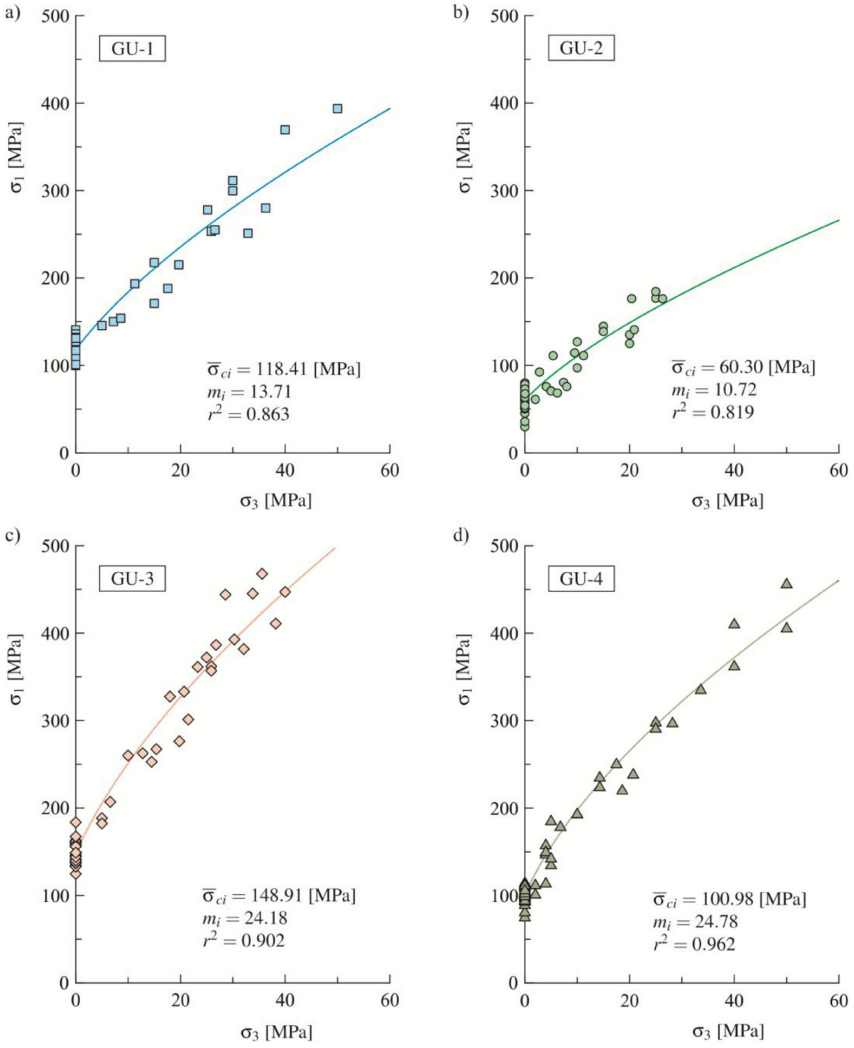



**Figure 3.** Test results and best fit Hoek-Brown shear failure curves for the four geotechnical units summarized in Table 1 and Table 2.

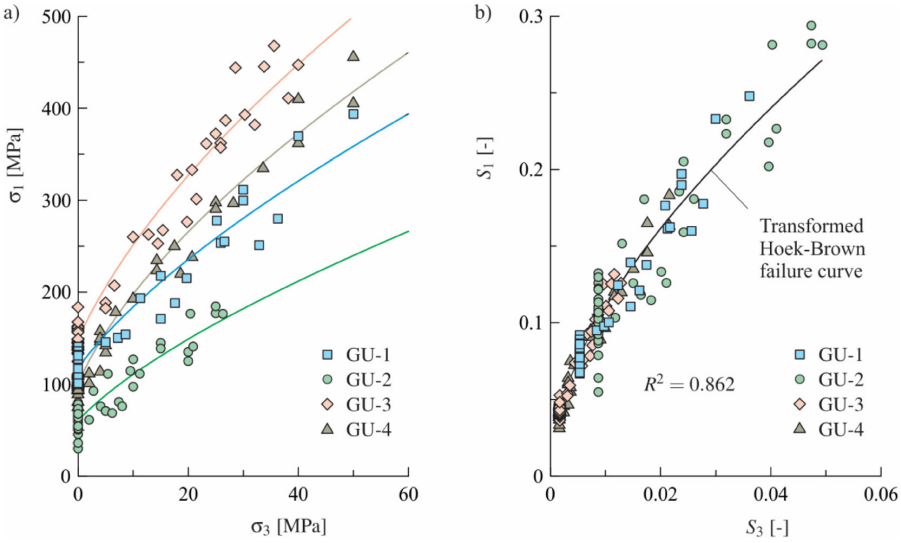



**Figure 4.** a) Test results and Hoek-Brown shear failure curves for the four geotechnical units in Table 1 and Table 2. b) Same results plotted in terms of the transformed principal stresses given by Equations (10), (11) and (12).

coefficient of determination, $R^2$, using similar equations as presented earlier on, i.e.:

$$R^2 = \frac{S_t - S_r}{S_t} \tag{13}$$

where

$$S_r = \sum_{i=1}^{n} \left[ S_{1,\,i} - S_{3,\,i} - \sqrt{S_{3,\,i}} \right]^2 \tag{14}$$

and

$$S_t = \sum_{i=1}^{n} \left[ S_{1,\,i} - \overline{S}_1 \right]^2 \tag{15}$$

with

$$\overline{S}_1 = \frac{1}{n} \sum_{i=1}^{n} S_{1,\,i} \tag{16}$$

In Equations (13) through (16), capital letters have been used for the coefficient of determination, for the sum of the squares of the estimate residuals, and for the squares of the data residuals, respectively; to indicate that these values correspond to transformed UCS-test and TX-test results, according to Equations (10) and (11).

Considering that the coefficient $R^2$ given by Equation (13) is intended to be representative of both best fit values of the intact rock properties $\sigma_{ci}$ and $m_i$ in the Hoek-Brown failure criterion, the values $S_{3,\,i}$ and $S_{1,\,i}$ in Equations (14), (15) and (16) are values corresponding to both UCS-tests and TX-tests.

Application of Equations (13) through (16) allows to compute the value $R^2 = 0.862 > 0.75$ indicated in Figure 4b, which according to what was mentioned earlier on, indicates that the fitting for all units considered together is also acceptable.

## 4. Reliability analysis of estimated intact rock properties

In view of the normal variability observed in UCS-test and TX-test results, it is also possible to assess the reliability (or level of confidence) of the parameters $\sigma_{ci}$ and $m_i$ obtained with the regression analysis discussed in previous sections. This sort of analysis involves using probability density functions (PDFs) to compute the probability (in percentage) of getting parameters $\sigma_{ci}$ and $m_i$ within specified ranges of values.

An example of PDFs applied to principal stress values from intact rock testing is shown in Figure 5 —after Contreras & Brown (2019). The diagram in the figure shows t-distributions applied to results from uniaxial compressive strength tests (UCS) and triaxial compressive strength tests (TCS) to assess the reliability of Hoek-Brown parameters in slope design. As mentioned in earlier sections, in typical situations there is most frequently more results available from UCS-tests than from TX-tests for any particular fixed value of confining pressure, with the data from TX-tests being typically scarce and incomplete (e.g., not covering a full sequence of confining pressures). The TX-test results shown in Figure 5 correspond then to the non-typical situation in which individual PDFs can be applied to results corresponding to different values of confining pressure.

For this reason, the simplified probability analysis presented in this section will focus on applying probability density functions to the UCS-test results only (i.e., to the parameter $\sigma_{ci}$), while treating the parameter $m_i$ as a dependent variable of $\sigma_{ci}$, to be computed using the regression analysis equations presented in Section 2. In the simplified probability analysis that follows, log-normal PDFs will be used, as these PDFs yield null probability of occurrence of negative values of $\sigma_{ci}$ and $m_i$. Also, the same geotechnical unit GU-1 considered in Section 2 (Figure 2, see also Table 1, Table 2 and Table 3) will be used to illustrate the analysis.

Figure 6 shows a histogram of normalized frequency (i.e., the vertical axis scaled so that the total area of the represented bars is equal to one) for the values of uniaxial compressive strength for GU-1 in Table 1. The histogram in Figure 6 also shows the log-normal PDF corresponding to the values of arithmetic mean and standard deviation of $\sigma_{ci}$ values computed already for GU-1 (see Table 3). The log-

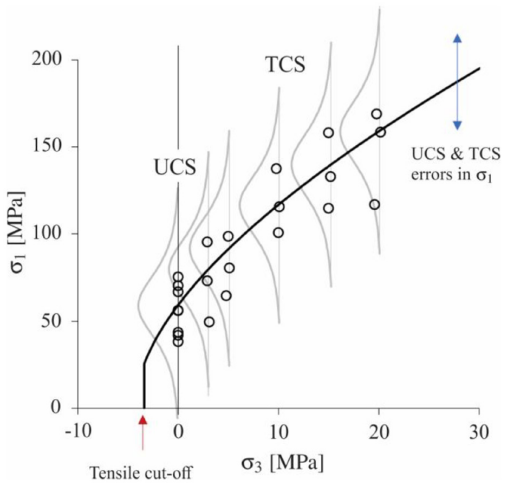



**Figure 5.** Example of probability density functions (PDFs) applied to results from uniaxial compressive strength tests (UCS) and triaxial compressive strength tests (TCS) - after Contreras & Brown (2019).

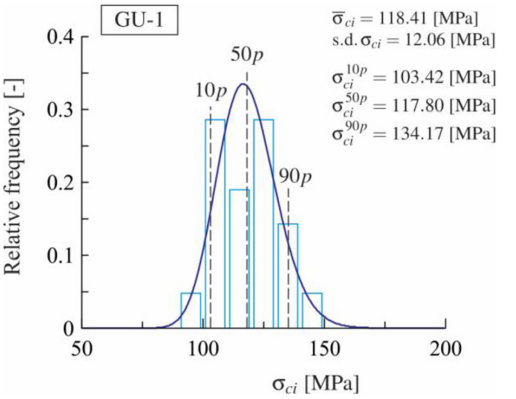



**Figure 6.** Histogram of uniaxial compressive test results for GU-1 in Table 1.

normal PDF represented in Figure 6 has been determined using the following equations —see, for example, Baecher & Christian (2002) and Montgomery & Runger (2015):

$$f(x)=\frac{\Delta b}{x\sigma_N\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln(x)-\mu_N}{\sigma_N}\right)^2\right] \tag{17}$$

where

$$\sigma_N=\sqrt{ln\left[1+\left(\frac{\sigma_x}{\mu_x}\right)^2\right]} \tag{18}$$

and

$$\mu_N=ln\left(\mu_x\right)-\frac{1}{2}ln\left[1+\left(\frac{\sigma_x}{\mu_x}\right)^2\right] \tag{19}$$

In Equation (17), $x$ is the independent variable (i.e., the variable $\sigma_{ci}$ in the case of Figure 6) and $\Delta b$ is the interval (or 'bin') chosen for constructing the histogram of the test results (i.e., the value 10 in the case of Figure 6). In Equations (18) and (19), $\mu_x$ and $\sigma_x$ represent the mean values and standard deviation of the test results —i.e., the values obtained with Equations (2) and (3), respectively. Since the area below the log-normal PDF defined by Equations (17) through (19) is equal to one, the probability of occurrence of measured value $x$ is defined as follows:

$$p(x)=\frac{1}{\Delta b}\int_0^x f(x)dx \tag{20}$$

For the values of uniaxial compressive strength represented in Figure 6, application of Equations (17) through (19) (with $\mu_x\,\sigma_{ci}$ and $\sigma_x\,s.d.\,\sigma_{ci}$ with values from Table 3) gives the following 10, 50 and 90 percentiles, interpreted to be the values of $\sigma_{ci}$ with a probability of occurrence of 10%, 50% and 90%, respectively:

$$\sigma_{ci}^{10p}=103.42\,MPa;\sigma_{ci}^{50p}=117.80\,MPa;\sigma_{ci}^{90p}=134.17\,MPa \tag{21}$$

The next step in the simplified probability analysis is to run a Monte-Carlo simulation that consists in generating a large number of random values of $\sigma_{ci}$ drawn from the log-normal PDF represented in Figure 6, and computing and analyzing the corresponding values of the parameter $m_i$.

To illustrate the process, Figure 7a shows the input histogram for a total of 2,000 randomly generated values of $\sigma_{ci}$; the figure also includes the same log-normal PDF from Figure 6. For each of the 2,000 randomly generated values of $\sigma_{ci}$, a value of $m_i$ was computed using the proposed fitting scheme introduced in Section 2. The arithmetic mean and standard deviation of the computed values of $m_i$ resulted:

$$\overline{m}_i=13.92\;;s.d.m_i=2.29 \tag{22}$$

The histogram of values of $m_i$ from the Monte-Carlo simulation is shown in Figure 7b; the figure also shows the log-normal PDF constructed with Equations (17) through (19), using the resulting values of arithmetic mean and standard deviation in Equation (22).

Application of Equations (17) through (19) (this time with $\mu_x\,m_i$ and $\sigma_x\,s.d.\,m_i$ as in Equation 22) give the following 10, 50 and 90 percentiles, interpreted to be the values of $m_i$ with a probability of occurrence of 10%, 50% and 90%, respectively:

$$m_i^{10p}=16.94\;;m_i^{50p}=13.74\;;m_i^{90p}=11.14 \tag{23}$$

The Hoek-Brown failure curves corresponding to the respective pairs $\sigma_{ci}$ and $m_i$, for the per- centiles 10, 50 and 90, are presented in Figure 8. Note that because in this simplified probability analysis the parameter $m_i$ has been considered to depend on the parameter $\sigma_{ci}$ through the proposed regression analysis introduced in Section 2, the failure curves for the three different percentiles intersect each other somewhere in the middle of the interval of principal stresses, meaning that the high values of $\sigma_{ci}$ correspond low values of $m_i$, while to low values of $\sigma_{ci}$ correspond high values of $m_i$. Although at first sight this seems to be counter-intuitive (i.e., one would expect to see high values of $\sigma_{ci}$ being paired with high values of $m_i$ for high values of percentiles, and vice versa), it has to be remembered that in this analysis the resulting value of $m_i$ is directly controlled by the given value of $\sigma_{ci}$ on the assumption that the $m_i$ value will be the best fit to all available TX-test results.

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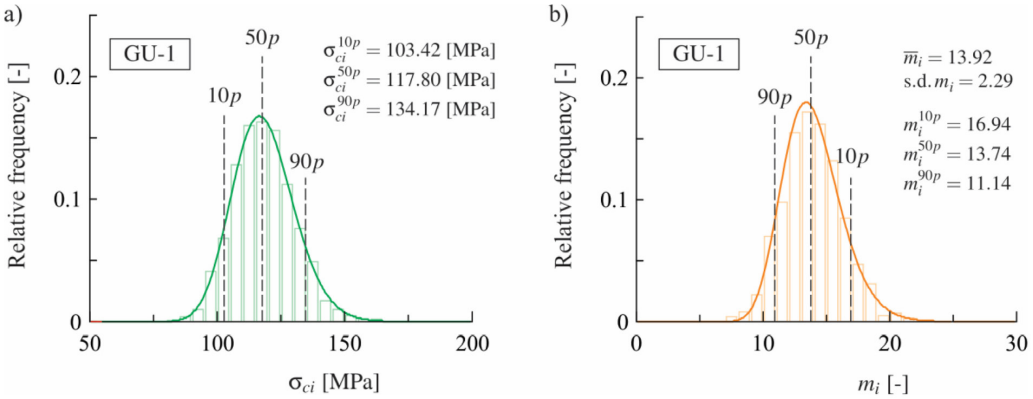



**Figure 7.** Histograms of a) uniaxial compressive strength ($\sigma_{ci}$), and b) resulting Hoek-Brown coefficient $m_i$, generated through a Monte-Carlo simulation of 2,000 randomly generated values.

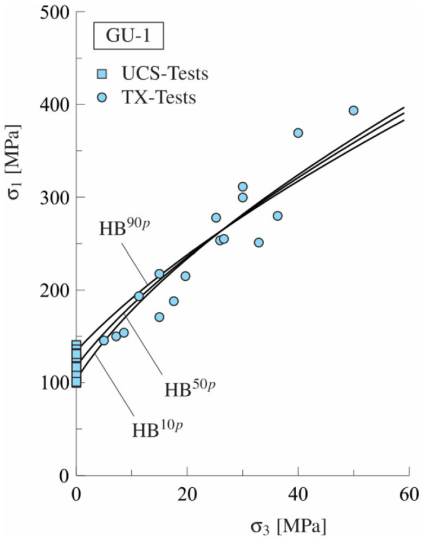



**Figure 8.** Best fit Hoek-Brown shear failure curves for the percentiles 10, 50 and 90 obtained from the Monte-Carlo simulation in Figure 7.

## 5. Final comments

The regression analysis presented in this paper can aid in the interpretation of UCS-test and TX-test results and in the computation of the parameters $\sigma_{ci}$ and $m_i$ in the Hoek-Brown failure criterion for intact rock. For cases in which the quality of UCS-test results is known to be lower than that of TX-test results, the computation of the parameters $\sigma_{ci}$ and $m_i$ should be done based on the recommendation by Hoek & Brown (2019), discussed in Section 1. In other instances, including cases in which the UCS-test results are known to be of similar quality as the TX-test results, a regression analysis that treats UCS-test results separately from TX-test results, as presented in Section 2, can aid in computing more representative values of the Hoek-Brown properties for intact rock.

The regression analysis presented in this paper can also aid in the interpretation of UCS-test and TX-test results, especially when geotechnical evaluation of intact rock strength involves many rock units. Application of the transformed version of the Hoek-Brown failure criterion allows to quantify the spread of all UCS-test and TX-test results about a unique reference target failure curve, through a single correlation coefficient, as shown in Section 3 of this paper.

## Declaration of interest

The authors have no conflicts of interest to declare. All co-authors have observed and affirmed the contents of the paper and there is no financial interest to report.

## Authors' contributions

Esteban Hormazabal: conceptualization, methodology, writing – original draft. Carlos Carranza-Torres: methodology, writing – review & editing.

## Data availability

The datasets generated analyzed in the course of the current study are available from the corresponding author upon reasonable request.

## Declaration of use of generative artificial intelligence

This work was prepared without the assistance of any generative artificial intelligence (GenAI) tools or services. All aspects of the manuscript were developed solely by the authors, who take full responsibility for the content of this publication.

## List of symbols and abbreviations

| | |
|---|---|
| $m_i$ | Hoek Brown intact rock constant |
| $r^2$ | Coefficient of determination |
| $x$ | Independent variable |
| PDF | Probability Density Function |
| $R^2$ | Global coefficient of determination |
| $S_1$ | Transformed major principal stress |
| $S_3$ | Transformed minor principal stress |
| TCS | Triaxial Compression test |
| TX | Triaxial Tests |
| UCS | Uniaxial Compression Tests |
| $\mu_x$ | Mean values of the test results |
| $\sigma_1$ | major principal stress |
| $\sigma_3$ | minor principal stress |
| $\sigma_{ci}$ | Intact rock uniaxial compressive strength |
| $\sigma_x$ | Mean values of the test results |
| $\Delta b$ | Interval chosen for constructing the histogram of the test results |

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