

Deterministic design optimization for soil nail walls in a Brazilian highway construction

Mélaine Anne-Fleur Bommertz¹ , Ana Luiza Mendes Halabi¹ ,

Adrian Torrico Siacara^{1#} 

Case Study

Keywords

Soil nail wall
Optimization
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Abstract

Soil nail walls are widely used for slope stabilization in transportation infrastructure; however, their design still relies predominantly on manual, trial-and-error procedures or simplified Limit Equilibrium Method (LEM) analyses, which do not ensure optimality. Existing optimization studies are typically limited to idealized problems, lack integration with industry-standard Finite Element Method (FEM) tools, and rarely incorporate realistic construction stages or current design codes. This study addresses these limitations by proposing a fully automated Deterministic Design Optimization (DDO) framework that integrates the Sequential Least Squares Programming (SLSQP) algorithm with the commercial FEM software Plaxis 2D via Python scripting. The choice of Plaxis 2D enables rigorous simulation of soil–structure interaction, staged construction, and advanced constitutive behavior, while the scripting interface allows seamless automation, iterative model updating, and efficient exploration of the design space. The framework is applied to a real Brazilian highway case study, incorporating the requirements of the national standard ABNT NBR 16920-2. Optimization variables include nail length, face inclination, and total construction cost. Results demonstrate robust convergence and highlight the critical role of cost-based objective functions in achieving economically efficient designs. Validation against LEM analyses performed in Slide2 shows good agreement in factors of safety, with differences observed in predicted failure mechanisms. The proposed approach provides a practical, code-compliant, and scalable framework for automated geotechnical design, with significant potential for cost reduction in large infrastructure projects.

1. Introduction

Soil nailing is widely used for stabilizing natural and cut slopes in highway and urban infrastructure. The technique consists of passively installing reinforcing elements (nails) into the soil mass to improve global stability (Cheung & Ho, 2021; Delmondes Filho et al., 2022; Sundaravel et al., 2023). Due to its versatility and effectiveness, the method is widely applied in cut slope construction, underground excavations, transportation infrastructure, slope stabilization, hill road widening, tunnel portals, landslide mitigation, and the rehabilitation of mechanically stabilized earth walls (Lazarte et al., 2015; Koerner, 2015; Sundaravel et al., 2023).

Design is traditionally performed through manual, trial-and-error adjustment of geometry and reinforcement until an admissible Factor of Safety (F) is achieved (Jin et al., 2025). This approach is time-consuming and does not ensure optimality.

Deterministic Design Optimization (DDO) enables automated and systematic exploration of the design space, aiming to identify cost-effective and mechanically efficient solutions (Arvin et al., 2022; Ouyang et al., 2026). Most previous studies on soil nail walls have focused on nail and facing design, pullout resistance (Ye et al., 2019), and stability analyses (Lin et al., 2017), while relatively few have investigated wall displacements (Liu et al., 2021).

Despite advances, current practice still relies on manual procedures or simplified Limit Equilibrium Method (LEM) analyses. Optimization studies are often restricted to idealized problems, lack integration with industry-standard Finite Element Method (FEM) tools, and rarely consider realistic construction stages (Villalobos & Villalobos, 2021; Zhou et al., 2009). The incorporation of modern design codes, such as ABNT NBR 16920-2 (ABNT, 2021), into automated frameworks remains limited, and validated real-case applications are scarce.

[#]Corresponding author. E-mail address: adrian.torrico@usp.br

¹Universidade de São Paulo, Escola Politécnica, Departamento de Engenharia de Estruturas e Geotécnica, São Paulo, SP, Brasil.

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In this study, a DDO framework is developed by coupling the FEM software Plaxis 2D with a Python-based optimization routine using the Sequential Least Squares Programming (SLSQP) algorithm. The procedure begins by defining design variables and an initial design. An automated script iteratively updates the variables and re-runs the FEM model, simulating staged excavation and nail installation. Outputs such as F and displacements are extracted to evaluate an objective function, typically based on construction cost. The SLSQP algorithm then updates the variables using gradient information to minimize the objective while satisfying all constraints. The framework is applied to a Brazilian highway case and validated against LEM, demonstrating its practicality and cost-efficiency. By demonstrating the measurable cost advantages of numerical optimization, this research contributes to the body of knowledge in two ways: it challenges the prevailing reliance on manual optimization and provides a replicable model for evidence-based design tool selection.

The structure of the paper is as follows. After introducing the theoretical background and problem definition in Section 2, Section 3 details the proposed DDO methodology. Section 4 describes the case study, while Section 5 reports the numerical results. These results are examined and validated in Section 6. Finally, Section 7 summarizes the main conclusions and contributions of the work.

2. Background and problem setting

2.1 Deterministic Design Optimization (DDO) of soil nail walls

Recent studies have investigated the influence of geometric parameters on soil nail wall performance using advanced numerical approaches. Finite element analyses have shown that variables such as nail length, inclination, and spacing significantly affect stability and deformation behavior (Bahramipour et al., 2025). Variations in these factors, alongside construction parameters like grouting pressure, influence the overall F and failure mechanisms of nailed slopes (Seo et al., 2014; Hong et al., 2017; Sharma & Ramkrishnan, 2020; Benayoun et al., 2021). While these studies provide valuable insights into the sensitivity of soil nail systems to individual design variables, they typically do not automate the search for the most economical combination that satisfies all safety constraints.

To move towards design automation, a variety of optimization algorithms can be employed (Yin & Jin, 2019; Hu, 2023). This study utilizes the Sequential Least Squares Programming (SLSQP) algorithm, a gradient-based optimization method. SLSQP operates by solving a sequence of subproblems, wherein a quadratic approximation of the objective function is optimized subject to a linearized version of the constraints. The algorithm employs a quasi-Newton approach to efficiently compute the Jacobian of the

objective function, guiding the search towards an optimal solution (Lindgård, 2020). At each iteration, SLSQP updates the design variables based on the estimated gradients and constraint sensitivities, generating a new candidate solution. This process continues until convergence criteria are satisfied, typically defined in terms of changes in the objective function, design variables, or constraint violations.

The efficacy of the SLSQP method is well documented, demonstrating high efficiency with fewer function evaluations and lower computational cost compared to many alternative optimization methods (Arora, 2017). This computational efficiency is particularly advantageous when the objective function evaluation is computationally expensive, as is the case when using finite element analysis. Consequently, the SLSQP algorithm is well-suited for integration with software like Plaxis 2D, offering the potential for faster convergence and a significant reduction in the overall computational effort required for design optimization (Yin & Jin, 2019; Rao, 2021).

2.2 Structural optimization formulation

Let $d = \{d_1, d_2, \dots, d_n\}$ represent the vector of design variables that define the geotechnical system to be optimized. Vector d contains n_{DV} design variables that represent all deterministic system parameters to be optimized. In this study, the geometry of the soil nail wall is optimized by adjusting the components and geometry of the geotechnical structure, represented by the design variable d . A typical formulation of DDO is as follows:

$$\text{Find } d^* \text{ which minimizes } C_{\text{construction}}(d) \quad (1)$$

subject to:

$$F(d) \geq F_T \quad d \in S \quad (2)$$

where: $F(d)$ is the factor of safety, F_T is the target F value, and $S = \{d_{\min}, d_{\max}\}$ are side constraints (Beck & Gomes, 2012; Siacara et al., 2022b; Zio & Li, 2022).

3. Methodology

The computational framework for DDO developed in this study integrates numerical modeling with an optimization algorithm. The numerical analyses were performed using the FEM software Plaxis 2D (Bentley Systems, 2024a, 2024b), which provides a programmable interface for external control. The optimization routine was implemented in Python using the PyCharm Integrated Development Environment (IDE). The SLSQP algorithm, accessed through the *minimize* function of the SciPy library (Virtanen et al., 2020), was employed to solve the optimization problem. This gradient-based algorithm is well-suited for the present application, as it can efficiently handle the objective function, constraints (e.g., F_T), and boundary conditions characteristic of geotechnical design optimization when coupled with Plaxis.

The methodology adopted in this study addresses the DDO for soil nail walls in Brazilian highway construction. It is divided into four stages, as illustrated in the flowchart of Figure 1.

The entire computational procedure was executed on a mid-range portable workstation (ASUS TUF705DT-AU197T). The system specifications included an AMD Ryzen 7 3750H processor, an NVIDIA GeForce GTX 1650 graphics card (4GB GDDR5 VRAM), and 8GB of DDR4 RAM.

4. Case study: Serra do Mar highway project

The primary application of the developed DDO framework is a case study involving a highway construction project located in the Serra do Mar region of Brazil. This infrastructure project employs various retaining systems, including soil nail walls, to accommodate slope excavations and backfills necessary for the roadway alignment. To support the geotechnical design, an extensive subsurface investigation campaign comprised numerous Standard Penetration Tests (SPT), mixed boreholes (percussion and rotary drilling) (SM), and Direct Shear (DS) tests on undisturbed samples were conducted.

For the present optimization study, a critical section of the CS04 soil nail wall (Section 13+10.00) was selected for detailed analysis. The geological profile of Section 13+10.00 is presented in Figure 2, which illustrates the subsurface stratigraphy interpreted from the adjacent SPT boreholes. The figure also depicts the configuration of the soil nail retention system, including the soil nails and the concrete facing, that forms the basis for the optimization study.

4.1 Soil material parameters

For the numerical analysis of the soil nail wall, the constitutive behavior of the geomaterials was represented using appropriate soil models. The residual soil layers—namely, the young residual soil (SRJ) and the mature residual soil (SRM)—were simulated using the Hardening Soil (HS) model. Barbosa et al. (2023) showed that advanced constitutive models, such as the HS model, provide more realistic predictions of soil nail wall behavior than the Mohr–Coulomb model, particularly in terms of displacements and stress distribution. Similarly, FEM studies with staged excavation indicate that the HS model can capture the progressive deformation and stress redistribution during construction when properly calibrated (Mohamed et al., 2023). Accordingly, the HS model was adopted for its ability to represent nonlinear, stress-dependent stiffness and loading–unloading behavior, which are essential for simulating staged excavation in soil nail walls.

In contrast, the remaining two soil units were modeled using the simpler Mohr–Coulomb (MC) failure criterion. The colluvium (COL) was represented with the MC model, as this material is completely removed during the initial excavation stage and therefore does not experience the subsequent stress paths that would necessitate a more complex model.

Similarly, the project fill material (AT), which serves as backfill for an access road, was also modeled using the MC criterion, given its secondary role in the overall system behavior and its location outside the primary zone of stress influence from the excavation-induced soil arching.

4.1.1 Unit weight and shear strength parameters of soils

The shear strength and physical properties of the geomaterials were characterized through a series of Direct Shear (DS) tests conducted on undisturbed samples. These laboratory investigations provided the basis for determining the effective cohesion (c'), effective friction angle (ϕ'), and unit weight (γ) parameters for each soil layer. Table 1 summarizes the geotechnical parameters adopted in the numerical simulations, including c' , ϕ' , saturated and unsaturated unit weights (γ_{sat} , γ_{unsat}), and dilatancy angle (ψ) for all soil materials considered in the analysis.

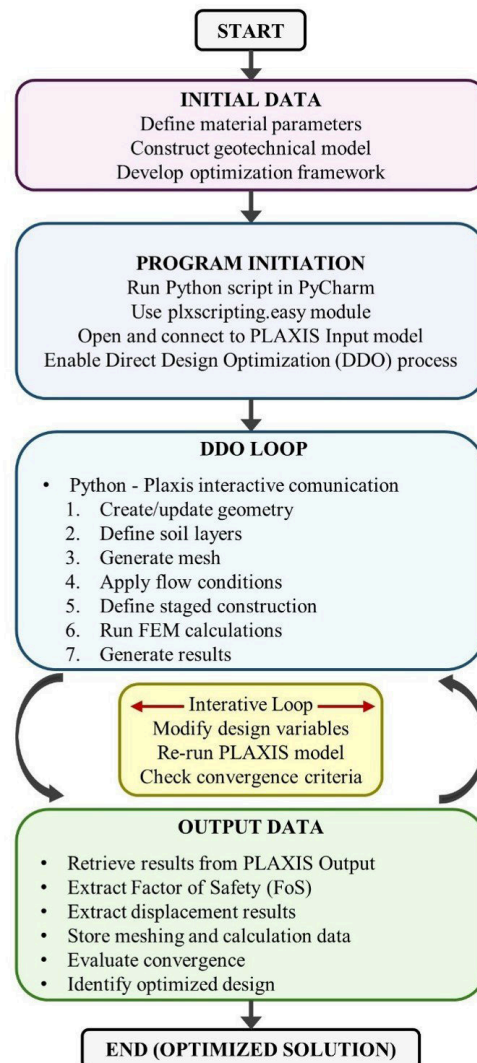


Figure 1. Flowchart of the DDO methodology.

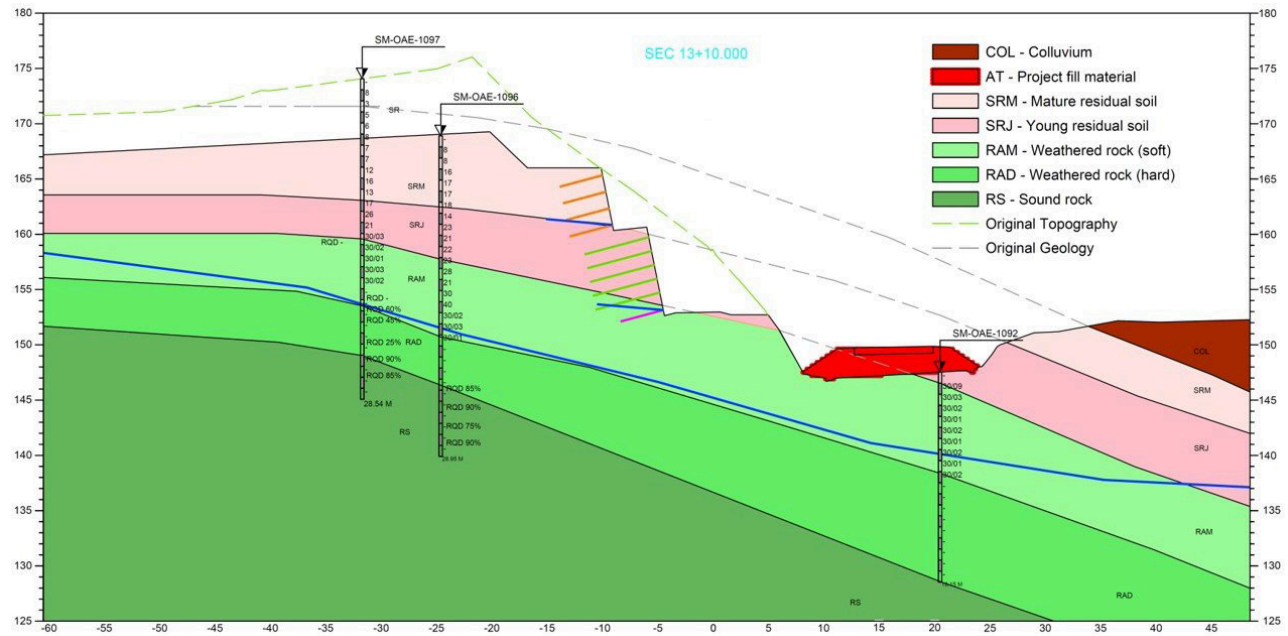


Figure 2. Geological profile of Section 13+10.00 with borehole locations and soil nail wall configuration.

Table 1. Summary of the simulated material's strength and stiffness parameters.

Soil - Shear Strength Parameters								
Color	Material	Higher N_{SPT}	γ_{unsat} (kN/ m ³)	γ_{sat} (kN/ m ³)	c' (kPa)	ϕ' (°)	ψ (°)	
	SRM	15	16.5	17.0	22.6	24.7	5	
	SRJ	30	17.4	17.9	26.0	28.5	5	
	COL	8	16.7	17.2	26.5	13.9	0	
	AT	8	18.0	18.5	10.0	27.0	0	
Soil - Stiffness Parameters								
Color	Material	Plaxis Soil Model	E_{50} (MPa)	E_{oed} (MPa)	E_{ur} (MPa)	ν	ν_{ur}	
	SRM	Hardening Soil	33.75	33.75	84.38	-	0.20	
	SRJ	Hardening Soil	67.50	67.50	168.75	-	0.20	
	COL	Mohr-Coulomb	11.20	-	-	0.30	-	
	AT	Mohr-Coulomb	26.40	-	-	0.30	-	
Hoek-Brown parameters of shear resistance of the rock layers								
Color	Material	γ_{unsat} (kN/m ³)	γ_{sat} (kN/m ³)	E_{rm} (GPa)	D	ν	σ_{ci} (MPa)	m_i
	RAM	22.0	22.5	0.64	0	0.27	5.0	28.0
	RAD	24.0	24.5	2.62	0.7	0.27	30.0	32.0
	RS	26.0	26.5	6.02	0.7	0.27	80.0	32.0
Nails - Strength and Stiffness Parameters								
Color	Material	E_{eq} (MPa)	γ_{eq} (kN/ m ³)	d_{DH} (m)	s_h (m)	T_{max} (kN/m)	M_p (kNm/m)	P_p (kN/m)
	Nail type 1	30.5	8.5	0.075	1.75	19.0	5.0	75.0
	Nail type 2	29.5	7.6	0.100	1.25	37.0	5.0	121.0
	Nail type 3	30.5	3.0	0.075	2.00	38.0	5.0	75.0
Shotcrete walls – Strength and Stiffness Parameters								
Color	Material	h (m)	EA (kN/m)	EI (kN/m ² /m)	d_{eq} (m)	w (kN/m/m)	ν	
	Shotcrete 1	5.7	1.85 x 10 ⁷	2.22 x 10 ⁴	0.12	2.01	0.20	
	Shotcrete 2	8.0	2.59 x 10 ⁷	3.11 x 10 ⁴	0.12	1.96	0.20	

4.1.2 Stiffness parameters of soils

Empirical correlations based on SPT results, such as those proposed by Teixeira and Godoy (Teixeira & Godoy, 1996), are widely used in Brazil for estimating stiffness parameters. Their use in the HS model is common in practical geotechnical applications, often combined with literature values and numerical back-analysis when laboratory data are unavailable. Although such approaches introduce uncertainty, they remain a practical and consistent solution for preliminary analyses and engineering design.

In this study, the Young's modulus (E) and Poisson's ratio (ν) were estimated using empirical correlations established for Brazilian soils through the following expression:

$$E = \alpha K N_{SPT} \quad (3)$$

where N_{SPT} is the SPT blow count, and α and K are empirical constants selected according to soil type. The constants α and K were assigned based on the granulometric characteristics of each soil layer. For the residual soils (SRM and SRJ), classified as sandy silt, values of $\alpha = 5$ and $K = 0.45$ MPa were adopted. For the COL soil and AT material, classified as silty clay, values of $\alpha = 7$ and $K = 0.20$ MPa were employed. Poisson's ratio of $\nu = 0.30$ was adopted for all materials in the MC analyses.

For the HS model employed for the SRM and SRJ layers, additional stiffness parameters were required following the recommendations of the Plaxis Reference Manual (Bentley Systems, 2024a). The reference secant modulus (E_{50}) was calculated using the empirical correlation of Teixeira and Godoy (Teixeira & Godoy, 1996) (Equation 3), with the highest representative N_{SPT} values from the investigation to obtain realistic, yet conservative, stiffness estimates. Consistent with the Plaxis guidelines, the oedometer modulus (E_{oed}) was taken as equal to E_{50} . The unloading-reloading modulus (E_{ur}) was estimated as 2.5 times E_{50} , reflecting the stiffer response of the soil upon unloading.

Poisson's ratio for unloading-reloading (ν_{ur}) in the HS model was set to 0.2. This value, lower than the standard Poisson's ratio of 0.3 adopted for primary loading, is recommended to represent the reduced lateral strain and stiffer behavior characteristic of soil during unloading conditions. All stiffness parameters adopted in the numerical simulations are summarized in Table 1.

4.2 Rock material parameters

The rock materials encountered in the Serra-type geology were classified into three categories based on weathering degree: weathered rock (soft) (RAM), weathered rock (hard) (RAD), and sound rock (RS). All three rock types were simulated using the Generalized Hoek-Brown failure criterion, which is well-established for modeling the behavior of jointed rock masses.

The stiffness parameters, particularly E and ν , are notoriously difficult to determine directly for rock materials. In the absence of site-specific laboratory or in-situ test data, representative values were adopted based on published literature for analogous lithologies. Given the geological similarities between the gneiss documented in the project and granitic rocks, reference values from Bowles (1997) were employed. Typical modulus values for granite range from 14 to 83 GPa, with ν between 0.26 and 0.30. Conservative estimates were adopted for the numerical simulations: E was set to 14 GPa for the RAM layer and 16 GPa for both RAD and RS layers, while $\nu = 0.27$ was uniformly applied across all rock categories. The unconfined compressive strength (σ_{ci}) is 5 MPa for RAM, 30 MPa for RAD, and 80 MPa for RS materials. The Hoek-Brown Constant (m_i) is 28 for RAM and 32 for RAD and RS.

To ensure numerical stability in the finite element analyses, a disturbance factor of $D = 0.7$ was introduced for both RAD and RS materials. This adjustment was necessary to prevent numerical issues arising from excessive contrasts in rock mass deformation modulus (E_{rm}) between adjacent materials—specifically, the approximately three-order-of-magnitude difference between the intact rock modulus (≈ 15 GPa) and the softest soil layers (≈ 12 MPa). It should be noted that this stiffness reduction does not compromise the validity of the results, as the affected rock layers are located well below the zone of influence of the soil nail retention system and still retain a representative stiffness on the order of 1 GPa. Table 1 presents both the undisturbed and disturbed modulus values, with the values employed in the simulations clearly indicated for each rock layer.

4.3 Project overloads

Vertical surcharge loads (σ_v) were applied to the numerical models to represent traffic and construction loading in accordance with the Brazilian standard NBR 7188:2013 (ABNT, 2013). The following distributed loads were considered:

- Highway traffic load at the top of the slope: $\sigma_v = -25$ kPa/m.
- Service road load at the mid-slope: $\sigma_v = -10$ kPa/m.
- Service road load at the slope base on new fill material: $\sigma_v = -10$ kPa/m.

4.4 Soil nails' structural parameters

The soil nails were modeled using the embedded beam element available in Plaxis, which provides a robust formulation for simulating the load-transfer behavior of piles and ground anchors (Bentley Systems, 2024a). This approach captures the interaction between the reinforcement elements and the surrounding soil mass. All relevant input parameters for the grouted soil nails, modeled as embedded beam structures, are summarized in Table 1, where the Equivalent Stiffness Parameter (E_{eq}), equivalent unit weight (γ_{eq}), Drill Hole Diameter (d_{DH}), horizontal spacing (S_h), Maximum Nail Tensile Capacity (T_{max}), Plastic Moment Capacity (M_p), and Plastic Axial Capacity (P_p) are presented.

4.5 Shotcrete wall's structural parameters

The soil nail wall incorporates two distinct facing sections with heights (h) of 5.7 m and 8.0 m, respectively. In accordance with the construction project specifications, both facings have a design thickness (d_{eq}) of 12.0 cm and are constructed of C30 reinforced concrete. The E for this concrete class is approximately 27.0 GPa.

The facing was simulated in the numerical model using plate elements, which are appropriate for capturing the flexural behavior of slender structural components. All relevant parameters employed for the plate elements representing both facing sections are summarized in Table 1, where Distributed Load (w), Axial Stiffness per Unit Length (EA), and Bending Stiffness per Unit Length (EI) are presented.

5. Numerical analysis of the Brazilian highway project

5.1 Geometry of the soil nail wall design

The geometric configuration adopted for the slope at Section 13+10.00 follows the specifications established in the Geotechnical Highway Project. The design parameters for the shotcrete facings and soil nail reinforcements are illustrated in Figure 3, which presents the complete reinforcement layout for the double-wall system.

5.2 Mesh generation and staged construction simulation

The numerical model in Plaxis was designed to simulate all construction phases sequentially, beginning from the initial in-situ stress state and progressing through each excavation

stage until the final configuration was achieved. The complete construction sequence is depicted schematically in Figure 3.

A graduated mesh refinement strategy was employed to optimize computational efficiency while maintaining accuracy in zones of interest (Nazem & Moavenian, 2021; Siacara et al., 2022a). The coarseness factor was set to 2.0 for the RS rock layer at the model base, 1.25 for the RAD rock layer in contact with the RAM, and 0.5 for the RAM, SRJ, and SRM materials within the zone of influence of the retention system. A medium global mesh size was adopted as a compromise between solution accuracy and computational time constraints.

5.3 Results of the soil nail wall analysis

The baseline analysis was performed for the Section 13+10.00 configuration as originally designed in the Geotechnical Highway Project, which is scheduled for construction in the coming years. The geometrical characteristics of the soil nailing system are summarized in Table 1.

The LEM analysis conducted with Slide2 software yielded a global F of 1.56, with the critical slip surface presented in Figure 4a. The FEM simulation performed in Plaxis produced an identical global F of 1.56 for the final construction phase (Phase 13). The critical rupture mechanism obtained from the FEM analysis, visualized through the color-coded total displacement contours, is illustrated in Figure 4b. For comparative purposes, the critical slip surface from the LEM analysis is superimposed as a green line on the FEM results.

The results from both analytical approaches demonstrate strong agreement, yielding identical global F values of 1.56. Although the calculated rupture surfaces differed slightly between the two methods, both indicated a similar failure mechanism.

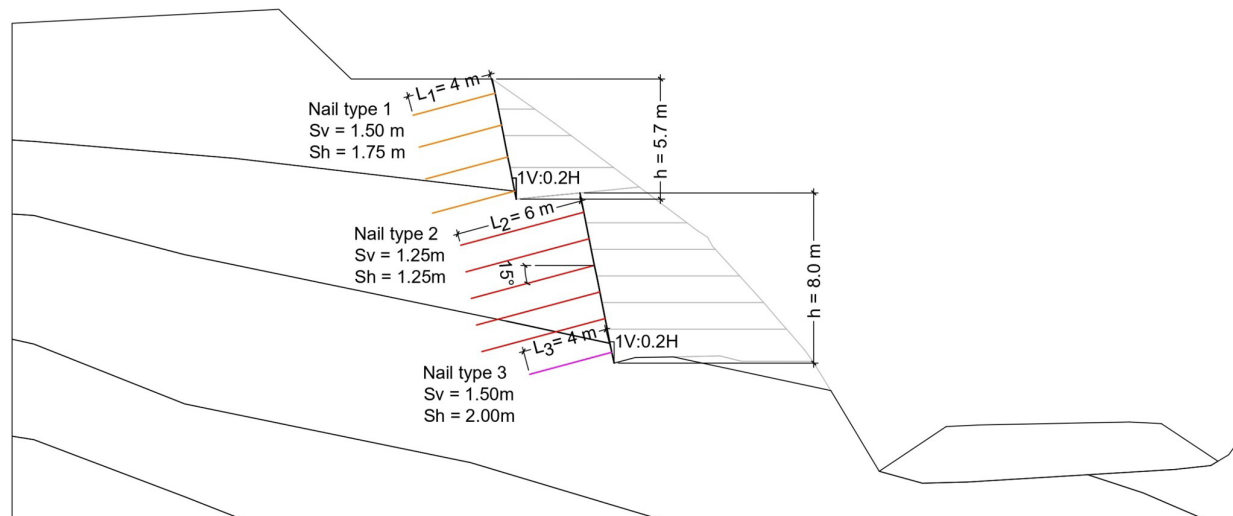


Figure 3. Geometry of the soil nail reinforcement layout for Section 13+10.00.

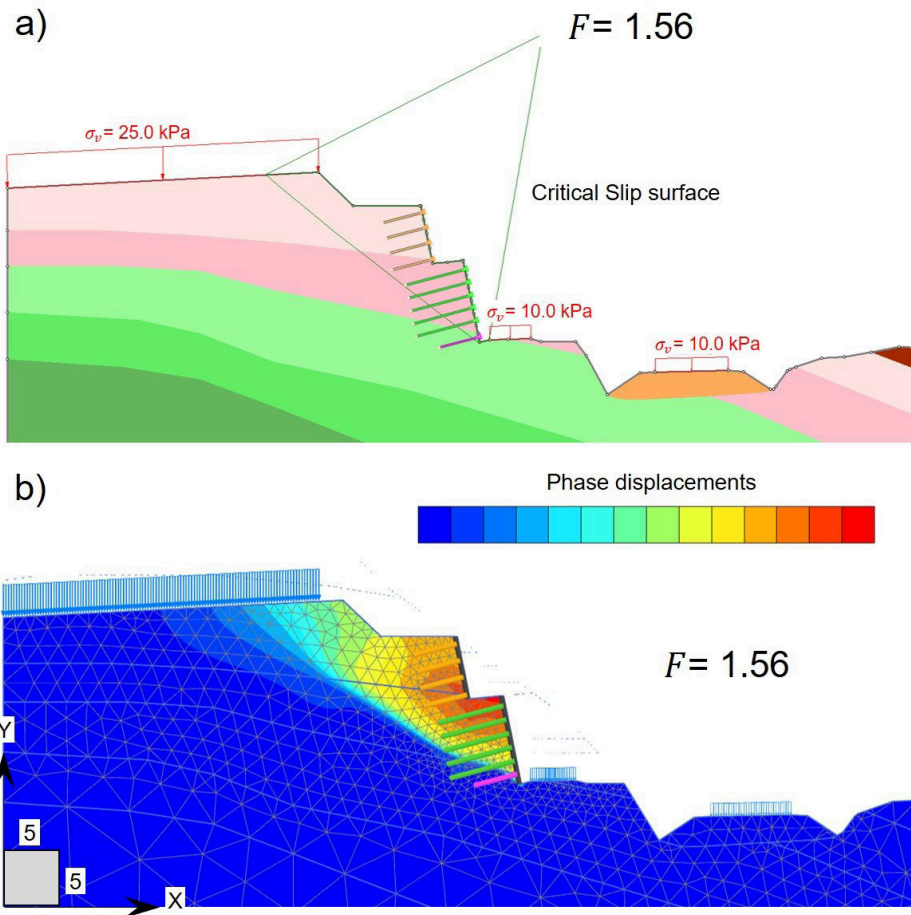


Figure 4. Comparison of (a) LEM analysis results from Slide2 and (b) FEM analysis results from Plaxis showing total displacements and critical rupture surface for the original Section 13+10.00 configuration.

The global rupture surface tends to either intersect or induce pullout failure of the third to fifth nails (counting from the top) of the second retention wall, ultimately terminating at the toe of that same wall. Similar findings were reported by Villalobos and Villalobos for different geometries and in comparisons between FEM and LEM (Villalobos & Villalobos, 2021). However, caution should be exercised when extrapolating these results to other configurations, as the F may vary significantly depending on the geometry, soil conditions, and reinforcement layout.

Because the critical rupture surface intercepts the Type 2 and Type 3 nails, these elements are identified as the most highly mobilized in resisting the applied loads. Consequently, these nail types represent the primary variables for optimization, as their length and spacing exert the greatest influence on the global F .

6. Deterministic design optimization of the soil nail wall

The design of a soil nail wall involves numerous interdependent parameters that must be specified by the geotechnical engineer, including:

- Vertical inclination of the retaining wall face;
- Number, length, and horizontal inclination of grouted nails;
- Horizontal and vertical nail spacing (nail mesh);
- Borehole diameter and steel bar reinforcement characteristics;
- Shotcrete facing thickness and structural reinforcement (if applicable).

For the present study, two primary geometric parameters were selected for optimization using the Python-Plaxis automated framework: the inclination between the vertical and the shotcrete wall (β_i) and the nail lengths (L_i). This selection was motivated by the significant influence of these parameters on both the global F and the overall construction cost of soil nailing systems.

Although the variables were treated as continuous for numerical simplification, construction constraints render them inherently discrete, with a minimum practical increment. Consequently, prior to field implementation, the optimization should be refined to account for this execution constraint, thereby improving both cost-effectiveness and constructability.

6.1 Optimization of nail lengths

An initial optimization case study was conducted to determine the optimal nail lengths for Section 13+10.00 as originally designed in the Geotechnical Highway Project. The baseline geometry of the soil nail retention system corresponds to that described in Section 5, with the three nail types characterized by distinct lengths. Let the vector $d = \{L_1, L_2, L_3\}$ represent the nail length design variables. The initial FEM simulation for the project configuration employed nail lengths of $L_1 = 4.0$ m (Type 1), $L_2 = 6.0$ m (Type 2), and $L_3 = 4.0$ m (Type 3), as illustrated in Figure 5. This configuration yielded a global F of 1.56, suggesting that the optimal nail lengths should be near these baseline values, with potential for reduction.

The proper formulation of the design optimization problem is critical to the solution process, typically accounting for approximately 50% of the total effort required (Arora, 2017). The nail length optimization problem is formally defined as: find the optimal vector d^* that minimizes the total nail length $f(L_1 + L_2 + L_3)$ subject to $F(d) \leq F_{max}$ and $F(d) \geq F_{min}$ where $F_{max} = 1.51$ and $F_{min} = 1.50$, with the constraint that $d \in S$ (the feasible design space). The boundary conditions for each design variable are summarized in Table 2, with initial values specified for the optimization. The allowable length ranges

for the three nail types were established following the recommendations of the French standard NF P94-270 (NF, 2020).

The lower bound $F_{min} = 1.50$ corresponds to the minimum acceptable global F for soil nail walls in Brazil (ABNT, 2021), and the upper bound $F_{max} = 1.51$ serves as a convergence tolerance, directing the optimization algorithm to approach the target value of 1.50 with a deviation not exceeding 1%. Both inequality constraints are necessary because an equality constraint requiring $F(d) = 1.50$ proved unworkable in this optimization context; the SLSQP algorithm cannot achieve an exact F value of 1.50 through Plaxis calculations, which would prevent convergence.

The optimization was implemented in Python using the SLSQP minimization algorithm from the SciPy optimization package (scipy.optimize). Initial values for all three nail length variables were set to 6.0 m, establishing a starting configuration with a F higher than the target optimum of 1.50 while remaining reasonably close to the expected solution. This initial value selection proved crucial to optimization success, as the SLSQP algorithm was observed to fail to converge when initial values were more distant from the optimum.

To execute the nail length optimization, a Python function $F(L_1, L_2, L_3)$ was developed to calculate the global F in Plaxis as a function of the three nail length variables.

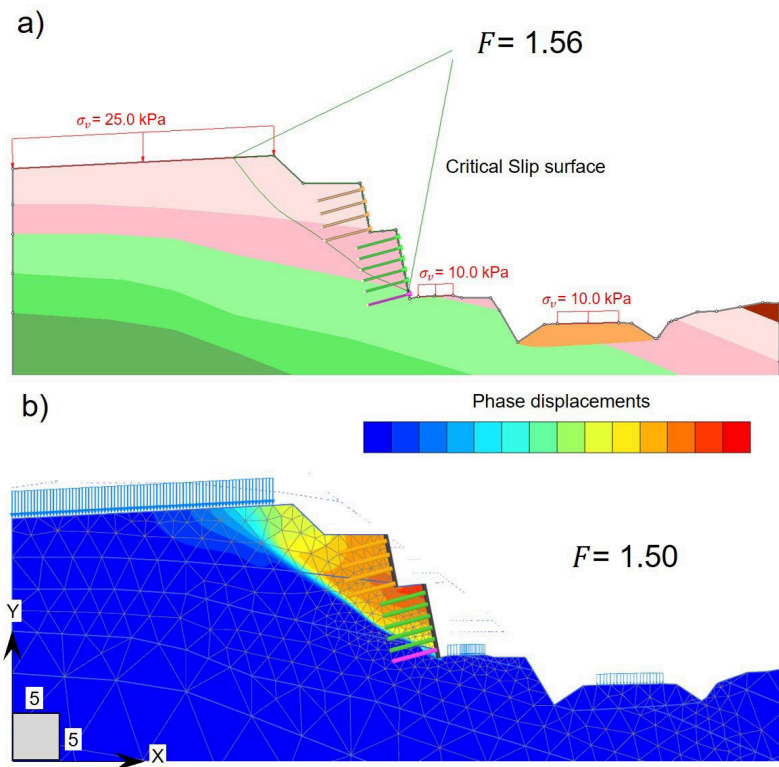


Figure 5. Comparison of critical rupture surfaces for optimized nail lengths: (a) LEM results from Slide2, and (b) FEM results from Plaxis showing total displacements.

Table 2. Summary of design variable bounds and optimization results for the Deterministic Design Optimization.

	Variable	Unit	Lower Bound	Upper Bound	Comments	Highway Design	Initial Design	Optimized Design
Nail DDO	L_1	m	4.0	7.0	$0.7h \leq x \leq 1.2h \quad L_2 \geq L_3$	4.0	6.0	5.4
	L_2	m	4.5	10.0		6.0	6.0	5.0
	L_3	m	4.0	10.0		4.0	6.0	5.0
	β_1	°	-	-	-	5	5	5
	β_2	°	-	-		5	5	5
	Total nail length	m	-	-		14	18	15.4 (14.5% reduction)
	Variable	Unit	Lower Bound	Upper Bound	Comment	Highway Design	Initial Design	Optimized Design
Slope DDO	L_1	m	-	-	Higher nail length to allow steeper facings	4.0	7.5	7.5
	L_2	m	-	-		6.0	7.0	7.0
	L_3	m	-	-		4.0	7.0	7.0
	β_1	°	0	30	Perfectly vertical face Geometrical limit	5	5	11
	β_2	°	0	30		5	5	0
	Excavation volume	m ²	-	-		4.22	4.22	3.16 (25.1% reduction)
					$\frac{5.7^2 \tan \beta_1}{2} + \frac{8.0^2 \tan \beta_2}{2}$			
	Variable	Unit	Lower Bound	Upper Bound	Comment	Highway Design	Initial Design	Optimized Design
Costs DDO	L_1	m	6.0	7.0	Allow steeper facings	4.0	8.0	6.0
	L_2	m	7.0	10.0	Highest density	6.0	8.0	7.0
	L_3	m	4.0	10.0	Secondary contribution	4.0	8.0	4.0
	β_1	°	0	30	Perfectly vertical face	5	5	2.5
	β_2	°	0	30	Geometrical limit	5	5	0
	Final cost	BRL	-	-	$f(d^*)$	-	-	BRL 784.00 (USD 156.80)

To minimize computational time for each optimization iteration, this function was designed to open a partially prepared Plaxis file containing the complete project geometry and all material properties, with the exception of the soil nails. The Python routine then generates and adds soil nails with the specified lengths, executes the model calculations, and retrieves the final F .

The optimization results indicate that the nail length configuration can be improved by reducing the second nail length while modestly increasing the first nail length. The optimized geometry yields nail lengths of $L_1 = 5.4$ m, $L_2 = 5.0$ m, and $L_3 = 5.0$ m. Notably, Type 2 nails are the most numerous in Section 13+10.00 (five nails) and feature the tightest spacing pattern, with both horizontal and vertical spacing of $s_v = s_h = 1.25$ m. Reducing their length was only possible by increasing the length of the Type 1 nails, effectively displacing potential rupture surfaces away from the retention walls. Since only four Type 1 nails are present with a wider spacing ($s_v = 1.50$ m, $s_h = 1.75$ m), this configuration achieves a reduction in total nail length while maintaining the required F of exactly 1.50. All results are presented in Table 2.

The critical rupture surface for the optimized configuration is illustrated in Figure 5a. The failure mechanism passes behind the nails of the upper retention wall and intersects

the three lowest nails of the lower retention wall. Maximum displacements occur at the toe of the lower facing, suggesting a potential foundation soil failure mechanism. The optimized nail length geometry was subsequently analyzed using Slide2 for limit equilibrium verification, with the resulting rupture surface presented in Figure 5b.

The Slide2 analysis yielded a slightly higher F of 1.56 compared to the FEM result of 1.50. Such differences between analytical methods are expected, and the results are considered satisfactory for validation of the nail length optimization procedure.

Several observations were made regarding difficulties encountered during the nail length optimization:

- Computational demand: The optimization required approximately 5 hours of computation time. This duration is highly dependent on mesh refinement, boundary conditions, and initial value selection, and convergence is not guaranteed. More robust optimization analyses would benefit from enhanced computational resources.
- Solution variability: Achieving a precise target F (e.g., exactly 1.50) is challenging due to inherent variability in Plaxis results. Successive calculations with identical input parameters may yield F variations of up to 0.05 due to mesh-dependent differences.

- Algorithm sensitivity: The choice of nonlinear optimization algorithm is critical to convergence behavior. Future work should consider testing multiple optimization algorithms to compare their methodologies, applicable domains, and solution quality.
- Facing displacements: The maximum horizontal displacements computed for both facings were approximately 2.0 mm, which falls below the recommended limiting values from the Clouterre project (Clouterre, 1991) and can therefore be considered acceptable for serviceability.

6.2 Optimization of facing inclination

The second DDO performed for Section 13+10.00 aimed to optimize the vertical inclinations (slopes) of the two soil nail facings. This optimization is a critical component of retention design, as it directly influences the volume of soil excavation required for the project. The objective was to identify the optimal facing inclinations that minimize environmental impact, respect geometric constraints, and achieve the required F_r despite steeper facing angles.

The slope DDO was applied to the same case study as nail length optimization. A new Plaxis file was created incorporating all soil and structural characteristics described in Section 5, with the exception that the SRM and SRJ soil layers—within which the two soil nail retention systems

are constructed—were left undefined. This base file was subsequently accessed by the Python code, which dynamically added the geometry of the soil nail facings, nails, and excavation polygons using specialized geometric functions.

To enable the optimization model to achieve F values of 1.50 for steeper facings (inclinations less than 11° from vertical), the nail lengths L_1 , L_2 and L_3 were increased by 2.0 m relative to the results of the previous nail length DDO. This additional length enhances the mobilized soil-nail friction within the passive zone, thereby stabilizing the near-vertical facings. The adopted nail length values and the optimization boundary conditions for facing inclinations are summarized in Table 2. The slope optimization problem is formally defined as: find the optimal vector d^* that minimizes the excavation volume $f\left(\frac{\tan(\beta_1) * 5.7^2}{2} + \frac{\tan(\beta_2) * 8.0^2}{2}\right)$ subject to $1.50 \leq F(d) \leq 1.51$.

The results of the slope DDO and the corresponding critical rupture surface are presented in Figure 6b. Maximum normal displacements for the facings were 2.4 mm, which remains below the acceptable limits recommended by the Clouterre research project. The slope optimization required approximately 4 hours of computation time on the laptop used for this study.

The optimization results demonstrate that increased nail lengths are necessary to accommodate steeper facing inclinations while maintaining a safety margin of $F = 1.50$.

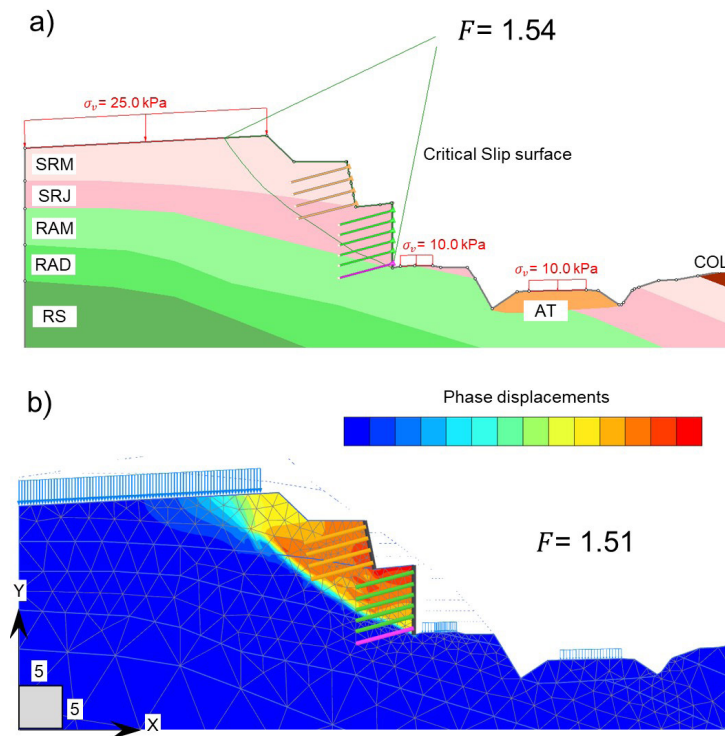


Figure 6. Optimized facing inclination results: (a) LEM analysis from Slide2, and (b) FEM analysis from Plaxis showing total displacements and critical rupture surface.

The increase in nail length was minimized for Type 2 nails, which feature the tightest spacing pattern. The slope DDO revealed that the optimal configuration for minimizing soil excavation is to maintain the upper retention wall at an inclination of approximately 11° while making the lower facing perfectly vertical (0° inclination). This geometry represents a combined optimization of nail lengths and facing inclinations that achieves the desired safety margin while producing a failure surface similar to that of the original Highway Project configuration. This motivated the final DDO, which simultaneously optimizes both nail lengths and facing inclinations.

The lower facing was optimized as perfectly vertical, thereby reducing excavation volume and maximizing the berm width. The upper retention wall, however, was maintained at a steeper inclination that allows nails to penetrate more deeply into the soil mass, effectively displacing potential rupture surfaces away from the facing and upper nail rows, which enhances overall system stability by engaging a larger volume of soil in the passive resistance zone. The optimized geometry from the slope DDO was subsequently analyzed using Slide2 for limit equilibrium verification. The resulting rupture surface is illustrated in Figure 6a. The Factors of Safety obtained from both analytical methods show close agreement, with values of 1.54 (LEM) and 1.51 (FEM), validating the slope optimization results.

6.3 Combined cost optimization

The final DDO aims to minimize construction costs by simultaneously optimizing both nail lengths and facing inclinations. This comprehensive approach enables the identification of the optimal combination of critical geometric parameters in the soil nail wall retention design.

Let $d = \{\beta_1, \beta_2, L_1, L_2, L_3\}$ represent the five design variables: the three nail lengths for each nail type and the two vertical facing inclinations. The optimization problem is formally defined as: find the optimal vector d^* that minimizes the total cost function $f(d^*)$ subject to $1.50 \leq F(d) \leq 1.51$, with constraints and initial values presented in Table 2.

The total cost function, expressed in Brazilian reais (BRL) per meter of out-of-plane width, incorporates three primary components—excavation, grouted nail installation, and facing construction—and is presented in Equation 4.

$$f = \left(\begin{array}{l} \text{Cost}_{\text{excavation+nails+facing}} = 100 * \\ \left(1 * \frac{\tan(\beta_1) * 5.7^2}{2} + 1 * \frac{\tan(\beta_2) * 8.0^2}{2} \right) + \\ \left(\frac{1000}{12} * L_1 * 4 + \frac{1000}{12} * L_2 * 5 + \frac{1000}{12} * L_3 \right) + \\ \left(1500 * 1 * \left(\frac{5.7}{\cos(\beta_1)} + \frac{8.0}{\cos(\beta_2)} \right) \right) \end{array} \right) \quad (4)$$

The unit costs employed in the optimization were approximated based on experience with similar structures in the São Paulo region of Brazil, with equivalent values in US dollars (USD) based on the December 2023 exchange rate:

- Excavation cost: BRL 100.00 (~ USD 20.00) for 1.0 m³.
- Grouted nail cost: BRL 1,000.00 (~ USD 200.00) for 12.0 m.
- Facing cost: BRL 1,500.00 (~ USD 300.00) for 1.0 m².

The boundary conditions for facing inclinations were established at 0° (vertical) and 30° (inclined). Initial nail length boundaries followed those defined in previous optimizations. However, the large number of design variables combined with wide boundary ranges resulted in excessive computation time (approximately 10 hours) and convergence difficulties. To address this, minimum nail length constraints were refined to values close to the expected optimum, facilitating algorithm convergence. The minimum nail lengths required to achieve a F of 1.40 or greater and enable convergence were established as $L_{1,\min} = 6$ m, $L_{2,\min} = 7$ m, and $L_{3,\min} = 4$ m. Initial values for the optimization algorithm were set to $\beta_1 = \beta_2 = 5^\circ$ for facing inclinations and $L_1 = L_2 = L_3 = 8.0$ m.

The critical failure surface for the combined cost optimization is presented in Figure 7b. Maximum normal displacements were 3.0 mm at the facing crest, which remains within acceptable serviceability limits. The optimized geometry was subsequently validated using Slide2 limit equilibrium analysis, with the resulting rupture surface illustrated in Figure 7a. Both analytical methods yielded closely matching results, with a F of 1.51 and similar failure surface geometries, confirming the reliability of the optimization procedure.

The combined optimization resulted in a nearly vertical upper facing ($\beta_1 = 2.5^\circ$) and a perfectly vertical lower facing ($\beta_2 = 0^\circ$). Optimized nail lengths converged to the prescribed minimum values ($L_{1,\min} = 6$ m, $L_{2,\min} = 7$ m, and $L_{3,\min} = 4$ m). While this outcome limits the conclusiveness of the nail length optimization, it suggests that grouted nail costs are the primary driver of the minimization. The adoption of these refined minimum bounds was necessary to achieve successful convergence on the available computational hardware. The optimization required approximately 18 hours of computation time on the laptop used for this study, highlighting the computational demands of high-dimensional geotechnical optimization problems. All results are presented in Table 2.

The total cost of the combined optimized design is substantially lower than that of the initial Highway Project configuration when considering excavation, nail installation, and facing construction costs for a 1.0 m out-of-plane width of Section 13+10.00. The cost reduction achieved is BRL 784.00 (approximately USD 156.80) per linear meter of wall. For the full CS04 retention system, which extends approximately 120.0 m in length, this translates to estimated savings of BRL 94,000.00 (approximately USD 18,800.00). These results demonstrate that the simultaneous optimization of facing inclinations and nail lengths represents a powerful approach for minimizing total retention costs in practical geotechnical applications.

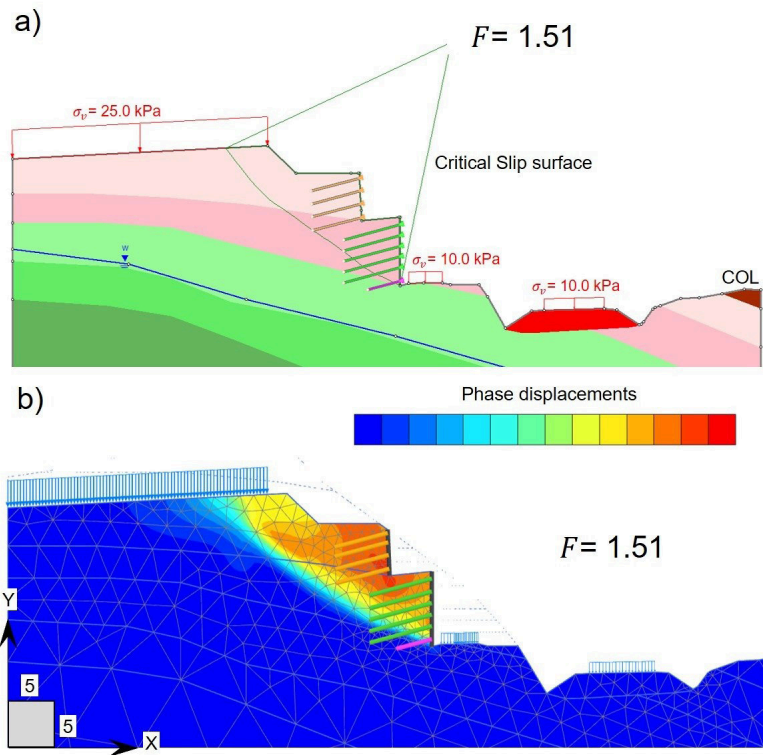


Figure 7. Combined optimization of facing inclination and nail lengths: (a) LEM analysis from Slide2, and (b) FEM analysis from Plaxis showing total displacements and critical rupture surface.

The use of total construction cost as the objective function amplified the importance of facing inclinations in the minimization process compared to the previous slope-only DDO. Under the adopted cost estimates, facing construction at BRL 1,500.00/m² accounts for approximately four-fifths of the total cost, with this component depending exclusively on the inclinations of both facings. Consequently, the optimization algorithm converges toward a configuration with near-vertical facings, with nail lengths adjusted to maintain the required F of 1.50. This finding underscores the critical importance of careful objective function selection in DDO for soil nail retention systems, as the weighting of geometric versus financial parameters fundamentally influences the optimal design solution. Engineers must therefore exercise judgment in formulating optimization problems that appropriately reflect project-specific priorities and constraints.

Future improvements in computational efficiency could be achieved through the use of surrogate models enhanced by active learning strategies (Siacara et al., 2024b); however, the development and implementation of such approaches are beyond the scope of the present study. Additional research directions include three-dimensional optimization under uncertainty (Shirgir et al., 2023), the explicit treatment of soil spatial variability through random field modeling (Siacara et al., 2024a), the incorporation of seismic loading conditions into optimization frameworks (Mollaie et al., 2022), calibration

and reliability-based design methodologies aligned with engineering codes and standards (Lin & Bathurst, 2022), and the assessment of climatic and environmental effects on the long-term performance of soil-nailed systems (Siacara et al., 2026). Collectively, these developments could enhance both the realism and practical applicability of optimization-based approaches for the design of soil-nailed retaining structures.

7. Conclusions

This paper has presented a fully automated Deterministic Design Optimization (DDO) framework that integrates a Python-based Sequential Least Squares Programming (SLSQP) optimization algorithm with finite element modeling in Plaxis 2D. By coupling the optimization routine with numerical analysis, the proposed methodology transforms conventional trial-and-error workflows into a systematic, reproducible, and computationally efficient design strategy for geotechnical structures.

The developed framework orchestrates the entire modeling sequence—including geometry generation, material assignment, mesh generation, hydraulic condition specification, staged construction simulation, numerical analysis, and results extraction—within a closed and convergence-controlled optimization loop. This seamless integration enables iterative modification of design variables,

automated execution of finite element simulations, and objective evaluation based on critical performance indicators such as Factor of Safety (F), material utilization, and construction costs. Consequently, design exploration becomes both rigorous and transparent, with decision-making grounded in quantifiable performance metrics.

The application of the combined slope and nail length DDO to a real-world case study—Section 13+10.00 of a Brazilian highway project in the Serra do Mar region—demonstrated significant economic benefits. The optimized configuration achieved cost savings of BRL 784.00 (approximately USD 156.80) per linear meter of out-of-plane wall width. For the full-scale CS04 retention system, extending approximately 120.0 m in length, this translates to estimated savings of BRL 94,000.00 (approximately USD 18,800.00). Extrapolated to large-scale highway infrastructure projects, the implementation of this methodology has the potential to reduce construction costs by millions of reais, substantiating the substantial economic value of automated optimization in geotechnical engineering practice.

Beyond economic benefits, this study has demonstrated that Python-Plaxis automation provides a powerful tool for parametric investigation of soil nailing systems. The implementation of variable facing inclinations directly within the finite element model through Python scripting proved particularly time-efficient for design scenarios requiring a target Factor of Safety (F_r) near 1.50. This parameter cannot be readily modified in conventional limit equilibrium software, highlighting a distinct advantage of the proposed approach. The DDO framework thus constitutes a valuable tool for advancing the understanding of soil nail retention mechanisms by enabling extensive simulations with precisely controlled parametric variations.

The optimization results also illuminated the principal limitations of the current automation framework. Computational time remains a significant constraint when employing finite element analysis within an optimization loop, necessitating access to adequate computational resources. The practical implementation of FEM-based automation requires the use of simplified yet representative geometry for the study section. To manage computational demands effectively, complex soil nail retention geometries can be optimized by focusing on a limited set of relevant design parameters, with cost optimization serving as an effective criterion for enhancing design efficiency while maintaining solution quality.

The results demonstrate that the proposed approach represents not merely a computational convenience, but a paradigm shift toward intelligent, data-driven geotechnical design. The framework provides a solid foundation for extending optimization strategies toward cost-effective design and multi-objective decision-making in geotechnical engineering. In this context, the presented DDO methodology contributes directly to the advancement of economic earth structure design, positioning automated optimization as a central component of next-generation geotechnical practice.

Future work should incorporate serviceability constraints, environmental impacts, and reliability-based approaches. Extending the framework to other geotechnical structures would demonstrate its versatility, while machine learning surrogate models could reduce computational demands for higher-dimensional optimization.

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Declaration of interest

The authors have no conflicts of interest to declare. All co-authors have observed and affirmed the contents of the paper and there is no financial interest to report.

Authors' contributions

Méline Anne-Fleur Bommertz: conceptualization, methodology, validation. Ana Luiza Mendes Halabi: literature review, writing – original draft. Adrian Torrico Siacara: supervision, writing – review & editing.

Data availability

All data produced or analyzed during this study are included in this article, and additional information will be available upon request.

Declaration of use of generative artificial intelligence

This work was prepared without the assistance of any generative artificial intelligence (GenAI) tools or services. All aspects of the manuscript were developed solely by the authors, who take full responsibility for the content of this publication.

List of symbols and abbreviations

c'	Effective cohesion
d	Vector of design variables
d_{DH}	Drill hole diameter
d_{eq}	Equivalent nail diameter
d_{max}	Maximum allowable value (upper bound constraint)
d_{min}	Minimum allowable value (lower bound constraint)
m_i	Hoek-Brown constant
s_h	Horizontal spacing
s_v	Vertical spacing
w	Distributed load
ABNT	Associação Brasileira de Normas Técnicas

AT	Fill material
BRL	Brazilian real (ISO 4217 currency code)
COL	Colluvium
D	Disturbance factor
DDO	Deterministic design optimization
DS	Direct shear test
E	Young's modulus
E_{50}	Secant stiffness for primary deviatoric loading
EA	Axial stiffness per unit length
E_{eq}	Equivalent stiffness parameter
EI	Bending stiffness per unit length
E_{oed}	Tangent stiffness for oedometer loading
E_{rm}	Deformation modulus of rock mass
E_{ur}	Unloading/reloading modulus
F	Factor of safety
F_{max}	Maximum factor of safety
F_{min}	Minimum factor of safety
F_T	Target factor of safety
FEM	Finite element method
H	Soil nail wall height
HS	Hardening soil
IDE	Integrated development environment
K	Empirical modulus coefficient (MPa), soil-type dependent (Teixeira & Godoy, 1996)
LEM	Limit equilibrium method
L_i	Nail lengths
MC	Mohr-Coulomb
M_p	Plastic moment capacity
NBR	Norma Brasileira
N_{SPT}	Blow count from SPT
P_p	Plastic axial capacity
RAD	Weathered rock (hard)
RAM	Weathered rock (soft)
RS	Sound rock
S	Side constraints
SLSQP	Sequential least squares programming
SM	Mixed borehole (percussion and rotary drilling)
SPT	Standard penetration test
SRJ	Young residual soil (saprolite)
SRM	Mature residual soil
T_{max}	Maximum nail tensile capacity
USD	United States dollar (ISO 4217 currency code)
α	Empirical coefficient, soil-type dependent (Teixeira & Godoy, 1996)
β_i	Facing inclination from vertical
γ	Unit weight
γ_{eq}	Equivalent unit weight
γ_{sat}	Saturated unit weight
γ_{unsat}	Unsaturated unit weight
ν	Poisson's ratio
ν_{ur}	Poisson's ratio for unloading/reloading
σ_{ci}	Unconfined compressive strength
σ_v	Vertical surcharge stress
φ'	Effective friction angle
ψ	Dilatancy angle

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