Soils and Rocks
An International Journal of Geotechnical and Geoenvironmental Engineering
www.soilsandrocks.com
ISSN 1980-9743
ISSN-e 2675-5475

Probabilistic safety assessment of geotechnical designs

Andrea Nascimento Vecci[1#], Alberto de Sampaio Ferraz Jardim Sayão[1]

Case Study

Keywords
Probability of failure
Spatial variability
FOSM
Monte Carlo simulation
Reliability index
Slope stability

Abstract

Slope stability calculations are frequently performed based only on deterministic methods, resulting in a value of Factor of Safety (*FS*). Deterministic procedures disregard the inherent variability of soil and rock properties. The probabilistic approach treats *FS* as a function of these properties and considers the impact of variability on the safety of geotechnical projects by estimating the values of the Reliability Index (β) and the Probability of Failure (*PF*). Deterministic and probabilistic analyses were carried out on two important geotechnical case studies: first, the main slope of Cauê open-pit mine, based on two probabilistic methods (FOSM - First Order Second Moment and MCS - Monte Carlo Simulation) applied with two classic Limit Equilibrium procedures (Simplified Bishop and Morgenstern-Price); the second case study is Santa Branca Dam, and was also based on the FOSM and MCS probabilistic methods, with the Simplified Bishop Limit Equilibrium method. The results from both safety analyses suggest that the FOSM method is simpler and more practical than the MCS, while provided lower *PF* values with more conservative β then the MCS method. The analyses also indicated that the FOSM technique should be applied associated with the Morgenstern-Price method, since this also provided a more conservative result for the Cauê Mine slope. The Santa Branca Dam was only analyzed with the Simplified Bishop method. In conclusion, it is recommended that probabilistic and deterministic analyses be carried out together to provide a relevant decision-making tool for geotechnical safety assessment by associating FOSM and Morgenstern-Price methods.

1. Introduction

In Geotechnical Engineering, slope stability analyses have been usually performed with basis on deterministic methods, by calculating the Factor of Safety (*FS*) considering the average values of geotechnical properties from a limited number of laboratory and field tests. Deterministic analyses can be oversimplified since *FS* values do not account for the inherent variability of geotechnical materials and can lead to unreliable *FS* results. Variability is an intrinsic property of natural materials, such as soils and rocks, and usually occurs as a result of various processes of formation and alteration of rocks, including diagenesis, fractional crystallization, changes by fluid circulation, and metamorphism (Sdvyzhkova et al., 2022). Probabilistic methods, based on statistical concepts, are gradually becoming popular in geotechnical practice as they incorporate the inherent uncertainties of natural materials.

The Factor of Safety can be defined as the ratio of resistance (*R*) overload or stress (*S*). Deterministic procedures use the mean values of the geotechnical variable properties when computing *FS*. These properties are frequently defined from a limited number of soils and/or rock samples. Average values of the geotechnical properties may, therefore, not represent the actual behavior of the field properties when the material has a strong spatial variability. Hence, deterministic analyses may result in unreliable *FS* values. Probabilistic analyses play a key role in reducing the need for human judgment to minimize the risk of geotechnical accidents. Assis (2020) emphasizes the non-existence of a *PF*-*FS* correlation when comparing 57 engineering structures (Figure 1), inducing further research on probabilistic analyses methods.

During the 1980s, the development of probabilistic concepts helped geotechnical engineers make safety decisions without relying exclusively on their empirical past experience. From a probabilistic perspective, the properties of natural materials can be considered as random variables (Sayão et al., 2012). Probabilistic studies on geotechnical safety have also been reported by Coelho (2023), Vecci (2023) and D'Hyppolito et al. (2024).

There are several probabilistic methods currently used to assess the stability of geotechnical projects, the most popular being known as FOSM (First Order Second Moment), Point Estimation, and MCS (Monte Carlo Simulation). In geotechnical practice, probabilistic and deterministic methods shall be understood as complementary procedures, for providing the design engineer with a clear idea of how reliable the

#Corresponding author. E-mail address: deiavecci@gmail.com
[1]Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Engenharia Civil, Rio de Janeiro, RJ, Brasil.

Submitted on March 12, 2025; Final Acceptance on October 17, 2025; Discussion open until May 31, 2026.
Editor: Renato P. Cunha
https://doi.org/10.28927/SR.2026.003625

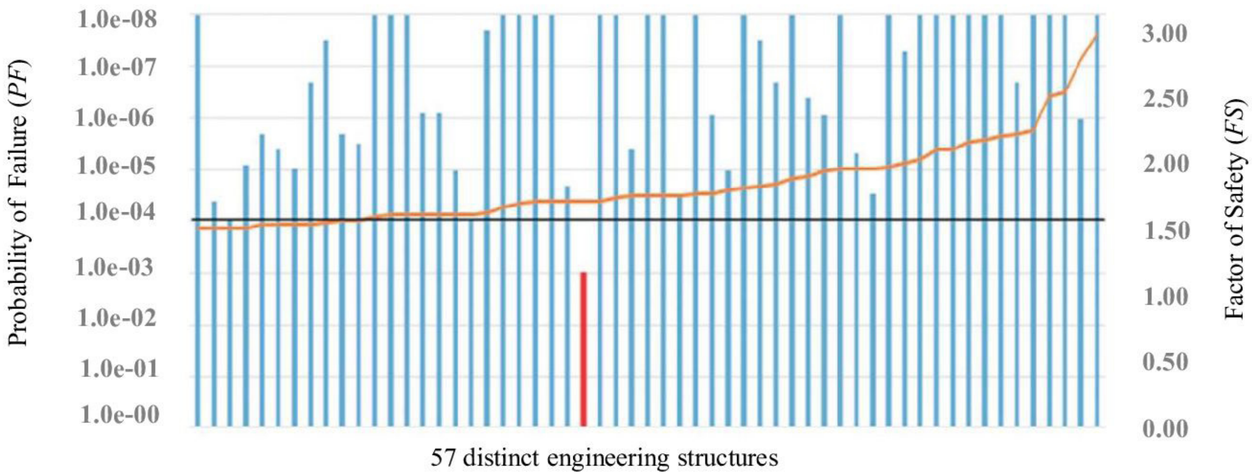

Figure 1. Non-existent *PF*-*FS* correlation as shown by Assis (2020) for various engineering structures.

deterministic *FS* may be. This paper presents results of the FOSM and MCS procedures, in association with the well-known deterministic limit equilibrium methods of Simplified Bishop (1955) and Morgenstern and Price (1965), applied to two important geotechnical cases in SE Brazil: the Cauê open-pit mine slope and the downstream slope of the Santa Branca embankment dam. The main objective of this paper is to evaluate which probabilistic and Limit Equilibrium methods could be best recommended to assess the slope stability. The acceptable *PF* value for each slope depends on the consequences of an eventual failure.

In operating open-pit mines, slopes are usually excavated to the steepest possible angle for economic reasons (Naghadehi et al., 2013), meaning production optimization. Therefore, a high probability of failure (*PF*) may be accepted in some open-pit facilities, whereas the same *PF* value might not be acceptable for a tailings dam, where there might be significantly higher PAR (population at risk) and debris would flow downstream of the facility. Well monitored slope stability (24/7) systems also allow for early warning, thus significantly reducing the consequences of failures (loss of life, equipment, production interruption, etc.) and opening room for higher acceptable *PF* but not increasing risk (risk = *PF* x Consequences)

1.1 Reliability index (β)

Probabilistic analyses of a given geotechnical problem aim to estimate the value of Probability of Failure (*PF*), defined as the probability of *FS* being below 1.0, or $P(FS<1)$. *PF* is an indirect result, attained after calculating the Reliability Index (β) of Equation 1 (Christian et al., 1992). In practice, the properties of natural materials may be considered as inherently variables, exhibiting a Normal (Gaussian) distribution (Sayão et al., 2012). As reported by Christian et al. (1992), the value of β expresses the number of standard deviations that separates the mean value of *FS* from the failure value (*FS* = 1.0).

$$\beta = \frac{E\left[FS\right]-1}{\sigma\left[FS\right]} \tag{1}$$

where:
$E[FS]$: mean value of *FS*.
$\sigma[FS]$: standard deviation of *FS*.

1.2 Probability of failure (*PF*)

Once β has been computed as described on Section 1.1, the Probability of Failure (*PF*) may be estimated directly from the graphical correlation shown in Figure 2. When presenting this correlation, Christian et al. (1992) acknowledged that the chosen limit state function (g) affects the calculated reliability index and defined the limit state function to be $g = R/S - 1$. Christian et al. (1992) and Sayão et al. (2012) show that the relationship between β and *PF* depends on the distribution function of *FS* only when $\beta > 0.8$. Figure 2 shows that the Normal distribution provides the highest *PF* values for any given β, so it can be considered the most practical and conservative assumption among the several possible distributions.

2. Methodology

2.1 First order second moment – FOSM

FOSM is a probabilistic method that provides good quality Reliability Index (β) results. It is also the simplest among the usual probabilistic methods applied in current geotechnical practice. FOSM can be considered an indirect

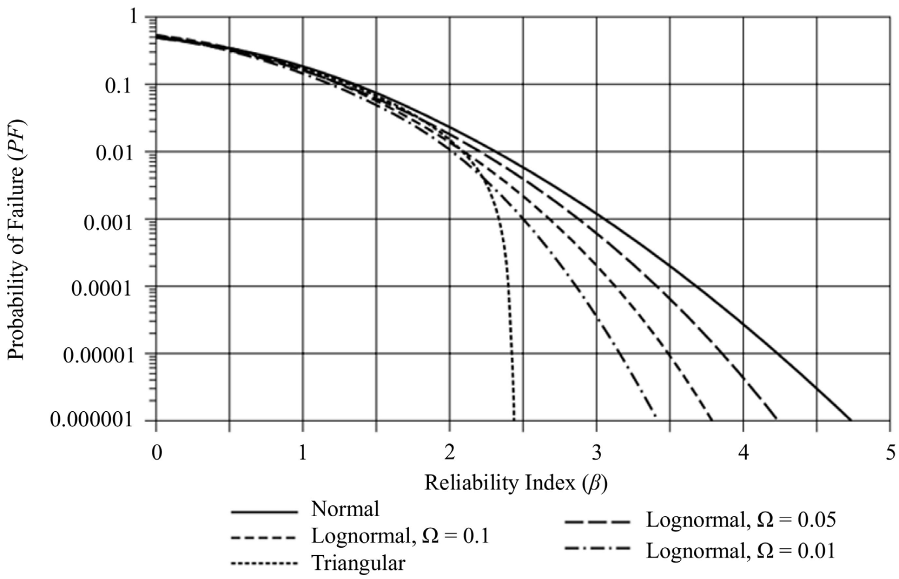

Figure 2. Probability of Failure x Reliability Index for various probabilistic distributions (adapted from Baecher & Christian, 2003).

probabilistic method, since there is no need to describe the probabilistic distribution curve of properties identified as random variables in a geotechnical problem. The mean and variance (V) of the dependent variable FS are calculated by applying the Taylor series around the mean values of the independent variables (Christian et al., 1992).

Harr (1987) suggested a simplification of the Taylor series as a function of random variables that are not intrinsically correlated, leading to Equation 2, which provides $V[FS]$:

$$V\left[FS\right] = \sum_{\mathrm{i}=1}^{n}\left(\frac{\delta FS}{\delta x_i}\right)^2 . \; V\left[x_i\right] \tag{2}$$

where:
$\delta FS / \delta x_i$ = partial derivative of FS with respect to x_i.

Equation 2 represents the variance of FS as $V\left[FS\right]$, and the variance of the independent random variable as $V\left[x_i\right]$, where the random variable is x_i. By definition, the Variance of x_i is the square of its standard deviation, $\left(\sigma\left[x_i\right]\right)^2$.

Christian et al. (1992) suggested the solution of approximating the partial derivative factor of Equation 2 by the method of divided differences (Equation 3) to solve the partial derivative components of this equation, without the need to make assumptions about the functions x_i.

$$\frac{\delta FS}{\delta x_i} = \frac{FS\left(x_i + \delta x_i\right) - FS\left(\overline{x_i}\right)}{\delta x_i} \tag{3}$$

where:
$\overline{x_i}$: mean value of variable x_i.
δx_i: small variation of x_i.
$FS\left(x_i + \delta x_i\right)$: Factor of Safety when x_i is varied one by one
$FS(x_i)$: Mean FS.

Dell'Avanzi & Sayão (1998) reported that, when using the FOSM method, a 10% variation for each variable (positive or negative) is satisfactory, in practical terms.

2.2 Monte Carlo simulation

Monte Carlo is a stochastic simulation procedure that generates datasets from a series of values that have as reference the statistical mean and deviation of geotechnical variables. From the random creation of thousands of these sets, the computer is able to trace and assign distribution curves to those variables, by simulating their possible probabilistic behaviors. The method is comprehensive and may be applied to various problems, when aiming to study uncertainties of a system. The nature of the method requires a high computational effort to obtain a proper probability distribution curve. The required number of simulations (N) depends on the number of independent random variables (n) considered in the problem, the accuracy (ε) and the desired confidence level (1-α) being expressed by Equation 4:

$$N = \left[\frac{\left(h_{\alpha/2}\right)^2}{4\varepsilon^2}\right]^n \tag{4}$$

where:
$h_{\alpha/2}$: confidence index, function of the confidence level (1- α).

Harr (1987) presented the values of the functions $h_{\alpha/2}$ associated to the values of confidence levels (1-α), as shown in Table 1.

Distinct from the FOSM method, Monte Carlo Simulation requires the knowledge of the probability density functions of the independent random variables involved in the problem, requiring the aid of a software to perform thousands of analyses, in which the independent soil variables are randomly varied within the possible range of values delimited by their characteristic probability density functions.

After a large number of simulations, the computer produces a histogram of all data producing a representative probability distribution curve of the dependent variable (*FS*). The area under this curve corresponding to the region with *FS* < 1 expresses the Probability of Failure (*FS*).

In general, the Probability of Failure computed by the Monte Carlo Simulation is obtained by the following Equation 5:

$$PF = \frac{1}{N_{mcs}} \sum_{i=1}^{N_{mcs}} I[FS_i\left(X_i\right) < 1] \quad (5)$$

where:
N_{mcs}: number of random generated samples.
X_i: vector *i* of random generated variables.

Table 1. Confidence index for normal distribution (adapted from Harr, 1987).

Confidence level (%) (1- α)	Confidence index ($h_{\alpha/2}$)
85	1.44
90	1.64
95	1.96
95.45	2.00
98	2.33
99	2.58
99.5	2.81
99.73	3.00
99.9	3.29
99.99	3.89
99.994	4.00

FS_i : i-th Factor of Safety (*FS*) given X_i;
I [ ]: Indicative function which is equals to a unit when FS_i (X_i) < 1 and zero otherwise.

3. Probabilistic analyses of the Cauê mine slope

3.1 Description

This paper presents deterministic and probabilistic studies of a 200m high slope in the Cauê Mine, located in Itabira, in the Iron Quadrangle of Minas Gerais, Brazil, based on two standard deterministic methods (Simplified Bishop and Morgenstern-Price) associated with two probabilistic procedures, known as FOSM - First Order Second Moment and MCS - Monte Carlo Simulation.

The Cauê Mine is an inactive iron ore open pit mine that was once the main active iron ore mine in Southeast Brazil. It is part of the Itabira Complex, located in the NE region of the Iron Quadrangle, in Minas Gerais state. The Cauê Mine (Figure 3) was one of the largest iron ore mines in the world during the 1970s (Vecci & Sayão, 2019).

Figure 4 shows a typical cross-section of the 200 m high slope of the Cauê Mine, which is studied in this paper. It had an overall inclination of 34° and was composed of weathered iron quartzite (saprolite). The phreatic surface is denoted by an inclined line that runs from the slope base to a point about 80m below its crest. This piezometric condition was inferred from more than 60 piezometers and water level indicators around the mine pit. The same slope was reported by Sayão et al. (2012), and its geotechnical and statistical characteristics were obtained from a comprehensive investigation including about 50 direct shear tests on undisturbed specimens prepared from block samples. Table 2 presents the soil properties and statistical parameters are used in these stability analyses.

Considering the geotechnical random variables of the Cauê Mine slope and calculating their mean, standard deviation, and variance (Table 2), the safety of the slope was investigated by applying the probabilistic methods FOSM and Monte Carlo Simulation, with the deterministic methods Morgenstern-Price and Simplified Bishop.

The FOSM method was applied based on the guidelines summarized by Vecci (2023).

The Monte Carlo Simulation was performed using RocScience Slide 6.0 software (RocScience, 2010).

Table 2. Cauê Mine slope. Geotechnical and statistical properties (Sayão et al., 2012).

Variable	Mean ($\bar{x}_i$)	Standard Deviation ($\sigma\,[x_i]$)	Variance ($V\,[x_i]$)
Unit weight (above WL) γ_{nat} (kN/m^3)	28.3	1.4	1.96
Unit weight (below WL) γ_{sat} (kN/m^3)	29.0	1.4	1.96
Cohesion, *c'* (kPa)	25.0	24.3	590.0
Tangent of friction angle, tan(*Φ'*)	0.781	0.085	0.007
Pore pressure (*u*)	120.0	20.0	400.0

Figure 3. Cauê Mine slope. 200 m high open-pit.

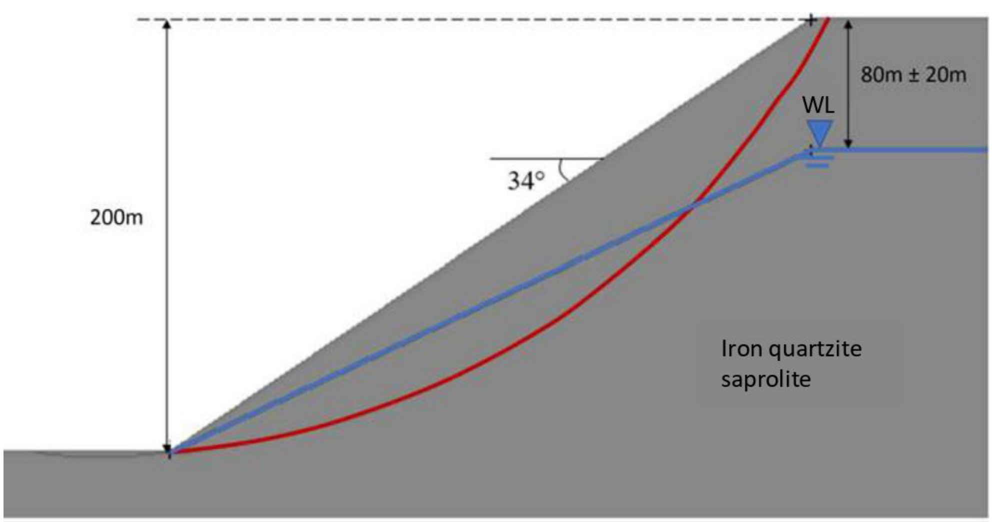

Figure 4. Cauê Mine slope. Typical cross-section.

3.2 Results

Deterministic analyses carried out with well-known limit equilibrium methods (Simplified Bishop and Morgenstern-Price) resulted in $FS = 1.264$ and $FS = 1.259$ for the methods respectively, and may be used in the deterministic analyses stage of this probabilistic assessment as the $E[FS]$ factor.

When the Simplified Bishop and Morgenstern-Price methods were associated to the FOSM procedure, maintaining fixed the critical failure surface from the initial deterministic analyses, the given results were $\beta = 1.62$ (Simplified Bishop), and $\beta = 1.54$ (Morgenstern-Price). These results show that, for equivalent FS, the resulting β (and PF), might vary slightly with the limit equilibrium method. These results highlight the importance of carrying out probabilistic analyses to help understand the meaning of conventional deterministic results.

The Monte Carlo Simulation gave identical results for both Simplified Bishop and Morgenstern-Price methods, with $\beta = 1.87$, showing less conservative *PF* results for the same case study, when compared to FOSM, where $\beta = 1.62$ (Simplified Bishop), and $\beta = 1.54$ (Morgenstern-Price). In this case study, Morgenstern-Price, when associated with FOSM, resulted in the most conservative β (or highest *PF*), when compared to MCS.

The results are shown in Table 3.

As usual, the random variables in Table 2 were assumed to have normal distribution functions and to be independent of each other (Sayão et al., 2012). By applying FOSM method, the β values were computed, and *PF* were respectively estimated from Equation 1 and the graphical correlation presented in Figure 2.

With the overall *FS* variance and all individual variable variances (shown in Table 4 and Table 5), the FOSM method offers the relative significance of each random variable in the overall stability. Figure 5 shows that, for the Cauê Mine slope, the FOSM method, with both Simplified Bishop and Morgenstern-Price indicates that natural and saturated unit weights (γ_{nat} and γ_{sat}) were not relevant to the stability analyses, with a very low significance on the *FS* result. This indicates that both parameters could be considered as non-variables in this case. The variance results also show that Φ' is the most significant parameter for the Cauê Mine slope, followed by c'.

These properties' significance provided by the FOSM method is a positive benefit for the geotechnical engineer when deciding on what actions should be taken, when *FS* needs to be increased. For example, the pore pressure accounts for only about 27% of the overall stability, and thus, in this case, further drainage would not be highly effective in increasing *FS*.

Table 3. Cauê Mine slope. Deterministic and Probabilistic analyses results.

Limit Equilibrium method	$E[FS]$	FOSM		Monte Carlo simulation	
		β	*PF*	β	*PF*
Bishop	1.264	1.62	1:20	1.87	1:43
Morgenstern-Price	1.259	1.54	1:16	1.87	1:44

Table 4. Cauê Mine slope. β, *PF* and *FS* variance in FOSM calculations carried out with Simplified Bishop.

x_i	$E[x_i]$	δx_i	$FS\ (x_i+\delta x_i)$	δFS	$\delta FS/\delta x_i$	$V\left[x_i\right]$	$(/\delta x_i)^2 V\left[x_i\right]$	$V[FS]$ %
c'	25.0	2.50	1.269	0.005	0.0020	590	0.00236	8.89
tanΦ'	0.781	0.113	1.437	0.173	1.5334	0.007	0.01693	63.79
γ_{nat}	28.3	-2.83	1.269	0.005	-0.0018	1.96	0.00001	0.02
γ_{sat}	29.0	2.90	1.274	0.010	0.0034	1.96	0.00002	0.09
u	120.0	12.00	1.213	-0.051	-0.0042	400	0.00722	27.22
					$V[FS]$*	0.0265		
					$\sigma[FS]$**	0.1629		
					β	1.62		
					PF	1:20		

$*V[FS] = \sum V\left[x_i\right]$. $** \sigma[FS] = \sqrt{V\left[FS\right]}$.

Table 5. Cauê Mine slope. β, *PF* and *FS* variance in FOSM calculations, carried out with Morgenstern-Price.

x_i	$E\left[x_i\right]$	δx_i	$FS\ (x_i+\delta x_i)$	$\delta\ FS$	$\delta\ FS/\delta x_i$	$V\left[x_i\right]$	$(\delta FS\ /\ \delta x_i)^2 . V\left[x_i\right]$	$V[FS]$ %
c'	25.0	2.50	1.265	0.006	0.0024	590	0.00340	12.08
tanΦ'	0.781	0.113	1.440	0.181	1.6044	0.007	0.01850	65.88
γ_{nat}	28.3	-2.83	1.264	0.005	-0.0018	1.96	0.00001	0.02
γ_{sat}	29.0	2.90	1.275	0.016	0.0055	1.96	0.00010	0.21
u	120.0	12.00	1.212	-0.047	-0.0039	400	0.00610	21.81
					$V[FS]$*	0.0281		
					$\sigma[FS]$**	0.1677		
					β	1.54		
					PF	1:17		

$*V[FS] = \sum V\left[x_i\right]$. $** \sigma[FS] = \sqrt{V\left[FS\right]}$.

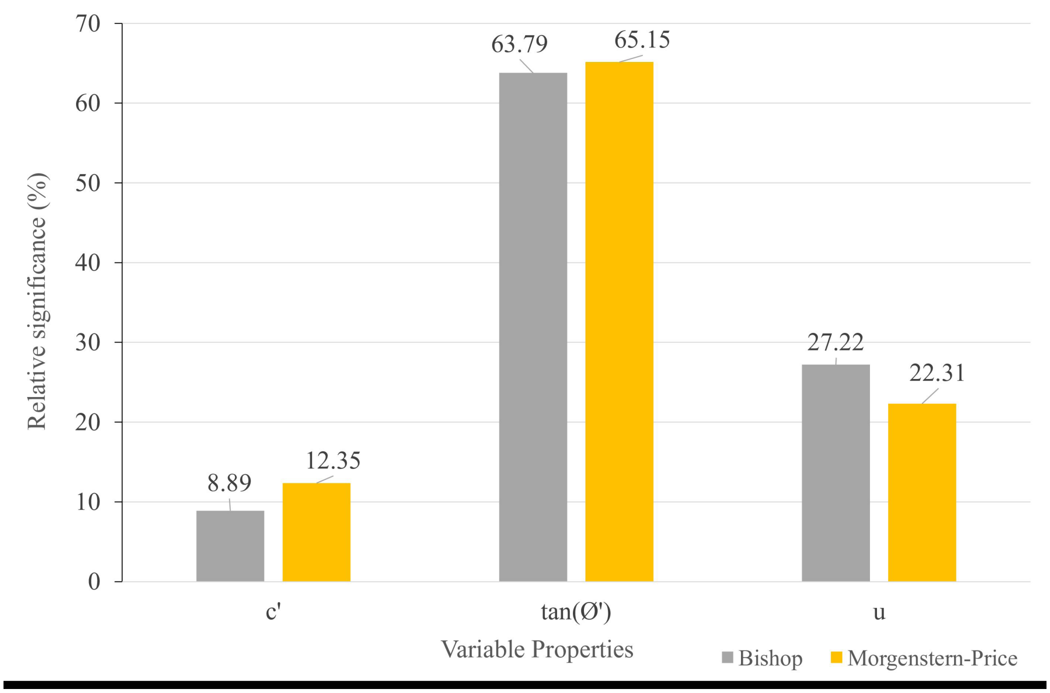

Figure 5. Cauê Mine Slope. Relative significance of each variable in the 200m slope safety, given by the FOSM method.

In slopes with lower height, where the pore pressure may be seen to be the most significant variable affecting safety, additional drainage would be expected to be the correct action to rapidly obtain a significant increase in *FS*.

On occasions where the Geotechnical Engineer will still apply the Monte Carlo Simulation procedure, applying FOSM before MCS is useful to narrow down the number of significant variables associated with the probabilistic analysis to substantially lower the required computational effort required for applying MCS.

On the other hand, as the Monte Carlo Simulation is an automated method, and provides *PF* results straight from the Slide software, it does not indicate the relative significance of each variable in the overall *FS* performance.

4. Probabilistic analyses of the Santa Branca earth dam

4.1 Description

The Santa Branca Dam has been built to provide a water reservoir for a hydroelectric power plant, located on the Paraíba do Sul River, in the eastern region of São Paulo State. The dam is composed of homogeneous soil embankment (Figure 6). Its downstream slope is protected by grass, and the upstream slope is covered by riprap (rockfill). Figure 7 shows the original typical cross-section of the dam.

The homogeneous embankment dam has a drainage mat and a vertical drain, the dam is 55m high and 320 m long. A reinforcement layer was added to its downstream slope as a technique used to solve the dam's internal drainage issues. Figure 7 shows the typical cross-section of the reinforced dam.

The average parameters of the materials that compose the Santa Branca Dam are presented in Table 6.

Reporting on Santa Branca Dam for distinct purposes, Pacheco (1990) explained how to obtain the mean and standard deviation of strength parameters from drained triaxial tests on undisturbed specimens from two other dams (Paraitinga and Paraibuna) with similar soil properties, also in São Paulo (Table 7).

FOSM analyses of previous studies indicated that, in this case, the specific weight and pore pressure showed insignificant variability in comparison to cohesion and friction angle (Araújo & Sayão, 2018). Therefore, only the two last parameters were treated as random variables in the present study of the Santa Branca Dam.

Considering the geotechnical random variables of the downstream slope of Santa Branca Dam and their mean, standard deviation, and variance (Table 7), the safety of the dam slope was studied by applying the two probabilistic methods (FOSM and MCS), associated with the deterministic Simplified Bishop method.

Figure 6. Santa Branca Dam. Aerial view of the downstream slope in mid-2017 (Araújo & Sayão, 2018).

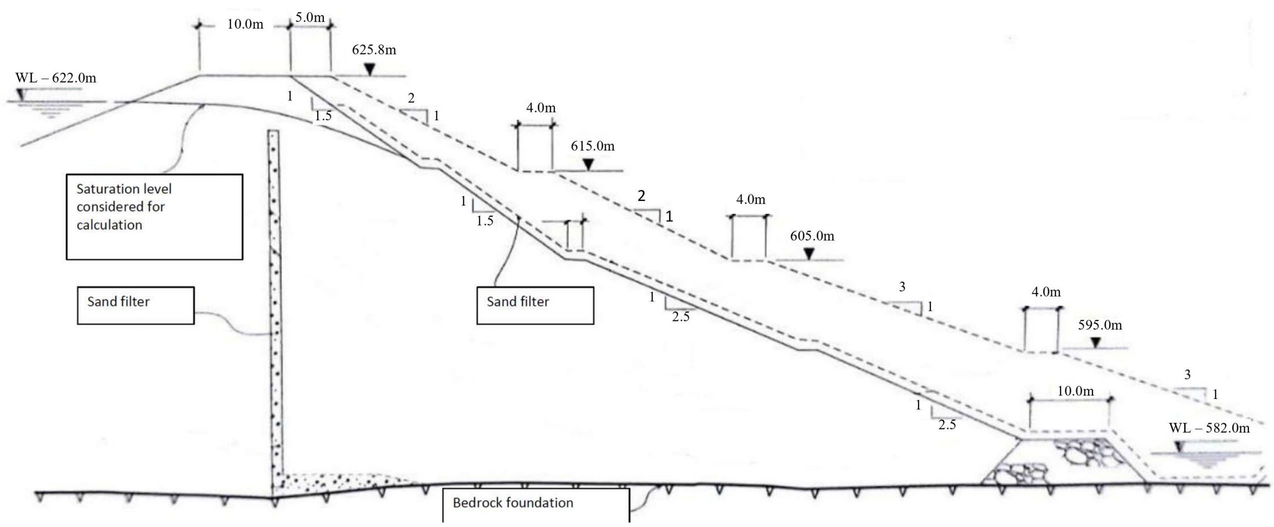

Figure 7. Santa Branca Dam. Typical cross-section (Araújo & Sayão, 2018).

The FOSM method was applied based on the guidelines proposed by Vecci (2023). The Monte Carlo Simulation was performed using RocScience Slide 6.0 software (RocScience, 2010).

4.2 Results

The deterministic analysis performed with the Simplified Bishop method resulted in $FS = 2.2$.

When the Simplified Bishop method was associated with the probabilistic method with a fixed critical failure surface, the FOSM method presented $\beta = 6.25$, while the Monte Carlo Simulation presented $\beta = 6.29$. As FS was high for this case, PF did not show a significant difference for the two distinct probabilistic methods (FOSM and MCS),

Table 6. Santa Branca dam. Mean values of soil properties.

Property	Mean ($\overline{x}_i$)
Compacted soil embankment	
Unit weight above water level - γ_{nat} (kN/m^3)	18.0
Unit weight below water level - γ_{sat} (kN/m^3)	20.0
Filter	
Unit weight γ (kN/m^3)	18.0
Friction angle, Φ'	35°
Rockfill	
Unit weight γ (kN/m^3)	20.0
Friction angle, Φ'	45°

Table 7. Santa Branca Dam. Statistical properties of the variables considered in the safety analysis.

Property	Mean ($\overline{x}_i$)	Standard Deviation ($\sigma[x_i]$)	Variance ($V[x_i]$)
Cohesion, *c'* (kPa)	83.61	17.05	290.70
Friction Angle, *Φ'*	24.43°	1.11°	1.23

Table 8. Santa Branca Dam. Deterministic and Probabilistic analyses results.

Limit Equilibrium method	*E*[*FS*]	FOSM		Monte Carlo Simulation	
		β	*PF*	β	*PF*
Simplified Bishop	2.2	6.25	$1{:}4.9x10^9$	6.29	$1{:}6.5x10^9$

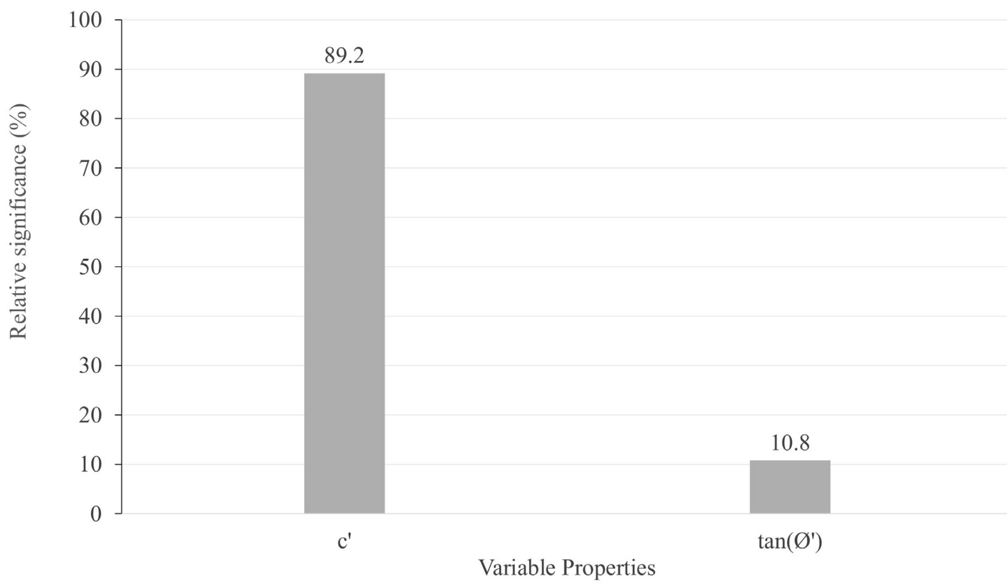

Figure 8. Santa Branca Dam - Relative significance of each variable in the dam's safety, as indicated by the FOSM method.

but FOSM presented a slightly more conservative value for *PF*. This case study also confirms the FOSM method to provide more conservative results than MCS, reinforcing the conclusion for the Cauê open-pit slope case study.

The results are summarized in Table 8.

The random variables in Table 7 were assumed to have Normal distribution functions and to be independent amongst each other, therefore, by FOSM, β was calculated from Equation 1, and *PF* was estimated from the correlation presented in Figure 2.

Since the implementation of FOSM associated with Simplified Bishop presented Probability of Failure tending towards zero, other methods of Limit Equilibrium were not considered for the Santa Branca Dam for, when *PF* approaches zero, there is little influence of the Limit Equilibrium method on the results of the low significant decimals of *PF*. The comparison of this study with other previous research's results, such as Araújo (2018) and Pacheco (1990), confirms this perception.

The FOSM method provided the relative significance of *c'* and *Φ*' in the *PF* result indicated by each variable property calculated variance (Table 9). Figure 8 shows that, for this case study, effective cohesion is the most significant parameter, with about 89% influence on the *PF* value, while the effective friction angle had the remaining significance, approximately 11%, as illustrated in Figure 8.

The Monte Carlo method is automated and provided *PF* results straight from the Slide software, which did not provide relative significance of each variable in the overall *FS* performance.

Table 9. Santa Branca Dam. β, *PF* and *FS* variance in FOSM calculations carried out with Simplified Bishop.

x_i	$E[x_i]$	δx_i	$FS\ (x_i+\delta x_i)$	δFs	$\delta FS/\delta x_i$	$V[x_i]$	$(\delta FS/\delta x_i)^2.V[x_i]$	$V[FS]$ %
Φ'	24.43	2.44	2.34	0.14	0.055	1.23	0.004	10.80
c'	83.61	8.36	2.29	0.09	0.010	290.70	0.033	89.20
					$V[FS]$*	0.037		
					$\sigma[FS]$**	0.193		
					β	4.150		
					PF	$1{:}4.9x10^9$		

*$V[FS] = \sum V[x_i]$. ** $\sigma[FS] = \sqrt{V[FS]}$.

5. Conclusions

Defining the degree of reliability of *FS* is essential for ensuring the safety of geotechnical projects and providing the geotechnical engineer with the chance to make better decisions on minimizing the chance of disasters, such as the recent ones with the Brazilian tailings dams Fundão (in Mariana) and B1 (in Brumadinho).

Analyzing the results of the two probabilistic studies reported herein, it is possible to conclude that the FOSM method is recommended over the Monte Carlo Simulation method, for being simpler, more practical, and more conservative for both Cauê Mine and Santa Branca Dam cases (Table 3 and Table 8).

Since FOSM provides the individual variances of each variable into the overall *FS*, the method is also recommended for providing the geotechnical engineer with the relative significance of each variable considered in the stability analyses of geotechnical projects. Knowing the relative significance is a relevant asset in the decision-making process, such as proper actions to be taken when increasing the *FS* is required.

FOSM is recommended to be performed together with a rigorous limit equilibrium procedure, such as Morgenstern-Price method, based on the results obtained for the Cauê Mine (Table 3), in which β was shown to be more conservative with Morgenstern-Price method, than with non-rigorous Simplified Bishop.

The relative significances of all variables may vary according to the specific geotechnical setting, as verified by the case studies reported herein. In the Cauê Mine slope, the effective friction angle (Φ') was the factor of greatest significance in *FS* value (Figure 5), while, in Santa Branca Dam, the effective cohesion (*c'*) was the most significant property of in the *FS* value (Figure 8).

Acknowledgements

The authors thank the Brazilian research agencies CNPq and CAPES for their financial support. The contributions of Drs E. Dell'Avanzi and S. Sandroni in previous probabilistic studies at PUC-Rio are also appreciated.

Declaration of interest

The authors have no conflicts of interest to declare. All co-authors have observed and affirmed the contents of the paper and there is no financial interest to report.

Authors' contributions

Andrea Nascimento Vecci: conceptualization, investigation, project administration, formal analysis, writing - original draft. Alberto Jardim Sayão: resources, funding acquisition, supervision, validation, writing - review & editing.

Data availability

The datasets generated and analyzed in the course of the current study are available from the corresponding author upon request.

Declaration of use of generative artificial intelligence

This work was prepared without the assistance of any generative artificial intelligence (GenAI) tools or services. All aspects of the manuscript were developed solely by the authors, who take full responsibility for the content of this publication.

List of symbols and abbreviations

c'	Effective cohesion
$h_{\alpha/2}$	Confidence index
u	Pore pressure
$\underline{x_i}$	Mean value of variable x_i
FOSM	First Order Second Moment
FS	Factor of safety
FS_i	i-th: Factor of Safety (*FS*) given X_i
g	Limit state function
I [ ]	Indicative function which is equals to a unit when
FS_i	$(X_i) < 1$ and zero otherwise.
MCS	Monte Carlo Simulation

N_{mcs} Number of random generated samples in the Monte Carlo Simulation
PF Probability of failure
R Resistance
S Stress
V Variance
WL Water level
X_i Vector i of random generated variables
β Reliability index
γ Unit weight
γ_{nat} Unit weight above water level
γ_{sat} Unit weight below water level
δx_i Small variation of x_i
ε Accuracy
$\sigma[FS]$ Standard deviation of FS
Φ' Effective friction angle

References

Araújo, M.B. (2018). *Considerations on probabilistic analyses of a dam slope stability* [Master's dissertation, Pontifical Catholic University of Rio de Janeiro]. Pontifical Catholic University of Rio de Janeiro's repository (in Portuguese).

Araújo, M.B., & Sayão, A.S.F.J. (2018). Análise probabilística da barragem de Santa Branca. *Revista Brasileira de Engenharia de Barragens*, 5(6), 6-12.

Assis, A.P. (2020). Risk management for geotechnical structures: consolidating theory into practice. *Soils and Rocks*, 43(3), 311-336.

Bishop, A.W. (1955). The use of the slip circle in the stability analysis of slopes. *Geotechnique*, 5(1), 7-17.

Baecher, G.B., & Christian, J.T. (2003). *Reliability and statistics in geotechnical engineering*. John Wiley & Sons.

Coelho, A.S. (2023). *Earth dam failure risk: quantitative analysis methodology* [Doctoral thesis, Pontifical Catholic University of Rio de Janeiro]. Pontifical Catholic University of Rio de Janeiro's repository (in Portuguese).

Christian, J.T., Ladd, C.C., & Baecher, G.B. (1992). Reliability applied to slope stability analysis. *Journal of Geotechnical Engineering*, 120(12), 2180-2207.

Dell'Avanzi, E., & Sayão, A.S.F.J. (1998). Avaliação da probabilidade de ruptura de taludes. In *Anais do 11º Congresso Brasileiro de Mecânica dos Solos e Engenharia Geotecnica* (Vol. 2). Brasília: Associação Brasileira de Mecânica dos Solos e Engenharia Geotécnica.

D'Hyppolito, L.C.B.S., Sayão, A.S.F.J., & Nunes, A.L.L. (2024). A simple method for assessing the probability of liquefaction of tailings dams. *Soils and Rocks*, 47(4), e2024000924.

Harr, M.E. (1987). *Reliability based design in civil engineering*. McGraw-Hill Inc.

Morgenstern, N.R., & Price, V.E. (1965). The analysis of the stability of general slip surfaces. *Geotechnique*, 15(1), 79-93.

Naghadehi, M.Z., Jimenez, R., KhaloKakaie, R., & Jalali, S.M.E. (2013). A new open-pit mine slope instability index defined using the improved rock engineering systems approach. *International Journal of Rock Mechanics and Mining Sciences*, 61, 1-14.

Pacheco, M.P. (1990). Probability and risk analyses concepts of geotechnical studies and projects. In *Proceedings of the IX Brazilian Conference on Geotechnical Engineering (COBRAMSEF)* (Vol. 3, pp. 37-56). Salvador: Brazilian Society for Soil Mechanics and Geotechnical Engineering.

Sdvyzhkova, O., Moldabayev, S., Bascetin, A., Babets, D., Kuldeyev, E., Sultanbekova, Z., Amankulov, M., & Issakov, B. (2022). Probabilistic assessment of slope stability at ore mining with steep layers in deep open pits. *Mining of Mineral Deposits Journal*, 16(4), 11-18.

Sayão, A.S.F.J., Sandroni, S.S., Fontoura, S.A.B., & Ribeiro, R.C.H. (2012). Considerations on the probability of failure of mine slopes. *Soils and Rocks*, 35(1), 31-37.

RocScience (2010). *Slide version 6.0.* Toronto: RocScience.

Vecci, A.N. (2023). *Probabilistic assessment methodology of geotechnical constructions* [Doctoral thesis, Pontifical Catholic University of Rio de Janeiro]. Pontifical Catholic University of Rio de Janeiro's repository (in Portuguese).

Vecci, A.N., & Sayão, A.S.F.J. (2019). Probabilistic analysis of mine slope stability. In *Proceedings of the XVI Pan-American Conference on Soil Mechanics and Geotechnical Engineering* (pp. 1757-1765). London: International Society for Soil Mechanics and Geotechnical Engineering.