

Laboratory studies to assess the stability of embankments supported by granular encased columns

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Article

Keywords

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Abstract

Granular encased columns (GECs) with geosynthetic are an effective technique for improving the soft soil foundation of embankments. One of the primary functions of GECs is to increase the bearing capacity of the soil, preventing foundation failure and eliminating the need for berms. GECs increase the soil's bearing capacity, preventing foundation failure, eliminating the need for berms, and increasing the factor of safety. Two methods are recommended to integrate the geosynthetic strength with the original strength of the ordinary granular column: a) using the increased angle of friction (ϕ'_{sub}), or b) using the increased cohesion (c'_{sub}). Laboratory (direct shear tests), numerical (3D modeling) and analytical studies were conducted to investigate the accuracy of using ϕ'_{sub} and c'_{sub} to represent the increased strength parameters attributable to, resulting from encasement. The results indicate that the strength of the composite column (sand + geosynthetic) can be adequately represented by the ϕ'_{sub} parameter for a wide range of geosynthetic stiffness (J). However, the approach using the c'_{sub} parameter did not yield satisfactory results, even for a broad range of J . Therefore, the use of the ϕ'_{sub} approach is recommended for stability analysis of embankments built on soft soils supported by GECs.

1. Introduction

The granular encased column (GEC) is a useful and alternative technique for stabilizing embankments on soft soils. This technique effectively improves the soft soil foundation of an embankment as noted by Almeida et al. (2018, 2023). According to EBGE0 (2012) the encasement of the granular column is necessary when the undrained strength (S_u) is equal to or less than 15 kPa. Three main functions of this technique in stabilizing the embankment can be highlighted: i) increase the bearing capacity of the soft soil, preventing foundation failure and the need for berms; ii) accelerate consolidation through radial drainage; and iii) reduce settlements during consolidation. Other functions include separation of granular material from the soft soil, preventing clogging of the column (Kousik & Shiyamalaa, 2016); reduction of the horizontal stress in soft clay (Almeida et al., 2018); improvement of foundation performance under seismic loads (Guler et al., 2014); and prevention of bulging, saving granular material preventing and increase in the diameter of the column (Hosseinpour et al., 2019).

The investigation of stability analysis of soft soil foundation against failure has been conducted using full-scale embankments (e.g., Hosseinpour et al., 2017) or reduced scale models (1g), such as those by Murugesan & Rajagopal (2009), Mohapatra & Rajagopal (2015), Mohapatra et al. (2016) and Alkhorshid et al. (2021).

Figure 1a illustrates the potential slip surface for a conventional embankment and Figure 1b shows the slip surface for an embankment built over GECs. The shear resistances of both the soft soil and the GEC column are incorporated into the stability analysis calculations. The use of a geosynthetic encasement provides additional shear resistance to the granular column. Stability analyses can be done using 2D or 3D models. However, 3D numerical modeling is more complex and requires considerably more time. In 3D modeling, the effect of the geosynthetic is explicitly considered by selecting the stiffness modulus (J) of the geosynthetic. The use of the 2D model is simpler, and can be performed using common programs based on limit equilibrium. For stability analyses using the 2D model, the additional shear resistance provided by the geosynthetic in the column must be considered.

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To calculate the increased angle of friction (ϕ'_{sub}) or the apparent cohesion (c'_{sub}), attributable to, resulting from encasement, the approach of Raithel & Henne (2000) can be applied. In the 2D model the column is represented as shown in Figure 1b with increased shear parameters (ϕ'_{sub} or c'_{sub}).

For this reason, therefore, several studies have been conducted to investigate the increase in shear parameters. Murugesan & Rajagopal (2009) performed direct shear tests to analyze the behavior of ordinary granular columns (without encasement) and geosynthetic encased columns (GECs). They observed that the encasement increased the shear resistance of the column. Mohapatra & Rajagopal (2015) conducted an experimental study to analyze the shear resistance of GECs using a direct shear apparatus. The improvement in the shear resistance of the column is attributed to the geosynthetic encasement. Mohapatra et al. (2016) conducted experimental tests on GECs in the laboratory to evaluate the effect of the geosynthetic on the lateral load capacity of the encased column. The tests were conducted by varying the normal stress (15–75 kPa), the diameter of the column, the material used for the encasement and the spatial arrangement of the columns. According to Mohapatra et al. (2016) the GEC and the surrounding soil behave as a composite material with the geosynthetic enhancing the overall strength. This factor needs to be considered in stability analysis.

The primary objective of this study is to evaluate the accuracy of the Raithel & Henne (2000) method in predicting the increased shear strength parameters (ϕ'_{sub} and c'_{sub}). This evaluation was carried out by comparing the calculated values with the experimentally measured values of ϕ'_{sub} and c'_{sub} , which were obtained through direct shear tests. To determine these parameters theoretically, two additional variables, as described by Raithel & Kempfert (2000), must also be calculated as outlined below. Other methodologies for the design of GECs have been proposed, including those by Pulko et al. (2011), Zhang et al. (2011), Castro & Sagaseta (2013) and Zhang & Zhao (2015). The methodology provided by Raithel & Kempfert (2000) was selected for this study because it demonstrated reliable analytical predictions

when compared to the numerical analysis conducted by Riccio et al. (2012). This approach provides the values of $\Delta\sigma_{3,geo}$ and $\sigma_{3,c}$, which are essential inputs for the stability evaluation, using the Raithel & Henne (2000) method.

The results presented in this paper were obtained from laboratory tests conducted using a wide range of values for the stiffness modulus of the columns and two relative densities of the soil (sand), which were used to simulate both the column material and the surrounding soil. Additionally, a computational 3D analysis of the direct shear tests was performed to assess the rotation of the column during the tests. This verification is important because the theoretical model used to analyze stability assumes the column is positioned vertically. The possibility of column rotation in the laboratory reduced scale model arises because its bottom was not fixed to the base of the direct shear cell. The numerical 3D analysis was conducted using the finite element software PLAXIS 3D.

The novelty of this paper lies in the comparison between the increased friction angle (ϕ'_{sub}) obtained from the direct shear test and the same value derived from theoretical calculations. A similar comparison was made for c'_{sub} . To our knowledge, no similar analysis, combining both laboratory and theoretical studies that address this crucial aspect of stability analysis for embankments supported by GECs, can be found in the literature.

2. Stability analysis – 2D model

To perform stability analyses using a 2D model based on limit equilibrium methods, the use of increased shear parameters is recommended. The approach proposed by Raithel & Henne (2000) offers two options: a) use ϕ'_{sub} (Equation 1); b) use c'_{sub} (Equation 2).

$$\sin(\phi'_{sub}) = \frac{\frac{1 + \sin(\phi')}{1 - \sin(\phi')} + \frac{\Delta\sigma_{3,geo}}{\sigma_{3,c}} - 1}{\frac{1 + \sin(\phi')}{1 - \sin(\phi')} - \frac{\Delta\sigma_{3,geo}}{\sigma_{3,c}} + 1} \quad (1)$$

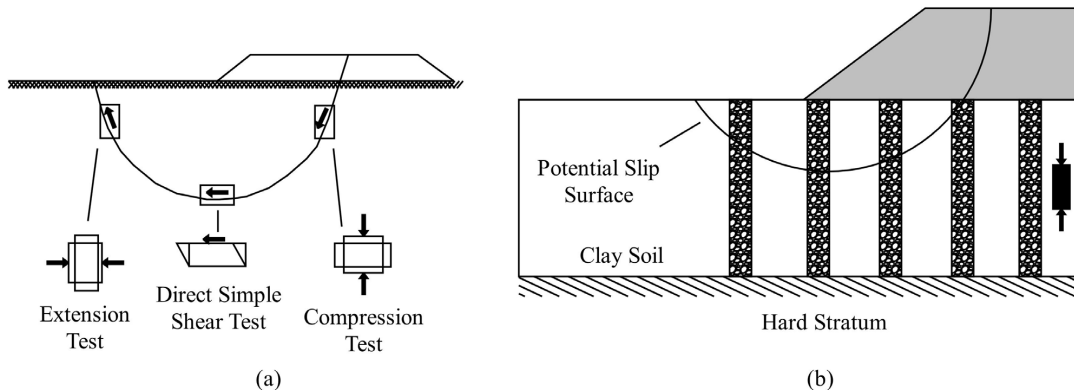


Figure 1. Shear solicitations along the failure surface of embankment on soft ground, (a) ordinary embankment; (b) embankment over GECs.

$$c'_{sub} = \Delta\sigma_{3,geo} \times \frac{1 + \sin(\phi')}{2 \times \cos(\phi')} \quad (2)$$

where: ϕ' = friction angle of the granular material of the column; $\Delta\sigma_{3,geo}$ = confining stress provided by geosynthetic; $\sigma_{3,c}$ = horizontal stress acting on the aggregate. The values of $\Delta\sigma_{3,geo}$ and $\sigma_{3,c}$ are determined using the GEC design method proposed by Raithel & Kempfert (2000).

To perform the stability analysis using a 2D model, it is necessary to transform the original (3D) condition into a 2D (plane strain) geometry, as shown in Figures 1a to 1c.

Tan et al. (2008) presented a simplified equation for this transformation. According to Equation 3, the column is replaced by an equivalent wall, as illustrated in Figure 2c.

$$b_c = B \times \frac{r_c^2}{R^2} \quad (3)$$

where: r_c = radius of the column; B = half-width of the equivalent wall (see Figure 2b); R = radius of the influence area of the column (see Figure 2a).

The procedure for analyzing the stability of the soft soil foundation improved by the GEC column is summarized in Table 1 according to the approach by Raithel & Henne (2000). A practical example of the application of this procedure is provided by Almeida et al. (2018).

3. Laboratory testing program

An extensive laboratory testing program involving direct shear tests was conducted to obtain the shear strength parameters of pure sand (without encasement) and sand reinforced with GEC (reduced scale model of GEC). Figure 3a shows the mesh pattern of the GECs at the instrumented test embankment built at the Thyssenkrupp steelworks located in the city of Itaguaí, in the state of Rio de Janeiro, Brazil (Almeida et al., 2015).

This configuration results in an area ratio $a_E = 12.5\%$, and the mesh used in this study serves as a typical example of the geometry to be reduced. The reduced model tested in the laboratory has the following dimensions (Figure 3b): $\varnothing_A = 6.0$ cm (diameter of the tributary area), $\varnothing_c = 2.12$ cm (diameter of the column = 80 cm / 37.67) and $h_c = 4.17$ cm (height of the GEC column). The relationship between the values of $\varnothing_A$ and $\varnothing_c$ and the corresponding values from the embankment built at Thyssenkrupp steelworks yields an RF (scaling factor or reduction factor) equal to 37.67 . The value of $\varnothing_A$ is derived from the influence area of each column in the field, where the spacing between axes is 2.00 m in a square pattern (2.00 m $\times$ 1.13 = 2.26 m / 37.67 = 0.06 m). To maintain the same value of a_E as in the field, the value of $\varnothing_c$ was set to 2.12 cm. Two materials were used to simulate the geosynthetic encasement: a) plastic film 1 (Figure 3c) and b) plastic film 2 (Figure 3d), both made of low-density polyethylene.

The use of two types of materials for the encasement was necessary to simulate varying stiffness modulus (J) of the geosynthetic. For the same material, the variation in J was achieved by changing the thickness of the encasement. Between one and 18 layers of plastic film were employed to increase the encasement thickness to produce different values of J for the same material. The value of J is given by Equation 4.

$$J = E \times A \quad (4)$$

where: E = encasement elastic modulus; $A = (e \times 1)$ is the encasement cross-sectional area, per meter; e = encasement thickness.

The relationship between J under prototype conditions (J_F) and the value of J for the laboratory model (J_M) is given by Equation 5 applying the scaling factor ($RF = 37.67$) to J_F :

$$J_M = \frac{J_F}{RF^2} \quad (5)$$

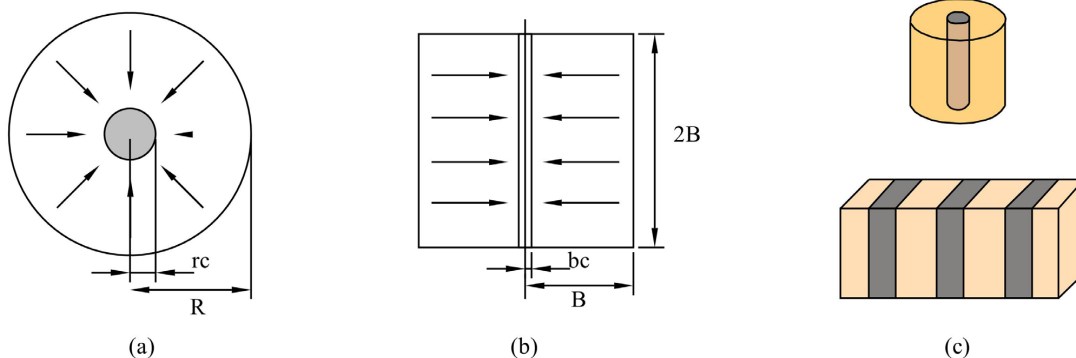


Figure 2. Transformation of the original condition (3D) into a plane-strain condition (2D). (a) and (b) adapted from Tan et al. (2008); (c) transformed condition, adapted from Castro (2017).

Table 2 presents the values of J_M and J_F , considering the material used for encasement and the number of film layers applied to each laboratory column. According to Almeida et al. (2018), the J_F value for commercially available products ranges from 1500 - 6500 kN/m. In Table 2, the encasements made of plastic film 1 replicate J_F values typically used in practical applications, while the encasements made of plastic film 2 reproduce significantly higher J_F values. Tensile stiffness values for prototype geosynthetic reinforcements, on the order of 30,000 kN/m, can be found,

and the encasements with higher values in Table 2 were tested for comparison purposes. Although stiffness moduli (J) greater than 30,000 kN/m are not commercially available, the purpose of using these values is to explore the range of predictions provided by the theoretical method. The main objective is to compare theoretical predictions and measurements, not to design a GEC foundation. The normal stresses applied to the specimens in the direct shear tests were 25 kPa, 50 kPa, 75 kPa and 100 kPa, and the sand was fully saturated ($S=100\%$) in all tests.

Table 1. Summary of parameters used in the stability analysis of GECs for soil improvement.

Type of analysis	Parameter of granular material	Variables	References
I	Use ϕ'_{sub} to replace ϕ . Use $c' = 0$.	$\Delta\sigma_{3,geo}$; $\sigma_{3,c}$; ϕ'	Raithel & Henne (2000) Raithel & Kempfert (2000)
II	Use c'_{sub} and ϕ'	$\Delta\sigma_{3,geo}$; ϕ'	

Note: ϕ'_{sub} is the increased friction angle (due to the geosynthetic) of the granular material in the column; c'_{sub} is the apparent cohesion due to the geosynthetic; ϕ' is the friction angle of the granular material in the GEC; $\Delta\sigma_{3,geo}$ and $\sigma_{3,c}$ are parameters from the Raithel & Kempfert (2000) method.

Table 2. Values of encasement and stiffness modulus under model (J_M) and prototype (J_F) conditions.

Encasement	Layers	Encasement material	t (mm)	E (MPa)	J_M (kN/m)	J_F (kN/m)
1	7	PF 1	0.08	24	1.34	1907
2	18	PF 1	0.08	24	3.46	4904
3	1	PF 2	0.10	200	20.00	28381
4	2	PF 2	0.20	200	40.00	56761
5	4	PF 2	0.40	200	80.00	113522
6	8	PF 2	0.80	200	160.00	227045

Note: PF1 = plastic film 1; PF2 = plastic film 2; E = Young's modulus; t = thickness of the encasement; J_M = stiffness modulus of encasement in the laboratory; J_F = equivalent stiffness modulus of encasement in the field.

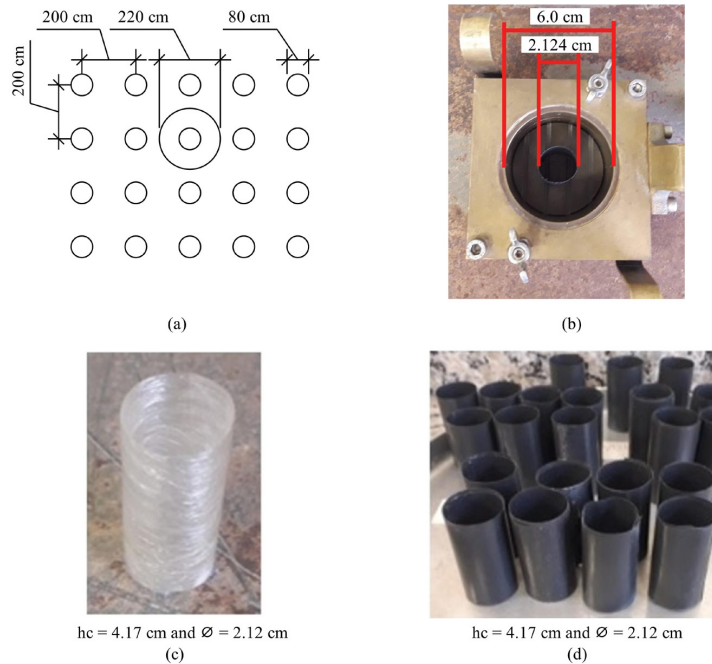


Figure 3. Field mesh of the GEC and reduced model of one unitary cell of GEC: (a) mesh in the test embankment; (b) direct shear cell with column at its center; (c) encasement made of plastic film 1; (d) encasement made of plastic film 2. Obs: h_c = height of the column; $\varnothing_c$ = diameter of the column.

Sand was used to simulate, in a realistic scenario, the compressible soil surrounding the column. This sand was collected at São Francisco beach located in the city of Niterói, state of Rio de Janeiro, Brazil. Table 3 summarizes the properties of this sand.

The specimens were prepared by pluviation, using a device similar to the one employed by Miura & Toki (1982). The grain shape ranged from subrounded to subangular. Compression tests under oedometric conditions were conducted, using direct shear test equipment. Vertical loads were applied to the top cap on the cell (Figure 3b), and the vertical displacements were measured without any movement of the cell. The elastic modulus (E) of the soil was determined using Equation 6, with the Poisson's ratio (ν) set to 0.30 and the oedometric modulus (E_{oed}) determined experimentally. Table 4 summarizes the properties of the São Francisco sand.

$$E = E_{oed} \frac{(1 + \nu)(1 - 2\nu)}{1 - \nu} \quad (6)$$

where: E_{oed} = oedometric modulus; E = elastic modulus; ν = Poisson's ratio.

All the direct shear tests were carried out by imposing a velocity of 0.08 mm/min, in accordance with the recommendations of the ASTM D3080/D3080M (ASTM, 2014) standard. The normal stress values applied to the top cap were 25 kPa, 50 kPa, 100 kPa and 200 kPa.

The maximum horizontal displacement applied to the specimens was 7.0 mm. The laboratory test program included 8 direct shear tests to determine the effective friction angle of the sand without encasement (ϕ'). In addition, 24 direct shear tests were conducted on sand with GEC to determine the increased effective friction angle (ϕ'_{sub}) and the increased apparent cohesion (c'_{sub}). Moreover, 12 direct shear tests were carried out to determine the friction angle between the interfaces sand-plastic film 1, sand-plastic film 2, and sand-steel box. The results from these interface friction angles were used in the numerical analysis and can be found in Table 4, in Section 4.

4. Numerical analysis – 3D program

Some of the direct shear tests carried out in the laboratory were simulated using numerical modeling. The software PLAXIS 3D was used to perform these simulations, using the finite element method. The primary goal of the analysis was to verify and quantify the rotation of the GEC column inside the cell (Figure 3b) while shear stress was applied. Verifying rotation is crucial to the analysis, as in the field, the column is embedded approximately 1.0 m within a firm stratum, where no rotation occurs. In the direct shear tests carried out, the base of the column was not fixed to the base of the cell. This practice of not fixing the base was also employed in other studies such as Murugesan & Rajagopal (2009) and Mohapatra et al. (2016).

Table 3. Properties of the São Francisco sand.

Properties	Value
Shape of grains	**Subrounded to subangular
*Grain size smaller than 0.06 mm (%)	0.1
*Grain size of fine sand (0.06 - 0.20 mm) (%)	95.15
*Grain size of medium sand (0.20 - 0.60 mm) (%)	4.75
C_u [-]	1.7
C_c [-]	0.9
USCS (Unified Soil Classification System)	SP (sand poor)
G_s [-]	2.65
e_{max} [-]	0.86
e_{min} [-]	0.53
RD (%)	40 and 100

Note: C_u = coefficient of uniformity; C_c = coefficient of curvature; G_s = specific gravity; e_{max} = maximum void ratio; e_{min} = minimum void ratio; RD = relative density.
*according to the Brazilian standard NBR 7181 (ABNT, 2016); **according to Oliveira Filho (1987).

Table 4. Values of R_{inter} for different interfaces of sand-column-box.

Sand	R_{inter} (sand-plastic film 1)	R_{inter} (sand-plastic film 2)	R_{inter} (sand-steel box)
$RD = 40\%$	0.632	0.778	0.671
$RD = 100\%$	0.545	0.629	0.563

Note: E_{oed} was determined for a vertical load stress equal to 53.2 kPa; E_{oed} = oedometric modulus; ν = coefficient of Poisson; E = elastic modulus; γ_d = dry unit weight; γ_s = unit weight of grains; γ_{sat} = saturated unit weight; c' = effective cohesion; ϕ' = effective friction angle; Ψ = angle of dilation; e_o = initial void ratio; k_x , k_y , k_z = coefficients of permeability in the X, Y and Z directions, respectively.

For numerical modeling, a mesh consisting of tetrahedral elements with 10 nodes was used. Mesh refinement was set to medium, since using a very fine mesh would require 7 days of computational time, while a medium mesh required only 24 hours. As demonstrated in the following section (Section 5.2), the use of a medium mesh provided highly satisfactory results, in terms of shear stress versus horizontal displacement, leading to close agreement between laboratory and numerical results. The mesh of the numerical model was composed of 36,519 elements and 70,745 nodes. To account for the friction angle of interface between the soil and the encasement, as well as the friction angle of the interface between soil and the steel box of the direct shear cell, 12 direct shear tests were conducted (4 tests for each interface). Thus, the parameter R_{inter} considered in PLAXIS 3D was determined using Equation 7 and is presented in Table 4.

$$R_{inter} = \frac{\tan \phi'_{int}}{\tan \phi'} \quad (7)$$

where: ϕ' = effective friction angle of the soil; ϕ'_{int} = effective friction angle of the interface.

The adopted values of R_{inter} are derived from the direct shear tests carried out on the sand plastic film 1, sand plastic film 2 and sand-steel-box interfaces. The material properties and constitutive models used in the numerical analysis are summarized in Table 5.

Figures 4a and 4b show the geometry of the model implemented in the PLAXIS 3D software, while Figures 4c and 4d depict two surfaces prepared for direct shear tests to determine the friction angle ϕ'_{int} . The velocity of horizontal displacement applied to the lower box was the same as that used in the laboratory tests, as was the maximum horizontal displacement imposed during tests. In total, eight numerical simulations were conducted to assess the influence of certain parameters on the deformed shape of the encasement. Generally, the shape of the encasement is associated with a horizontal displacement of 7.0 mm. However, in some cases, the analysis was halted before reaching 7.0 mm due to excessive plastic deformation points in the column and sand. The parameters considered for this purpose included the stiffness modulus of the encasement in the laboratory (1.34 kN/m or 20.0 kN/m), the normal stress applied to the specimen (25 kPa or 100 kPa), and the relative density of the sand (40% or 100%). Table 6 provides a summary of the numerical analyses, which also included the

Table 5. Material properties, shear box apparatus, and constitutive models used.

Parameter	Sand ($RD = 40\%$)	Sand ($RD = 100\%$)	Steel box	Top cap
Constitutive model	Elastic-plastic	Elastic-plastic	Linear elastic	Linear elastic
E_{oed} (kPa)	23034	30875	-	-
ν [-]	0.30	0.30	0.20	0.20
E (kPa)	17110	22940	$120 \cdot 10^6$	$70 \cdot 10^6$
γ_d (gf/cm ³)	1.42	1.69	8.7	2.7
γ_{sat} (gf/cm ³)	1.88	2.05	-	-
γ_s (gf/cm ³)	2.65	2.65	-	-
c' (kPa)	0	0	-	-
ϕ' (°)	36.8	47.2	-	-
Ψ (°)	1.38	16.3	-	-
e_o [-]	0.86	0.57	-	-
k_x (cm/s)	6000	6000	0	0
k_y (cm/s)	6000	6000	0	0
k_z (cm/s)	6000	6000	0	0

Table 6. Characteristics of the models used to simulate the direct shear tests carried out in the laboratory.

Model	J_M (kN/m)	Material of encasement	Number of layers	Normal stress (kPa)	Sand RD (%)
J20N1S25D40	20.00	Plastic film 2	1	25	40
J20N1S100D40	20.00	Plastic film 2	1	100	40
J20N1S25D100	20.00	Plastic film 2	1	25	100
J20N1S100D100	20.00	Plastic film 2	1	100	100
J1.34N7S25D40	1.34	Plastic film 1	7	25	040
J1.34N7S100D40	1.34	Plastic film 1	7	100	40
J1.34N7S25D100	1.34	Plastic film 1	7	25	100
J1.34N7S100D100	1.34	Plastic film 1	7	100	100

Note: J = stiffness modulus; N = number of layers of material used to build the column; S = normal stress applied to the specimen; RD = relative density of the sand.

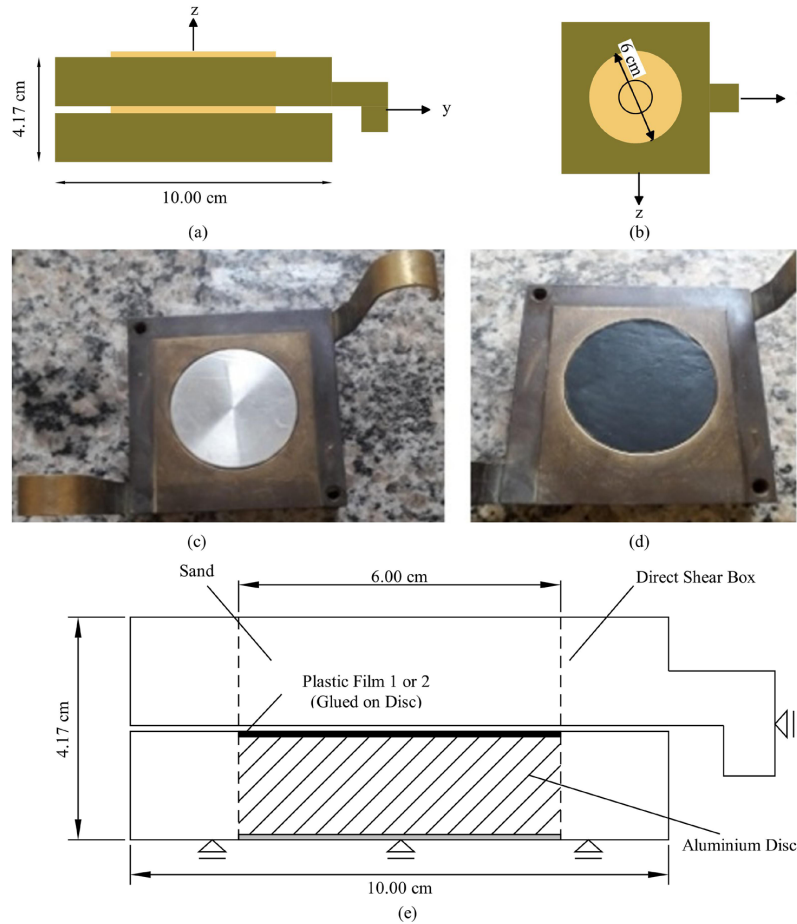


Figure 4. Geometry of the numerical model implemented in PLAXIS 3D; (a) cross-sectional view; (b) plan view; (c) surface for sand-plastic film 1 interface test; (d) surface for sand-plastic film 2 interface test; (e) boundary conditions and characteristics of the tests for sand-plastic film interfaces.

Table 7. Stages of the numerical simulation.

Stage	Horizontal displacement (mm)	Accumulated horizontal displacement (mm)	Applied normal stress (kPa)	Observation
1	0.0	0.0	25 or 100	Load application
2	1.0	1.0	25 or 100	Displacement
3	1.0	2.0	25 or 100	Displacement
4	1.0	3.0	25 or 100	Displacement
5	1.0	4.0	25 or 100	Displacement
6	1.0	5.0	25 or 100	Displacement
7	1.0	6.0	25 or 100	Displacement
8	1.0	7.0	25 or 100	Displacement

determination of the vertical strain (ϵ) at the top of the column and the angular distortion (θ) at the end of each analysis. The exhumation of the encasements after the direct shear tests did not allow for conclusions about their rotation. This was due to the specimens being dismantled at the end of test in order to analyze the deformed shape of the encasement.

Table 6 presents the characteristics of the 3D numerical analyses performed to simulate all eight direct shear tests

carried out in the laboratory, allowing the quantification of the angular distortion (θ) of the column.

Table 7 presents the stages involved in the numerical simulation and the characteristics of each stage. The first stage was used solely to apply the load to the specimen and impose the total horizontal displacement (0.7 cm). The subsequent stages were used to impose the displacement in steps of 0.1 cm each.

5. Results and discussion

In this section, the results of both routines (laboratory tests and numerical simulations) are presented and discussed. A comparison between the results of laboratory tests (c' and ϕ') and analytical predictions (ϕ'_{sub} and c'_{sub}) is made, based on the equations provided by Raithel & Kempfert (2000) and Raithel & Henne (2000).

5.1 Laboratory tests and analytical predictions

The comparison of c' and ϕ' (from the tests) with c'_{sub} and ϕ'_{sub} (obtained through analytical formulations) is presented below. The results of apparent cohesion and friction angle obtained from direct shear tests, and calculated using Raithel & Kempfert (2000) and Raithel & Henne (2000), are provided in this section. A summary of the direct shear tests conducted is presented in Table 2, which includes scenarios both without encasement ($J_M = 0$) and with different encasements (see Table 2 for J_M and J_F values). Additionally, the analysis considered both conditions regarding the relative density of the sand ($RD=40\%$ and $RD=100\%$) and the plastic film 1 and plastic film 2 materials. All the results presented pertain to the peak condition.

According to Equations 1 and 2, it is necessary to determine the values of $\sigma_{3,c}$ and $\Delta\sigma_{3,geo}$ to calculate ϕ'_{sub} , as well as the value of $\Delta\sigma_{3,geo}$ to calculate c'_{sub} . The parameters $\sigma_{3,c}$ and $\Delta\sigma_{3,geo}$ were calculated using the equations provided by Raithel & Kempfert (2000), considering the values summarized in Table 8.

Figure 5 and Figure 6 show the results for the friction angle and apparent cohesion, respectively, considering $RD=40\%$. In Figure 5, it is possible to observe the values of increased peak friction angle (ϕ'_{sub}) obtained from tests and compare them with the increased peak friction angle (ϕ'_{sub}) obtained from analytical methods (Raithel & Kempfert, 2000 and Raithel & Henne, 2000). A good agreement can be seen in these results, particularly for the values of J_F equal to 1.907 kN/m, 4.904 kN/m and 28.381 kN/m. The values

of 1.907 kN/m and 4.904 kN/m fall within the range of commercially available geosynthetics. This behavior was consistent across all normal stress applications (25 - 200 kPa). The increase in the value of ϕ'_{sub} due to the increase in the stiffness J_F is the most commonly observed behavior.

Figure 6, focusing on c'_{sub} and $RD=40\%$, shows good agreement between laboratory tests and theoretical predictions only when the normal stress is low (25 kPa) for all stiffness modulus of the geosynthetic (J_F). However, for the other normal stresses (50 kPa, 100 kPa and 200 kPa) the good agreement is observed only when J_F equals 1.907 kN/m and 4.904 kN/m, that is, a value similar to typically available in the market. For these cases, the values of c'_{sub} are practically null, indicating that no significant improvement to the soil was achieved. For normal stresses equal to 50 kPa, 100 kPa and 200 kPa, the theory predicts values of c'_{sub} higher than those seen in the tests. Consequently, the theoretical approach yields non-conservative values of c'_{sub} in these cases. Therefore, it is advisable to avoid using the c'_{sub} model for these cases.

Figures 7 and 8 show the results for the apparent cohesion and friction angle, respectively, corresponding to the condition with $RD=100\%$.

Figure 7 demonstrates a good agreement between tests and theoretical predictions for normal stresses equal to or greater than 50 kPa, particularly for J_F values up to 56,761 kN/m. For low normal stress (25 kPa), however, the values of ϕ'_{sub} obtained from theoretical model are conservative up to $J_F = 56,761$ kN/m (as shown in Table 2).

Figure 8 indicates that the values obtained from theoretical model for c'_{sub} are conservative up to a normal stress of 50 kPa. For normal stresses exceeding 50 kPa (specifically 100 kPa and 200 kPa) the values of c'_{sub} do not exhibit good agreement, remaining conservative for J_F values equal to 1.907 kN/m and 4.904 kN/m. At a normal stress of 100 kPa, the values of c'_{sub} are non-conservative for $J_F = 113.522$ kN/m and $J_F = 227.045$ kN/m. Similarly, for a normal stress equal to 200 kPa, the values of c'_{sub} are non-conservative for J_F greater than 113.522 kN/m.

Table 8. Input data for the calculation of $\sigma_{3,c}$ and $\Delta\sigma_{3,geo}$.

Variable	Sand ($RD=40\%$)	Sand ($RD=100\%$)
a_E (%)	0.125	0.125
h (cm)	4.17	4.17
$r_c = r_{geo}$ (m)	0.01062	0.01062
J_M (kN/m)	1.34/3.46/20.00/40.00/80.00/160.00	1.34/3.46/20.00/40.00/80.00/160.00
ϕ'_s (°)	36.8 to 36.3	48.2 to 43.5
c'_s (kPa)	0.0	0.0
$E_{oed,s,ref}$ (kPa)	23034	30875
γ (gf/cm ³)	1.42	1.69
p_{ref} (kPa)	53.2	53.2
m^* [-]	0.55	0.69
ν [-]	0.30	0.30

Note: a_E = area ratio; h = height of column; $r_c = r_{geo}$ = radius of column and encasement; J_M = stiffness modulus of encasement (model); ϕ'_s = effective secant friction angle of the soil; c'_s = cohesion; $E_{oed,s,ref}$ = reference oedometric modulus of soil; γ = apparent unit weight of sand; p_{ref} = reference stress; m = modulus exponent; and ν_s = Coefficient of Poisson of the soil. *Modulus exponent obtained from oedometric tests.

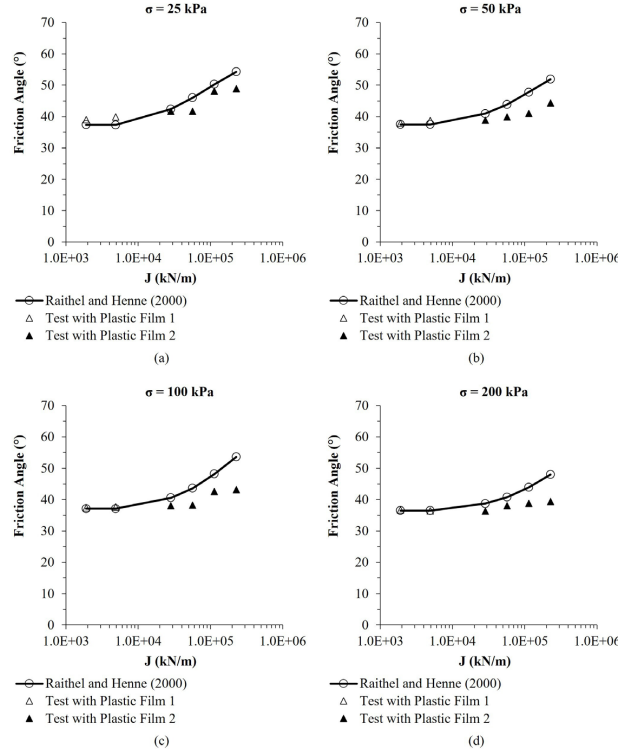


Figure 5. Comparison between test results (ϕ') and theoretical predictions (ϕ'_{sub}) for the friction angle, $RD = 40\%$, peak condition and field-equivalent stiffness modulus, J_F : (a) $\sigma'_v = 25$ kPa; (b) $\sigma'_v = 50$ kPa; (c) $\sigma'_v = 100$ kPa and (d) $\sigma'_v = 200$ kPa.

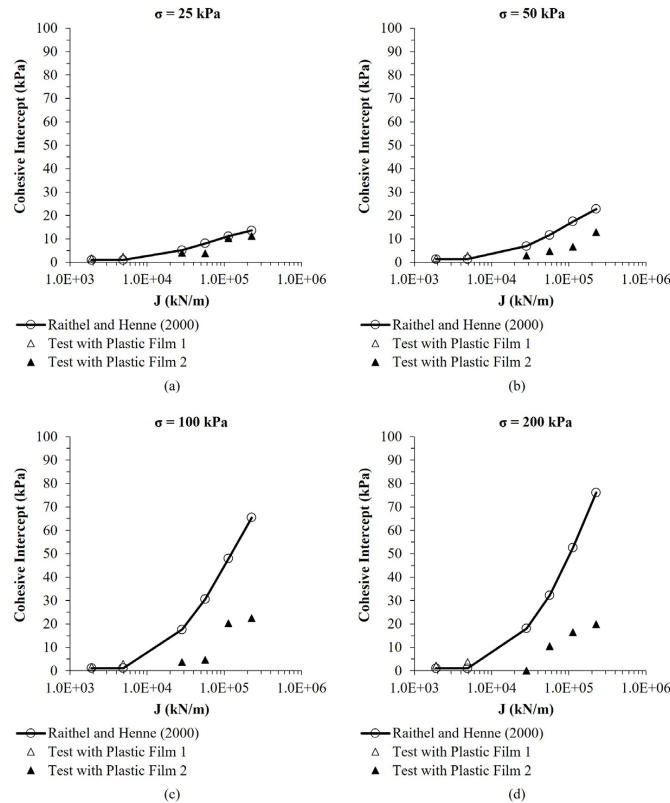


Figure 6. Comparison between test results (c') and theoretical predictions (c'_{sub}) for apparent cohesion, $RD = 40\%$, peak condition and field-equivalent stiffness modulus, J_F : (a) $\sigma'_v = 25$ kPa; (b) $\sigma'_v = 50$ kPa; (c) $\sigma'_v = 100$ kPa and (d) $\sigma'_v = 200$ kPa.

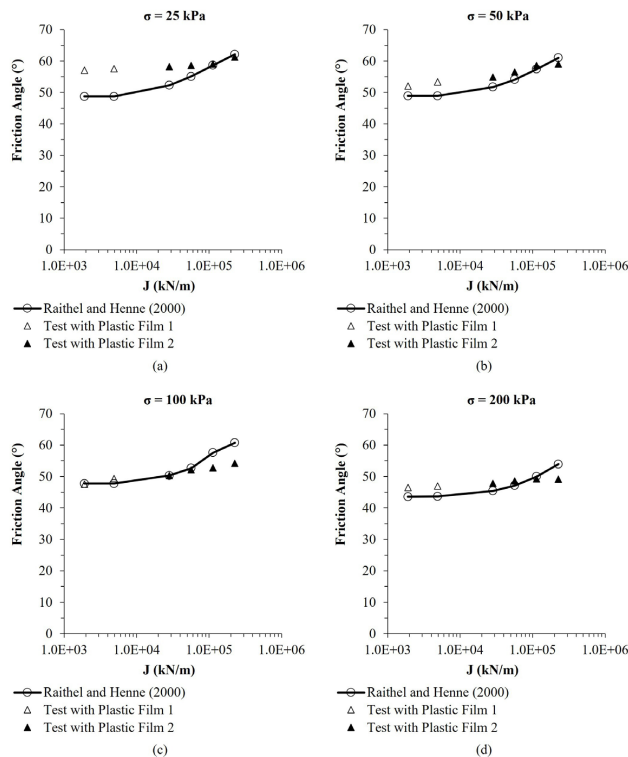


Figure 7. Comparison between test results (ϕ') and theoretical predictions (ϕ'_{sub}) for friction angle, $RD = 100\%$, peak condition and field-equivalent stiffness modulus J_F : (a) $\sigma'_v = 25$ kPa; (b) $\sigma'_v = 50$ kPa; (c) $\sigma'_v = 100$ kPa and (d) $\sigma'_v = 200$ kPa.

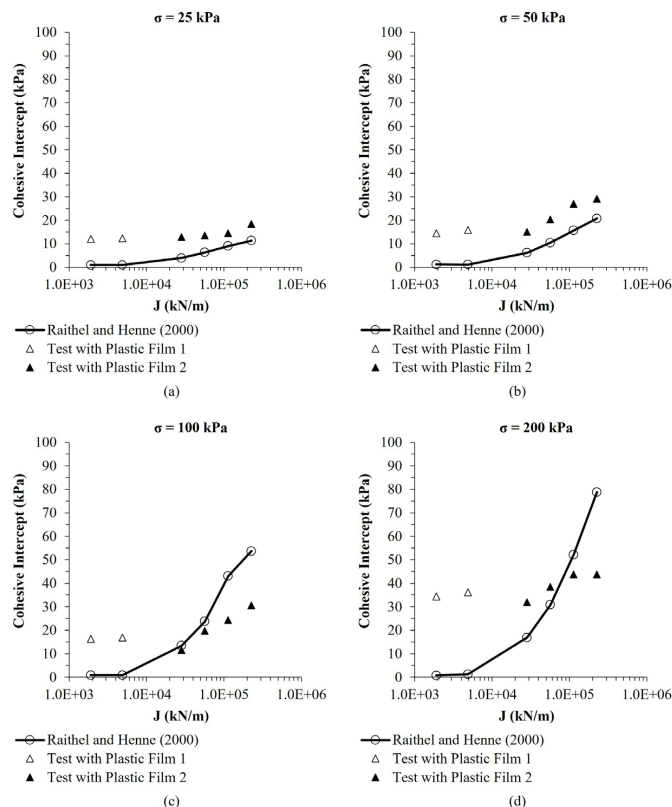


Figure 8. Comparison between test results (c') and theoretical predictions (c'_{sub}) for apparent cohesion, $RD = 100\%$, peak condition and field-equivalent stiffness modulus J_F : (a) $\sigma'_v = 25$ kPa; (b) $\sigma'_v = 50$ kPa; (c) $\sigma'_v = 100$ kPa and (d) $\sigma'_v = 200$ kPa.

Based on the results in Figures 5 to 8, ϕ'_{sub} is the more suitable approach for calculating the stability of foundations improved with GECs, as c'_{sub} provides non-conservative results (indicating potential safety concerns) for normal stresses equivalent to the height of the embankment, ranging from 50 kPa to 200 kPa, when $RD = 40\%$. For $RD = 100\%$, c'_{sub} is not recommended for normal stress exceeding 50 kPa and J_F values greater than 113.552 kN/m. It is important to note that these analyses are theoretical, as such high J_F values are rarely encountered in practical geosynthetic encasement applications.

The primary objective of this research is to evaluate the applicability of the theoretical approach proposed by Raithel & Henne (2000). To achieve this, geosynthetic encasements were chosen to cover a broad range of stiffness modulus, from 1.907 kN/m to 227.045 kN/m. Therefore, the tests were carried considering values of J_F found in the practical engineering and values of J_F higher than these ones. The tests allowed an assessment of how well the theoretical approach aligns with measured and calculated results. Consequently, the J_F values encompass those commonly employed in practical applications, as well as values that may not be frequently encountered.

5.2 Numerical analyses

This section presents the results of the numerical analysis concerning the angular distortion of the encasements, as outlined in Table 6. Figure 9 provides a visual representation of the angular distortion measurement used to assess the significance of encasement rotation. In Figure 9a, the distortion is defined as the arctangent of L/H , while in Figure 9b, the maximum specific axial deformation (ε) is calculated as $(h/2)/H$. The axial deformation (ε) corresponds to the specific deformation observed at the top and center of the GEC.

Table 9 presents results in terms of the deformed shape of the encasement, angular distortion (θ) and specific axial deformation (ε) for horizontal displacements of 1.0 mm, 2.0 mm, 3.0 mm, 4.0 mm, 5.0 mm and 7.0 mm. These values correspond to the moment when the peak shear strength was achieved during the direct shear tests.

The values of angular distortion provided in Table 9 ranges from 1.36° to 9.26° , indicating a relatively low degree of angular distortion. In these cases, the assumption of vertical alignment of the encasement was evaluated in most analyses. It is important to note that the base of the encasement remained in its original position, i.e., the position prior to the shearing process. The deformation of the encasement occurred primarily at the upper part of the reinforcement, while the lower part remained securely attached below the shear plane. This observed behavior justifies the decision not to fasten the base of encasement, as recommended by Mohapatra et al. (2016). This decision simplifies the operational process of the test. In real field conditions, this implies that the significant deformations of the GECs will occur just above the potential slip surface (as shown in Figure 1b).

The deformed shapes presented in Table 9 exhibit deformation patterns similar to those reported by Mohapatra et al. (2016), characterized as Type-2 failures, in which the GEC does not shear until failure, and no excessive bending is observed. The significance of this is that GECs with $J_F \geq 1907$ kN/m ($J_M \geq 1.34$ kN/m) typically do not fail under field conditions, even when there are high strains leading to geosynthetic rupture.

The effectiveness and reliability of the results presented in Table 9 can be verified by inspecting the results shown in Figure 10. In this figure, shear stress and horizontal displacements are compared between results obtained from laboratory tests and numerical analyses conducted using PLAXIS 3D. Based on these results, shown in Figure 10, the numerical model is considered to accurately represent the shear stresses as a function of horizontal displacements.

The good agreement regarding shear stress is verified at the peak condition. Regarding the maximum specific axial deformation (ε), the results presented in the Table 9 show that the encasement does not rotate significantly at the base of the shear box, and the vertical deformation remains minimal. Based on these results, it can be stated that fixing the encasement at the bottom of the shear box was not necessary for the test conditions used in this study.

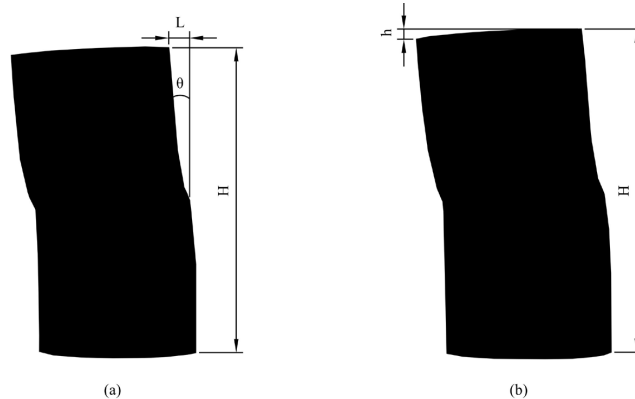
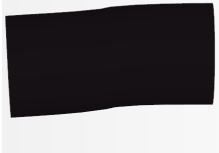
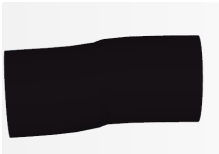
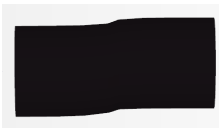


Figure 9. Definition of θ and ε for the deformed encasement at the end of numerical analysis; (a) distortion (θ); (b) specific axial deformation (ε).

Table 9. Deformed shape of encasement, angular distortion and specific axial deformation (ε).

Model	J20N1S25D40	J20N1S100D40	J20N1S25D100	J20N1S100D100	J1.34N7S25D40	J1.34N7S100D40	J1.34N7S25D100	JN7S100D100
Deformed shape of encasement								
Horizontal displacement (mm)	3.0	4.0	7.0	7.0	1.0	5.0	2.0	5.0
Angular distortion (°)	4.46	5.59	9.26	9.23	1.36	6.81	2.73	6.78
Specific axial deformation (%)	0.61	0.82	0.98	1.01	0.10	0.30	0.60	1.36
Relative density of sand – RD (%)	40	40	100	100	40	40	100	100
Material of the encasement	Plastic film 2	Plastic film 2	Plastic film 2	Plastic film 2	Plastic film 1	Plastic film 1	Plastic film 1	Plastic film 1
J_M (kN/m)	20	20	20	20	1.34	1.34	1.34	1.34

Note: J_M is the stiffness modulus of the laboratory model; Simulation codes are as per Table 6; J = stiffness modulus; N = number of layers of material used to build the column; S = normal stress applied to the specimen; RD = relative density of the sand.

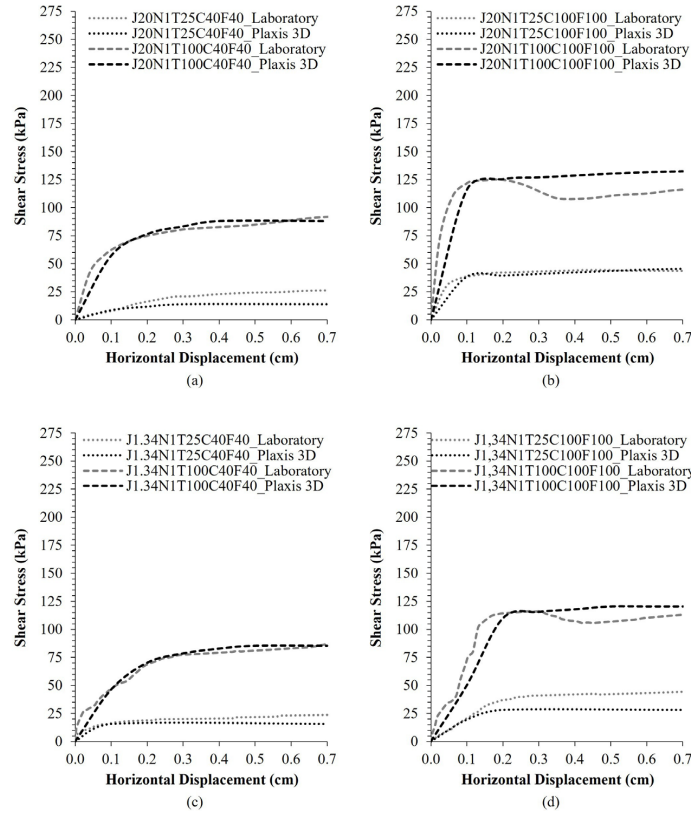


Figure 10. Comparison of shear stress curves obtained from laboratory and numerical analyses (PLAXIS 3D): (a) $J_M = 20$ kN/m and $RD = 40\%$; (b) $J_M = 20$ kN/m and $RD = 100\%$; (c) $J_M = 1.34$ kN/m and $RD = 40\%$; (d) $J_M = 1.34$ kN/m and $RD = 100\%$.

6. Conclusions

Laboratory shear tests were carried out on sand with and without encasement to assess the increase in shear strength parameters (friction angle and apparent cohesion). Additionally, the theoretical predictions of increased friction angle and apparent cohesion when reinforcing the sand with encasement were examined. Two types of materials for encasement were employed: plastic film 1 and plastic film 2. Based on the findings from experimental, numerical and analytical studies, the following conclusions are presented:

- The use of plastic film 1 and plastic film 2 proves itself effective in simulating the shear stresses on the GECs using a reduced model, compatible with the direct shear test apparatus. By employing these two materials, it was possible to simulate encasement stiffness values across a broad range (encompassing both commercial and non-commercial values);
- In general, the higher the encasement stiffness, the higher the shear strength parameters, i.e., the increased friction angle (ϕ'_{sub}) and the apparent cohesion (c'_{sub}). A similar trend was observed for the sand density, where higher density corresponded to higher strength parameters (c'_{sub} and ϕ'_{sub});

- The comparison between the increased friction angle (ϕ'_{sub}) and apparent cohesion (c'_{sub}) obtained through laboratory tests and based on the approach of Raithel & Henne (2000) shows good agreement for ϕ'_{sub} . However, for the c'_{sub} , the theoretical predictions are either conservative or unsafe, depending on factors such as applied normal stress, sand density, and encasement stiffness. For stability analyses of soft soil foundations improved with GECs, it is advisable to rely on ϕ'_{sub} rather than c'_{sub} ;
- Direct shear strength tests were also simulated in PLAXIS 3D software, considering both plastic film 1 and plastic film 2 as encasements. Based on the numerical modeling, it was concluded that the encasement does not rotate significantly at the base of the shear box, and its fixation was deemed unnecessary for the test conditions. Furthermore, the analysis of angular distortion indicates a slight leaning of the encasement in the direction opposite to the shear box movement;

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Declaration of interest

The authors declare that there are no conflicts of interest to declare. All co-authors have reviewed and affirmed the content of the paper and there are no financial interests to report.

Authors' contributions

Mario Vicente Riccio Filho: conceptualization, formal analysis, writing – review e editing. Sabrina Medeiros Penasso: investigation, data curation, formal analysis. Luiz Fernando Nicodemus Rodrigues: investigation, data curation, formal analysis. Ennio Marques Palmeira: conceptualization, formal analysis.

Data availability

The datasets generated and analyzed during the study are available from the corresponding author upon request.

Use of generative artificial intelligence for textual, visual, and data enhancement

This work was prepared without the assistance of any generative artificial intelligence (GenAI) tools or services. All aspects of the manuscript were developed solely by the authors, who take full responsibility for the content of this publication.

List of symbols and abbreviations

1g	Reduced model
2D	Two dimensions
3D	Three dimensions
a_E	Area ratio
b_c	Diameter of the column
c'	Effective cohesion
c'_{sub}	Apparent cohesion
e_0	Initial void ratio
h_c	Height of the GEC column
k_x	Coefficient of permeability in the X direction
k_y	Coefficient of permeability in the Y direction
k_z	Coefficient of permeability in the Z direction
r_c	Radius of the column
t	Thickness of the encasement
A	Cross-sectional area
B	Half width of the equivalent wall
Cc	Coefficient of curvature
Cu	Coefficient of uniformity
E	Elastic modulus
E_{oed}	Oedometric modulus
GEC	Granular encased column
G_s	Specific gravity
J	Stiffness modulus

J_F	Equivalent stiffness modulus of encasement in the field
J_M	Stiffness modulus for the laboratory reduced model
N	Number of columns
PF1	Plastic film 1
PF2	Plastic film 2
R	Radius of the influence area of the column
RD	Relative density
RF	Scaling factor (or reduction factor)
R_{inter}	Relationship between the angle of friction of the soil and the interface material
S	Saturation
S_u	Undrained strength
SP	Poorly graded sand
T	Normal stress applied in the test
USCS	Unified Soil Classification System
γ_d	Dry unit weight
γ_s	Unit weight of grains
γ_{sat}	Saturated unit weight
ε	Vertical strain
θ	Angular distortion
ν	Poisson's ratio
$\sigma_{3,c}$	Horizontal stress acting on the column
ϕ	Friction angle of the granular material of the column
ϕ_{int}	Effective friction angle of the interface
ϕ_{sub}	Increased friction angle of the soil
$\Delta\sigma_{3,geo}$	Confining stress provided by geosynthetic
$\emptyset_A$	Diameter of influence area
$\emptyset_C$	Diameter of the column
Ψ	Angle of dilatancy

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