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Principle of pole point method of Mohr's circle of strain and its applications in geotechnical engineering

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Article

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Mohr's strain circle
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Two-dimensional stress-strain problems
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Abstract
Mohr's circle is a convenient geometric method to solve two-dimensional stress and strain problems in geotechnical engineering and materials engineering. The pole point is such a special point that can help readily find stresses and strains on any specified plane by using a diagram instead of complex computations. This paper first presents two conventional pole point methods of a Mohr's strain circle, i.e., the parallel line method and the normal line method. A new method of determining the pole point of strain, called the ray method, is then proposed. It was found that the parallel line method and the normal line method are two special cases of the ray method; however, the parallel line method was proved the most efficient way to determine the strains for a specified plane. The uniqueness of the pole point was proved by using an indirect proof; and the pole point method was verified by a theoretical method. Results also show that Mohr's strain and stress circles can be drawn in a concentric circle. Based on the relationship between the pole points of stress and strain, any relatively complex stress and strain states can be determined by using the pole point method rather than using the theoretical method.

1. Introduction

Mohr's law is very useful to tackle stress and strain problems in materials engineering. The German bridge engineer Karl Culmann (1821-1881) is the first to put forward the graphical means of representing stresses (Timoshenko, 1983). He introduced the stress circle in considering longitudinal and vertical stresses in horizontal beams during bending, and it is a precursor and a particular case of Mohr's circle analysis. Some details on constructing Culmann's circle of stress are referred to Timoshenko (1983) and Allison (1984). In 1882, German engineer Chrisitan Otto Mohr extended the stress circle to the three-dimensional case and proposed the strength criterion of the stress circle (Allison, 1984). The circle is called Mohr's circle of plane stress worldwide. In addition, Culmann constructed a point on the Mohr's circle for a specific element from which draw a straight line parallel to an arbitrary plane on the element. The coordinates of the intersection of the parallel line with the Mohr's circle represent the stresses acting on the plane (Cutler & Elliott, 1983). The point is now called the pole point. It has been proved that the pole point method can easily obtain the stresses of a soil element instead of using complex formulae (Li et al., 2013).

In addition, it has been proved that Mohr's circle diagram and the pole point method are widely used to solve two-dimensional and three-dimensional stress and strain problems in geotechnical engineering (Means, 1983; Lisle & Ragan, 1988; Lisle & Robinson, 1995; Treagus, 1995; Bui et al., 2014). Many soil mechanics books illustrate that the pole point for Mohr's stress circle can be obtained by using the parallel line method (Terzaghi, 1943; Lambe & Whitman, 1969; Budhu, 2011; Das, 2010; Holtz & Kovacs, 1981).

A pole is a unique point located on the circumference of Mohr's circle. If a line is drawn through the pole parallel to a given plane, the point of intersection of this line and Mohr's circle will give the stresses on the plane. The procedure for finding the pole is shown in Figure 1. Point *B* on Mohr's circle represents the stress conditions on plane *AB* (Figure 1a). If a line is drawn through *B* parallel to *AB*, it will intersect Mohr's circle at *P*, point *P* is the pole for Mohr's circle. We could also have found pole *P* by drawing a line through *C* parallel to plane *AC*. To find the stresses on plane *BC*, we draw a line through *P* parallel to *BC*, it will intersect Mohr's circle at *F*, and the coordinates of point *F* will give the normal and shear stresses on plane *AB*.

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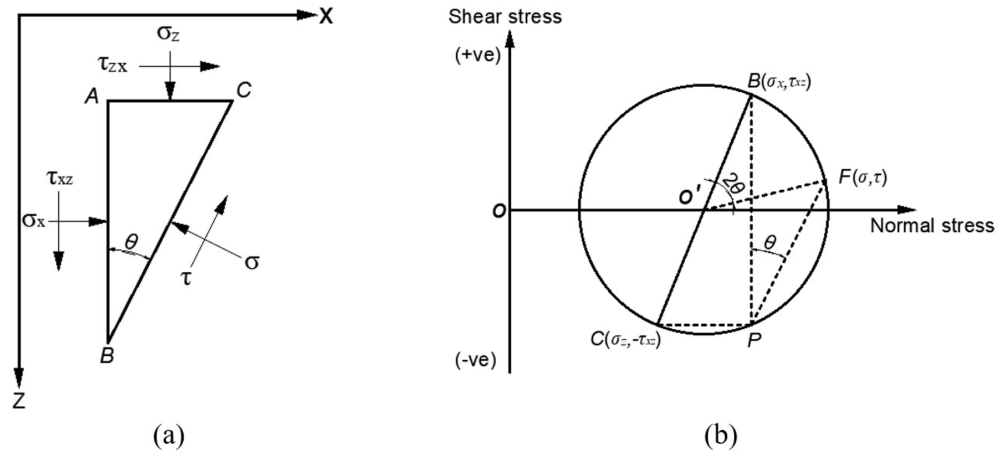


Figure 1. Pole method of finding stresses on an inclined plane (a) stress element, and (b) corresponding Mohr's circle (Das, 2010).

Li et al. (2013) verified the uniqueness of the pole point for Mohr's circle of stress by using an indirect proof and pointed out that the positive shear stress should conform to a counterclockwise rotation for an infinitesimal soil element. However, the pole point for Mohr's circle of strain has not been addressed in the aforementioned literature. By now, for a soil element, the most popular method in obtaining the strain state of an arbitrary plane is the rotation method (Gere & Goodno, 2009; Hearn, 1997; Hibbeler, 2010; Beer et al., 2012).

This paper first presents two methods that are the parallel line method and the normal line method to determine the pole point for strain. The ray method is then proposed to obtain the pole point of the Mohr's Circle of strain. In addition, the uniqueness of the pole points obtained by the three methods is verified. Based on the relationship between pole points of stress and strain, the advantage of the pole point method in obtaining the complex stress and strain states of a soil element is shown by using an example.

2. Pole point methods for Mohr's strain circle

Considering a soil element subjected to the two-dimensional stresses (Figure 2a), we should first define a sign convention for the normal and shear strains to determine the pole point of the Mohr's strain circle. As shown in Figure 2b, a positive value for a normal strain component indicates compression in the corresponding direction, otherwise a negative value for tension. In addition, the shear strain is considered positive if the interior angle diminishes under shear stresses.

2.1 The parallel line method

The following steps are required to obtain the pole point of the Mohr's strain circle by using the parallel line method:

1. Establish a coordinate system. The abscissa and ordinate represent the normal strain ε and half the value of the shear strain, γ/2, respectively (Figure 2c).
2. Plot two reference points *C* (ε_x, $\gamma_{xy}/2$) and *D* (ε_y, $-\gamma_{xy}/2$) in Figure 2a where ε_x and $\gamma_{xy}/2$ represent the normal and shear strains of plane *ad*, respectively and ε_y and $-\gamma_{xy}/2$ represent the normal and shear strains of plane *cd*. Then draw a Mohr's strain circle using line *CD* as its diameter.
3. Draw a line from point *C* parallel to *ad* (Figure 2c). The point where the parallel line intersects the Mohr's circle is the pole point *P*. In addition, the intersection of the Mohr's circle with the line parallel to line *cd* from point *D* can also establish the same pole point *P*.

To obtain the strain state of a specific plane *de* (Figure 2b), a line parallel to plane *de* from the pole point *P* is drawn (Figure 2c). The intersection of the line with the Mohr's strain circle is point *E*, and the coordinates of point *E* denote the normal and shear strains of plane *de*.

2.2 The normal line method

The following steps are required to obtain the pole point of the Mohr's strain circle by using the normal line method:

1. As shown in Figure 2c, a straight line from point *C* is pictured parallel to the normal line of plane *ad* (denoted as n_1 in Figure 2b), and intersection of the line with the Mohr's circle is the pole point *P'*.
2. To obtain the normal and shear strains of plane *de*, a line is plotted from point *P*' parallel to the normal line of plane *de* (denoted as n_2 in Figure 2b). The parallel line intersects the Mohr's strain circle at point *E*, and its coordinates represent the strain state of plane *de*.

Since line *PC* (Figure 2c) and plane *ad* (Figure 2a) are parallel, line *PC* is perpendicular to the line *P'C*, indicating that the inscribed angle of arc *PP'* equals 90°. Therefore, the

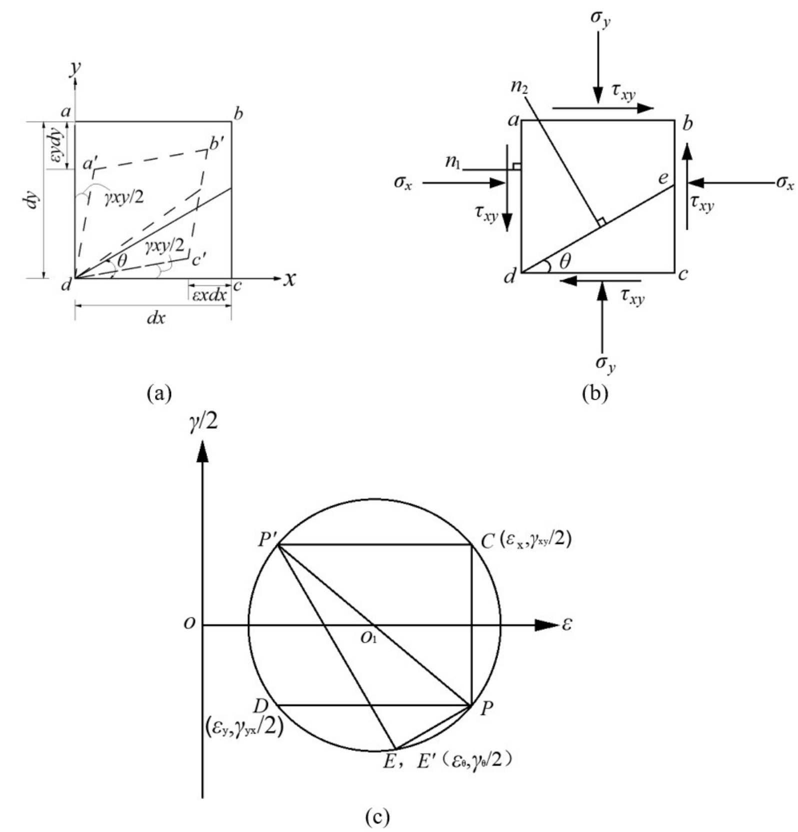


Figure 2. Plane strain element of soil and the Mohr's strain circle for the parallel line method. (a) Stress element, (b) Strain element, and (c) Mohr's strain circle.

central angle of arc *PP'* equals 180°, indicating that pole points *P* and *P'* are on the same diameter of the Mohr's strain circle.

2.3 The ray method

A new method called the ray method is first proposed to define the pole point:

1. As shown in Figure 3a, an arbitrary point *f* on plane *bc* is positioned. Then a ray *fg* is obtained by rotating the line *fb* about point *f* clockwise to angle α.
2. A line is drawn from *C* parallel to ray *fg*, which intersects *h* the Mohr's strain circle at the pole point *P"* (Figure 3b).
3. To obtain the strains of the plane *de*, an arbitrary point *h* is first selected on this plane, and ray *hi* is obtained by rotating line *dh* about *h* clockwise to angle α (Figure 3a). As shown in Figure 3b, a straight line is then drawn from the pole point *P"* parallel to the ray *hi*, which intersects the Mohr's strain circle at point *E"*, reading the strain state of plane *de*.

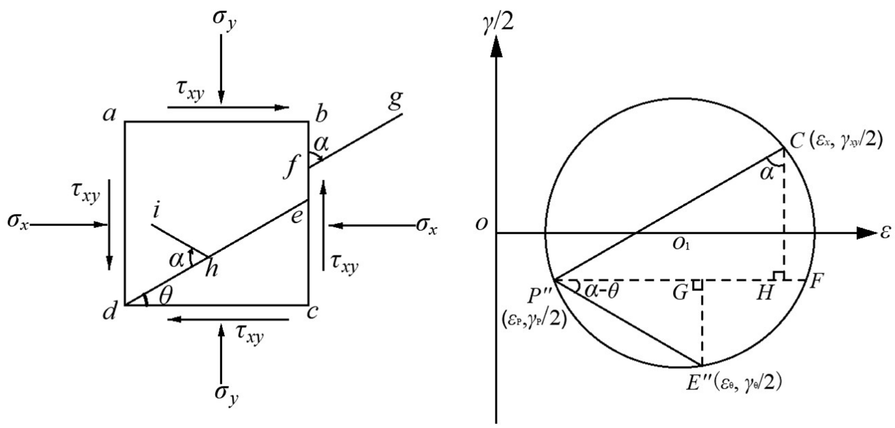
Comparing Figures 2 and 3, when the angle α equals 0°, the pole point *P"* will coincide with point *P*, i. e., the ray method becomes the parallel method. Furthermore, when the angle α equals 90°, the pole point *P"* may coincide with the point *P'*, indicating that the ray method is equivalent to the normal line method. From this viewpoint, the parallel line and normal line methods are two special cases of the ray method.

In addition, it can be also concluded that the parallel line method is easier to obtain the strain and stress states of a soil element.

3. Uniqueness of the pole point of Mohr's strain circle

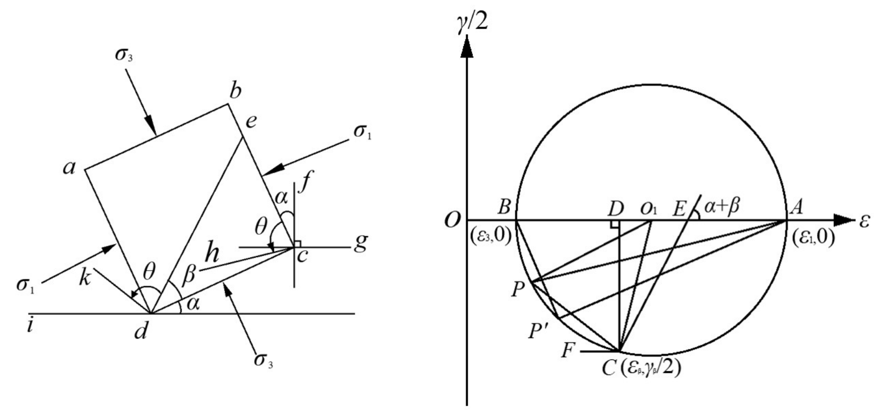
Since the parallel line and normal line methods are two special cases of the ray method, the uniqueness of the pole point obtained by the ray method is proved by using an indirect method herein.

As shown in Figure 4a, the maximum and minimum principal strains of the soil element are ε_1 and ε_3, and the normal and the shear strain of a specific plane *de* are ε_β and $\gamma_\beta/2$. In addition, the included angle between ray *dk* and plane *de*, as well as the included angle between ray *ch* and plane *bc*, are θ counterclockwise. Then establish a coordinate system and construct points *A* and *B* with the coordinates of (ε_1, 0) and (ε_2, 0), respectively. Therefore, the corresponding Mohr's strain circle for the soil element can be drawn using line *AB* as its diameter. (Figure 4b). Points *A* and *B* on the Mohr's circle represent the strain state of planes *ad* and *cd*, respectively and point *C* denotes the strain state of plane *de*.



(a) (b)

Figure 3. Plane strain element of soil and the Mohr's strain circle for the ray method. (a) Stress element, and (b) Mohr's strain circle.



(a) (b)

Figure 4. Plane strain element of soil and corresponding Mohr's strain circle. (a) Stress element, and (b) Mohr's strain circle.

A straight line is drawn from point *C* parallel to line *dk*, which intersects the Mohr's strain circle at pole point *P*. The pole point *P'* can also be determined by drawing a straight line from point *A* parallel to line *ch* (Figure 4b). Provided point *P* overlaps point *P'*, the pole point is said to be unique. The proof is as follows:

In Figure 4a, a straight line (*cg*) is drawn horizontally, and a straight line (*cf*) is drawn vertically both through point *c*, and then it can be concluded that the values of $\angle bcf$ and $\angle gch$ are α and α+θ-90°, respectively. Besides, the normal and the shear strains of plane *de* can be formulated by

$$\varepsilon_\beta = \frac{\varepsilon_1 + \varepsilon_3}{2} + \frac{\varepsilon_1 - \varepsilon_3}{2}\cos 2(90° - \beta) \tag{1}$$

$$\frac{\gamma_\beta}{2} = -\frac{\varepsilon_1 - \varepsilon_3}{2}\sin 2\left(90° - \beta\right) \tag{2}$$

In addition, as shown in Figure 4b, a straight line is constructed from point *C* parallel to plane *de*, which intersects the abscissa at point *E*. Then ray *CF* is drawn parallel to the abscissa, and a line is drawn from point *C* perpendicular to the abscissa intersecting the abscissa at point *D*.

From Equations 1 and 2, the coordinates of points *C* $((\varepsilon_1 + \varepsilon_3)/2 - (\varepsilon_1 - \varepsilon_3)\cos 2\beta/2, -(\varepsilon_1 - \varepsilon_3)\sin 2\beta/2)$, O_1 $((\varepsilon_1 + \varepsilon_3)/2, 0)$ and E $((\varepsilon_1 + \varepsilon_3)/2 - (\varepsilon_1 - \varepsilon_3)\cos 2\beta/2, 0)$ are determined. Then the lengths of the straight lines *CD* and O_1C can be given as

$$\left|CD\right| = \frac{\varepsilon_1 - \varepsilon_3}{2}\sin 2\beta \tag{3}$$

$$\left|O_1C\right| = \frac{\varepsilon_1 - \varepsilon_3}{2}\cos 2\beta \tag{4}$$

The tangent of $\angle CO_1D$ in Figure 4b can be determined by Equations 3 and 4 as

$$\tan\angle CO_1D = \frac{\left|CD\right|}{\left|O_1D\right|} = \tan 2\beta \tag{5}$$

In addition, since chord P*C* is normal to ray *de*, then

$$\angle PCF = \angle kdi = 180° - \alpha - \beta - \theta \tag{6}$$

and,

$$\angle DCP = 90° - \angle PCF = \alpha + \beta + \theta - 90° \tag{7}$$

Thus, the angle PCO_1 can be determined by solving Equations 5-7 as

$$\angle PCO_1 = \angle DCP + \angle DCO_1 = \alpha + \theta - \beta \quad (8)$$

and,

$$\angle PO_1C = 180° - 2\angle PCO_1 = 180° - 2(\alpha + \theta - \beta) \quad (9)$$

then,

$$\angle PO_1B = \angle CO_1D - \angle CO_1P = 2(\alpha + \theta) - 180° \quad (10)$$

Because $\angle PAB$ and $\angle PO_1B$ are inscribed angles, the central angle of arc *PB* on the Mohr's strain circle is

$$\angle PAB = 0.5\angle PO_1B = \alpha + \beta - 90° \quad (11)$$

In addition, since line *AP*' parallel to ray *ch*, then

$$\angle P'AB = \angle hcg = \alpha + \beta - 90° \quad (12)$$

Thus, $\angle PAB$ equals $\angle P'AB$, meaning that point *P* overlaps point *P'*. Therefore, the pole point proves to be unique.

4. Validity of the pole point of strain

In this section, the validity of the pole point obtained by using the ray method will be verified by a theoretical method.

As shown in Figure 3b, straight lines are constructed from point *C* and point *E"* perpendicular to the line *P"F* to meet at the points *H* and *G*, respectively.

Therefore, the tangent of $\angle\alpha$ can be calculated by

$$tan\angle\alpha = \left| \frac{\varepsilon_x - \varepsilon_P}{\frac{\gamma_{xy}}{2} - \frac{\gamma_P}{2}} \right| \quad (13)$$

In addition, since the pole point is located on the Mohr's strain circle, the relationship between the coordinates of the pole point can be also expressed as

$$(\varepsilon_P - \frac{\varepsilon_x + \varepsilon_y}{2})^2 + \left(\frac{\gamma_P}{2}\right)^2 = (\frac{\varepsilon_x - \varepsilon_y}{2})^2 + \left(\frac{\gamma_{xy}}{2}\right)^2 \quad (14)$$

Solving Equations 13 and 14 gives the coordinates of the pole point as

$$\varepsilon_P = \cos^2\alpha\varepsilon_x + \sin^2\alpha\varepsilon_y - \sin\alpha\cos\alpha\gamma_{xy} \quad (15)$$

$$\gamma_P = (\sin^2\alpha - \cos^2\alpha)\gamma_{xy} - \sin 2\alpha(\varepsilon_x + \varepsilon_y) \quad (16)$$

In a similar way, the strain state of the oblique plane *de* can be determined by

$$\tan\angle(\alpha - \theta) = \left| \frac{\frac{\gamma_P}{2} - \frac{\gamma_\theta}{2}}{\varepsilon_P - \varepsilon_\theta} \right| \quad (17)$$

$$(\varepsilon_\theta - \frac{\varepsilon_x + \varepsilon_y}{2})^2 + \left(\frac{\gamma_\theta}{2}\right)^2 = (\frac{\varepsilon_x - \varepsilon_y}{2})^2 + \left(\frac{\gamma_{xy}}{2}\right)^2 \quad (18)$$

Then the normal and shear strains of plane *de* are

$$\varepsilon_\theta = \frac{1}{2}(\varepsilon_y - \varepsilon_x)\cos 2\theta - \frac{1}{2}(\varepsilon_x + \varepsilon_y) - \frac{1}{2}\gamma_{xy}\sin 2\theta \quad (19)$$

$$\gamma_\theta = (\varepsilon_x - \varepsilon_y)\sin 2\theta - \gamma_{xy}\cos 2\theta \quad (20)$$

The rectangular soil element *OLMN* in Figure 5a is considered. The strains along O_x and O_y are ε_x and ε_y, and γ_{xy} is the shearing strain.

Let the *OM* be of length *a*, then $ON = a\cos\theta$ and $OL = a\cos\theta$. Therefore, the strain of *ON* and *OL* are $-a\cos\theta\varepsilon_x$ and $-\ \cos\theta\varepsilon$, respectively.

If point *M* moves to point *M'*, then the movement of point *M* parallel to the abscissa is $-(a\cos\theta\varepsilon_x + a\sin\theta\cdot\gamma_{xy}/2)$. Moreover, the movement of point *M* parallel to the ordinate is $-(a\sin\theta\varepsilon_y + a\cos\theta\cdot\gamma_{xy}/2)$.

Since the strains are small, the movement of *M* parallel to *OM* is practically coincident with *MM'* which equals $-(a\cos\theta\varepsilon_x + a\sin\theta\cdot\gamma_{xy}/2)\cos\theta$ - $(a\sin\theta\varepsilon_y + a\cos\theta\cdot\gamma_{xy}/2)\sin\theta$ (Figure 5b). Thus, the strain of the oblique plane ε_θ can be given by

$$\begin{aligned}\varepsilon_\theta &= -\frac{\left(a\cos\theta\varepsilon_x + a\sin\theta\cdot\frac{1}{2}\gamma_{xy}\right)\cos\theta + \left(a\sin\theta\varepsilon_y + a\cos\theta\cdot\frac{1}{2}\gamma_{xy}\right)\sin\theta}{a} \\ &= -\left(\cos\theta\varepsilon_x + \sin\theta\cdot\frac{1}{2}\gamma_{xy}\right)\cos\theta - \left(\sin\theta\varepsilon_y + \cos\theta\cdot\frac{1}{2}\gamma_{xy}\right)\sin\theta \\ &= \frac{1}{2}(\varepsilon_y - \varepsilon_x)\cos 2\theta - \frac{1}{2}(\varepsilon_x + \varepsilon_y) - \frac{1}{2}\gamma_{xy}\sin 2\theta \end{aligned} \quad (21)$$

To determine the shear strain in the direction of *OM*, a straight line is first drawn from point *N* perpendicular to *OM*, which intersects *OM* at point *P*. Since the strain along *OM* and *OP* is ε_θ, it can be concluded that the extensions of *OM* and *OP* are $a\varepsilon_\theta$ and $a\cos\theta\varepsilon_\theta$, respectively. Since the normal and shear strains are small, the movement of *P* parallel to *OP* is practically coincident with *PP'*.

It can be seen from Figure 5c that, when point *P* moves to point *P'*, line *PN* rotates about point *P* clockwise through a small angle α, and α can be given by

$$\alpha = \frac{a\cos^2\theta\varepsilon_\theta - \left(a\cos\theta\varepsilon_x + a\cos\theta\cdot\frac{1}{2}\gamma_{xy}\tan\theta\right)\cos\theta}{a\cos\theta\sin\theta}$$

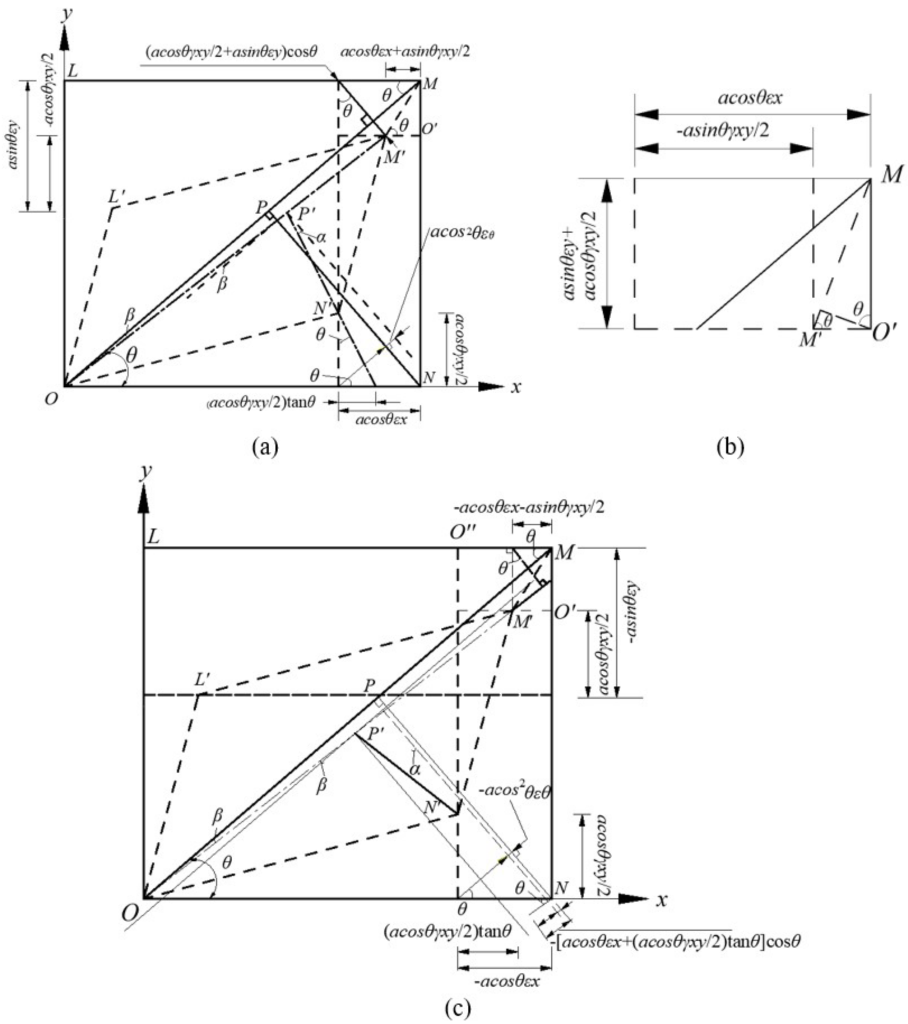


Figure 5. Strains on an inclined plane. (a) Strain element, (b) Enlarged view of O' M M', and (c) Strains on OM.

$$= \varepsilon_x \cot\theta - \varepsilon_\theta \cot\theta + \frac{1}{2}\gamma_{xy} \tag{22}$$

Line *OM* also rotates about point *O* through a small angle β counterclockwise, and β can be calculated by

$$\beta = \frac{\left(a\cos\theta\varepsilon_x + a\sin\theta\cdot\frac{1}{2}\gamma_{xy}\right)\sin\theta - \left(a\sin\theta\varepsilon_y + a\cos\theta\cdot\frac{1}{2}\gamma_{xy}\right)\cos\theta}{a}$$

$$= \left(\cos\theta\varepsilon_x + \sin\theta\cdot\frac{1}{2}\gamma_{xy}\right)\sin\theta - \left(\sin\theta\varepsilon_y + \cos\theta\cdot\frac{1}{2}\gamma_{xy}\right)\cos\theta$$

$$= \sin\theta\cos\theta\left(\varepsilon_x - \varepsilon_y\right) + \frac{1}{2}\gamma_{xy}\left(\sin^2\theta - \cos^2\theta\right) \tag{23}$$

Thus, the shear strain γ_θ in the direction *OM* equals the sum of α and β, and can be given by

$$\gamma_\theta = \alpha + \beta = \left(\varepsilon_x - \varepsilon_\theta\right)\cot\theta + \left(\varepsilon_x - \varepsilon_y\right)\sin\theta\cos\theta + \gamma_{xy}\sin^2\theta \tag{24}$$

Substituting ε_θ of Equation 21 into Equation 24 gives

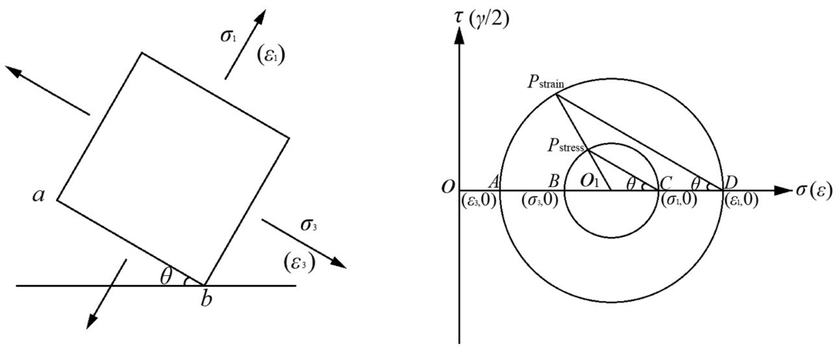
$$\gamma_\theta = \left(\varepsilon_x - \varepsilon_y\right)\sin 2\theta - \gamma_{xy}\cos 2\theta \tag{25}$$

It can be found that the normal and shear strains obtained by using the ray method (Equations 19 and 20) are the same as those calculated by the theoretical method (Equations 21 and 25), indicating that the pole point method is proved to be valid in obtaining the strain state for a specific oblique plane of a soil element.

5. Relationship between pole point of strain and pole point of stress

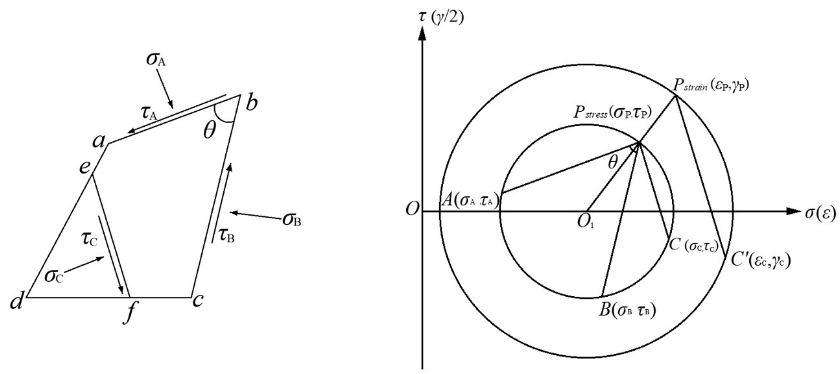
For a soil element subjected to two-dimensional principal stresses (Figure 6a), provided that suitable stress and strain scales are chosen, the stress and strain circles will have the same center (Figure 6b) (Hearn, 1997). Hearn (1997) found that the stress and strain scales can be expressed as:

$$K_{\text{stress}} = \frac{E}{1-\upsilon} \times K_{\text{strain}} \tag{26}$$



(a) (b)

Figure 6. Plane stress (strain) element of soil and the corresponding Mohr's strain and stress circles. (a) Stress (strain) element, and (b) Mohr's strain and stress circles.



(a) (b)

Figure 7. Arbitrary plane stress element of soil and the corresponding Mohr's stress and strain circles. (a) Arbitrary stress element, and (b) Mohr's strain and stress circles.

where K_{stress} and K_{strain} are stress and strain scales of a soil element, and K_{stress} and K_{strain} equal $2OO_1/(\sigma_1+\sigma_2)$ and $2OO_1/(\varepsilon_1+\varepsilon_2)$, respectively (Figure 6b). In addition, E and μ are the Young's modulus and Poisson's ratio of the element. Therefore, the relationship between the radius of the Mohr's strain and stress circles can be expressed by

$$R_{stress} = \frac{1-\upsilon}{1+\upsilon} \times R_{strain} \tag{27}$$

where R_{stress} and R_{strain} are the radius of the Mohr's stress and strain circles, respectively.

If the radius of a Mohr's circle of stress is known, the radius of the Mohr's circle of strain can then be determined from Equation 27.

A straight line is drawn from point C parallel to plane ab, and the intersection of the parallel line with the Mohr's stress circle is the pole point of stress (denoted as P_{stress} in Figure 6b). In a similar manner, a line is constructed from point D parallel to plane ab, and the intersection of the parallel line with the Mohr's strain circle is the pole point of strain (denoted as P_{strain} in Figure 6b). Therefore, line DP_{strain} is parallel to line CP_{stress}; consequently, it can be concluded that $\angle O_1P_{stress}C$ equals $\angle O_1P_{strain}D$, i.e., points O_1, P_{stress} and P_{strain} lie on the same diameter of the Mohr's circle.

Provided that the strain or the stress state of a plane, as well as the Poisson's ratio of an element are given, the stress and strain states of an arbitrary plane in the element can be quickly and easily determined by using the diagram based on the relationship between pole point of strain and pole point of stress.

6. Application of the pole point method in solving complex stress and strain problems

In this section, the parallel line method is used to show the advantage of the pole point method in solving the following complex stress and strain problems.

As shown in Figure 7a, each plane of the quadrilateral strain element is not perpendicular and only the normal and shear stresses of planes *ab* and *bc* are known. For such a case, since points *A* and *C* (Figure 7b) not lie on the same diameter of the Mohr's circle, the Mohr's stress and strain circles cannot be drawn by using the conventional method. Thus, the stress state, as well as the strain state of the oblique plane *ef* cannot be determined easily. However, as illustrated before, there must be a pole point on the Mohr's circle for the specific stress or strain state. Therefore, the three non-collinear points (the pole point, point *A* and point *C*) can construct a unique Mohr's circle. Such problem can be tackled as follows.

1. The σ-τ coordinate system is first constructed, then a line is drawn from point *A* parallel to plane *ab*, and in addition, a line is drawn from point *B* parallel to plane *bc*. The intersection of the two lines is the pole point of stress (denoted as P_{stress} in Figure 7b), since the pole point is unique under a specific pole point method. Then, Mohr's stress circle can be determined by using the three non-collinear points *A*, *B* and *P*.
2. By using the Equation 27 and the relationship between the pole points of Mohr's stress and strain circles, the Mohr's strain circle and the corresponding pole point of strain (denoted as P_{strain}) can also be determined.
3. A line is drawn from point P_{stress} parallel to plane *ef*, which intersects the Mohr's stress circle at point *C*, and the coordinates of point *C* give the stress state of plane *ef*. In a similar way, a line is drawn from P_{strain} parallel to plane *ef*, which intersects the Mohr's strain circle at point *C'*, and the coordinates of *C*' give the strain state of plane *ef*.

7. Conclusion

Mohr's circle and the pole point method diagram are very useful in representing the two-dimensional stress and strain states in materials engineering such as mechanical packaging and infrared window material engineering and geotechnical engineering. This paper first summarizes the parallel line, and the normal line pole point methods for Mohr's strain circle, then a new ray method is proposed to obtain the pole point of strain. This study emphasizes the proof of the uniqueness, and the validity of the pole point method, as well as the relationship between the pole points of strain and stress. A relatively complex case is given to illustrate the advantage of the pole point method in determining stress and strain states of a soil element. The following conclusions can be made.

The parallel line method and the normal line method are two special cases of the ray method. However, results show that the parallel line method is the most convenient to obtain the strain state of a soil element. It has been also proved that the pole point is unique under a specific method. The pole point is verified by a corresponding theoretical method.

For the given suitable stress and strain scales, the Mohr's strain and stress circles can be drawn as two concentric circles. The pole points of strain and stress on in the same diameter of the Mohr's circle, provided the Poisson's ratio of the material is known.

A case study shows that the pole point methods have significant advantage in obtaining the strain and stress states of an arbitrary plane in the soil element under complex stress and strain conditions in which each plane of a quadrilateral soil element is not perpendicular.

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Declaration of interest

The authors have no conflicts of interest to declare. All co-authors have observed and affirmed the contents of the paper and there is no financial interest to report.

Authors' contribution

Dayong Li: conceptualization, methodology, formal analysis, funding acquisition. Chuanping Sun: visualization, writing - original draft. Yukun Zhang: formal analysis, funding acquisition, validation, writing - review & editing.

Data availability

All data produced or examined in the course of the current study are included in this article.

Declaration of use of generative artificial intelligence

This work was prepared without the assistance of any generative artificial intelligence (GenAI) tools or services. All aspects of the manuscript were developed solely by the authors, who take full responsibility for the content of this publication.

List of symbols and abbreviations

CD	The diameter of the Mohr's strains circle for the parallel line method
E	Young's modulus
K_{strain}	Strain scale of a soil element
K_{stress}	Stress scale of a soil element
O_1	The center of the Mohr circle

P	Pole for Mohr's circle
R_{strain}	The radius of the Mohr's train circle
R_{stress}	The radius of the Mohr's stress circle
γ	Shear strain
ε	Normal strain
ε_i	Maximum principal strain of the soil element in the $i = 1, 2$ directions
ε_x	Normal strain on plane *ad*
ε_y	Normal strain on plane *cd*
θ	Included angle between ray *dk* and plane *de*
μ	Poisson's ratio

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