Soils and Rocks
www.soilsandrocks.com
An International Journal of Geotechnical and Geoenvironmental Engineering
ISSN 1980-9743
ISSN-e 2675-5475

Discussion of "Forks in the road: decisions that have shaped and will shape the teaching and practice of geotechnical engineering"*

Charles John MacRobert[1#]

Discussion

An alternative way to illustrate how an associated flow rule leads to the erroneous conclusion that frictional materials dissipate no energy upon plastic deformation is by resolving the stresses and displacements acting along a slip surface. The author of this discussion finds this an easy way to get the principle across at undergraduate level.

Prior to the derivation, it is helpful to illustrate the concept of dilation using the particle climbing analogy presented in Figure 4 of the article under discussion. Students are then asked to remember back to their physics lectures for an equation to determine energy dissipated:

Energy dissipated = Displacement · Force
Energy dissipated = Displacement · Stress · Length

$$E = \delta \cdot \sigma \cdot L \tag{1}$$

Figure 1 is then drawn or displayed on the board. The initial condition diagram is used to define the relationship between shear stress (τ), effective stress (σ'), and friction angle (ϕ') given by $\tau = \sigma' \cdot \tan\phi'$. The final condition diagram illustrates the displacement along the slip surface (δ_L), the displacements normal to the slip surface (δ_n) and how these are related by the angle of dilation (ψ) as $\delta_n = \delta_L \cdot \tan\psi$.

Students are then asked where energy is being dissipated, which displacements are associated with each stress, the lengths along which energy is dissipated and what happens to the sign of the work done if the displacement is in the opposite direction to the stress (as is the case for the normal stress and associated displacement). This discussion leads to the following equation:

Total energy dissipated = Energy dissipated along slip surface + Energy dissipated normal to slip surface

$$E = \tau \cdot L \cdot \delta_L - \sigma' \cdot L \cdot \delta_n \tag{2}$$

Equations for τ and δ_n are then substituted into Equation 2 to give:

$$E = \sigma' \cdot L \cdot \delta_L (\tan\phi' - \tan\psi) \tag{3}$$

Which reduces to $E = 0$, if an associated flow rule is assumed (i.e., $\phi' = \psi$). The work done along the slip plane is thus all used in increasing the volume normal to the slip

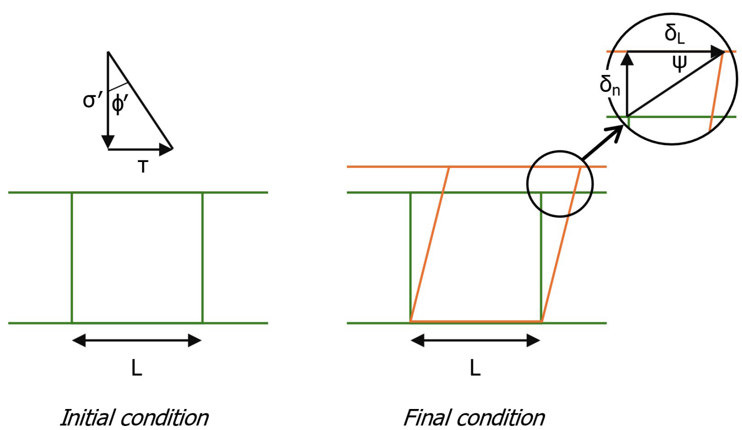

Figure 1. Sliding along a thin slip surface.

#Corresponding author. E-mail address: macrobert@sun.ac.za
[1]Stellenbosch University, Department of Civil Engineering, Stellenbosch, South Africa.

Submitted on August 28, 2024; Final Acceptance on October 25, 2024.
Editor: Renato P. Cunha
*Appears in Salgado, R. (2024). Soils and Rocks, 47(2), e2024010123.
https://doi.org/10.28927/SR.2025.007124

plane and there is no net energy dissipated in the direction of shearing. This equation can also be used to show that if a non-associated flow rule is assumed (i.e., $\phi' > \psi$) net energy is dissipated in the direction of shearing (i.e., $E > 0$).

The next concept to develop is that an associated flow rule results in a higher resistance compared to using a non-associated flow rule (i.e., Equation 9 and Figure 6 in the article under discussion). Conceptually this is complicated as this energy dissipation is being considered in the context of the upper bound theorem where one is searching for a more efficient mechanism that dissipates less energy. Surely, the case where no net energy is being dissipated is more efficient?

Reinforcing that dilation is related to volume change can be helpful in overcoming this conceptual barrier. For the larger the dilation angle, the larger the volume change during shear, and consequently the larger the mechanism associated with failure. Figure 7 from Loukidis & Salgado (2009) is helpful in illustrating this phenomenon. So, whilst a non-associated flow rule (i.e., $\phi' > \psi$) results in net energy being dissipated in the direction of shearing (i.e., $E > 0$) the mechanism is much more efficient resulting in a lower overall resistance.

Author’s Reply

Rodrigo Salgado

I thank the discusser for the interest in the paper and for contributing to one of the concepts discussed in the paper: the meaningfulness and applicability of an associated flow rule to the calculation of soil stress-strain response.

I find that the way in which the discusser proposes to illustrate to students the incorrect result from use of an associated flow rule—the absence of energy dissipation during shearing—is compelling. Most students will have been exposed to the notion that work done per unit volume on an element of soil can be calculated through the product of stress by strain increment. If not, they will easily relate the work done by a force on a body to the displacement that occurs during some interval of time to the analysis the discusser proposed. From that to the energy principle that the work done must be balanced or converted into energy that is either stored or dissipated is a short step. By properly applying these principles to the known kinematic response of the soil element in dilation—a strain increment normal to the direction of shearing that is a function of the dilatancy angle—it is possible, as the discusser has shown, to arrive at the absurd result that a frictional material would not dissipate energy (of course, only if an associated flow rule is assumed). Not expanded on by either the discusser or the original paper is the fact that even analyses based on nonassociated flow rule will necessarily lead to realistic results. For example, the dilatancy angle is not constant through shearing, so there is another “layer” of sophistication needed for realistic results to be obtainable. Finally, I believe that this result further raises awareness of the need for more sophisticated constitutive relationships to properly capture mechanical response of soils.

I conclude by commending the discusser for teaching soil mechanics by relying on fundamental mechanics. As I stated in the paper, I have found that—contrary to what some assume—students at the undergraduate level are capable of learning when this approach is used. Not only that, I believe that they thrive and enjoy more the learning of the subject because of the greater understanding of the subject and related subjects that the approach enables them to develop.

List of symbols

E	Dissipated energy
σ	Stress
σ'	Effective stress
τ	Shear stress
ϕ'	Friction angle
δ	Displacement
δ_L	Displacement along slip surface
δ_n	Displacement normal to slip surface
ψ	Dilation angle

References

Loukidis, D., & Salgado, R. (2009). Bearing capacity of strip and circular footings in sand using finite elements. *Computers and Geotechnics*, *36*(5), 871-879.