An Expedite Method to Predict the Shear Strength of Unsaturated Soils

Orencio Monje Vilar

Abstract. An expedite method to predict the shear strength of unsaturated soils is proposed and tested against many soils of different origins and subjected to different test types. The method uses an empirical hyperbolic function that has been successfully used to fit experimental data. The parameters of this function are obtained considering effective shear strength parameters from saturated soil and from test results of air dried samples tested without the need of suction control. Air dried samples can be, alternatively, replaced by test results of samples tested at suction larger than the maximum suction expected in the problem under analysis. The good agreement between the estimate functions and the experimental data shown by both alternatives makes them promising and reliable to estimate unsaturated shear strength parameters for preliminary purposes.

Key words: unsaturated soil, suction, shear strength, prediction, laboratory tests.

1. Introduction

The interest in unsaturated soil behavior has increased in recent years. Many advances in methods of testing and analyses have been proposed, however the major drawback when trying to characterize unsaturated soils in the laboratory still arises from the need to control or measure soil suction. Soil suction is usually imposed through the axis translating technique (Hilf, 1956), the osmotic technique (Kassif & Bem Shalon, 1971) and using saline solutions to control relative humidity (Soto, 2004). Measurement of suction is undertaken most commonly using mini tensiometers, psychrometers (Edil et al., 1981) or high capacity tensiometers (Ridley & Burland, 1995).

A common feature of all the experimental techniques devised to test unsaturated soils are that they are time consuming or complex or both, requiring specialized expertise. Thus, it is not surprising that there has been a significant research effort directed towards the development of methods to predict the basic soil properties, such as the unsaturated hydraulic conductivity (van Genuchten, 1980, among others) and the shear strength. A common approach of the methods of estimating the shear strength of unsaturated soils is to use the soil water retention curve (SWRC) and effective shear strength parameters for saturated soil (Oberg & Salfors, 1997; Vanapalli et al., 1996; Fredlund et al., 1996). Khalili & Khabbaz (1998) used a traditional effective stress approach that is also based on effective stress parameters, $c'$ and $\phi'$, and a single stress variable, $\chi$, proposed by Bishop (1959) and defined as

$$\sigma' = (\sigma - u_a) + \chi (u_a - u_w)$$

(1)

where $\sigma$ is the total stress, $u_a$ is the pore air pressure, $u_w$ is the pore water pressure, $c'$ is the effective cohesion intercept and $\phi'$, the effective angle of internal friction of the saturated soil. These authors proposed a unique relationship between the effective stress parameter $\chi$ and the ratio between suction $(u_a - u_w)$ and air entry value $(u_a - u_w)$, the suction ratio.

Rassam & Cook (2002) have proposed a method where the shear strength envelope can be obtained using effective shear strength parameters for saturated soil, the SWRC and the test results from one unsaturated specimen at the residual water content. The method uses a power additive function and the predicted envelopes were in close agreement with experimental measurements for a variety of soil types.

In this paper an expedite method to predict the shear strength of unsaturated soils is proposed. The method uses data from saturated samples (effective stress parameters) and from air dried samples or, alternatively, from samples tested at a known suction that is larger than the maximum expected suction in the problem.

2. Fundamentals

Many authors, such as Bishop et al. (1960), Fredlund et al. (1978), Ho & Fredlund (1982), Escario & Saez (1986) have dealt with the shear strength of unsaturated soils. Most of the proposed experimental techniques use the axis translating technique to impose or to control soil suction. Triaxial compression tests and direct shear tests are commonly used and the usual drainage control include the drained test (CD), in which the pore air and pore water pressures are kept constant during all the test and the constant moisture test (CW) in which the pore air pressure is controlled and the pore water pressure is measured, as the flow of water is impeded.

Test results have been analyzed using an effective stress approach (Bishop, 1959) or independent stress state variables (Fredlund et al., 1978). The former is based on
Eq. (1), while the later uses independent stress state variables, the net normal stress \((\sigma - u)\) and the matric suction \((u_a - u_w)\). The Fredlund et al. (1978) approach can be expressed as

\[
\tau = c + (u_a - u_w) \tan \phi' + (\sigma - u) \tan \phi
\]

(2)

where \(\tau\) = the shear strength of the unsaturated soil and \(\phi'\) = the angle of internal friction with respect to matric suction.

Equation (2) can be expressed as

\[
\tau = c + (\sigma - u) \tan \phi'
\]

(3)

where the total intercept of cohesion, \(c\), is equivalent to

\[
c = c' + (u_a - u_w) \tan \phi
\]

(4)

Thus, the model relates an increase in matric suction to a linear increase in shear strength, more specifically by increasing cohesion. However many test results have shown that the influence of suction on shear strength is not linear (Escario & Saez, 1986; Rohm & Vilar, 1995; Satija, 1978). In fact, in spite of the different assumptions in the derivation of effective stress approach and independent stress state variables, it is easy to show that they are related since

\[
\chi = \frac{\tan \phi b}{\tan \phi'}
\]

(5)

As it is known that \(\chi\) is non linear with the degree of saturation, it is not surprising that \(\phi'\) should not be constant with suction.

Some empirical functions have been proposed to deal with the non linearity of the shear strength envelope of unsaturated soils. For instance, Abramento & Carvalho (1989) have proposed a potential function and de Campos & Carrillo (1995) a fourth order polynomial function.

The following hyperbolic equation is being used by the author to represent the influence of matric suction on the unsaturated shear strength of some Brazilian soils (Rohm & Vilar, 1995; Teixeira & Vilar, 1997 and Machado & Vilar, 1998).

\[
c = c' + \frac{\psi}{a + b\psi}
\]

(6)

where \(a\) and \(b\) are fitting parameters and \(\psi = u_a - u_w\).

The use of Eq. (6) can be illustrated by applying it to test data of some soils available in the literature. The characteristics and properties of these soils are presented in Table 1. This table also summarizes data from other soils that will be used later. Figure 1 shows the experimental data and fitting curves for some of the listed soils. The chosen soils comprise a wide range of soil types and test conditions that include undisturbed and compacted samples tested in direct shear tests, triaxial compression tests and unconfined compression tests performed with suction control. The equation matches the experimental data with little scatter, illustrating its usefulness in representing the effect of suction on shear strength of unsaturated soils. The fitting parameters and coefficient of determination \((R^2)\) are presented in Table 2.

3. The Proposed Procedure

The good agreement between the fitting equation and the experimental data suggests that it can be used to predict the unsaturated shear strength if the fitting parameters could be obtained from other sources of data, such as the effective stress envelope.

Figure 2 shows a sketch of the hyperbolic function to represent the shear strength variation with suction and its link with SWRC. The qualitative features of the relationship between both curves are discussed and used to establish the assumptions used to derive the \(a\) and \(b\) hyperbolic parameters.

It is known that below the air entry value the soil remains saturated and the effective stress principle still remains valid. If the Fredlund et al. (1978) equation is considered, it is easy to show that in the saturated state \(\phi' = \phi\). The air entry value depends on many aspects such as the void ratio and confining stress. To keep the proposed procedure as simple as possible and to avoid the introduction of additional parameters in the proposed model, it is considered that the slope of the relationship between \(c\) and \(\psi\) (Eq. (7)), as \(\psi\) approaches zero, is \(\tan \phi'\), that is

\[
\frac{dc}{d\psi}_{\psi \to 0} = -\frac{a}{b} = \tan \phi'
\]

(7)

As suction increases the soil begins to desaturate and most of the results published so far shows an increase in shear strength with suction up to a maximum. After that, shear strength remains almost constant (Escario, 1988, de Campos & Carrillo, 1995, Machado & Vilar, 1998) and some particular soils have shown that shear strength drops off to a lower value as suction is increased (Escario, 1988; Gan & Fredlund, 1996). Considering the experimental errors inevitably present and for the sake of simplicity it will be assumed that shear strength reaches an ultimate value in any case. So, as \(\psi\) approaches infinity, shear strength approaches an ultimate or unsaturated residual value, \(c_{\omega_i}\) or \(\tau_{\omega_i}\), depending on how shear strength is represented, value that is probably related to the residual water content of the soil.

Assuming that shear strength will reach an ultimate value at the residual water content, the following statement can be written:

\[
\lim_{\psi \to \infty} c = c_{\omega_i} = c' + \frac{1}{b}
\]

(8)

or that
An Expedite Method to Predict the Shear Strength of Unsaturated Soils

Table 1 - Properties and characteristics of some unsaturated soils.

<table>
<thead>
<tr>
<th>Soil type</th>
<th>φ’ (°)</th>
<th>c’ (kPa)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandy clay (compacted)</td>
<td>30</td>
<td>30</td>
<td>LL = 33%; PI = 10%; γ = 16 kN/m³; w = 15.5%; tricaldial CD tests</td>
</tr>
<tr>
<td>Copper tailing dam (compacted)</td>
<td>38.7 to 40.1</td>
<td>9.2</td>
<td>Sand (SM); γ’ = 18.4 kN/m³; NP; w = 13%; CD and CW tests</td>
</tr>
<tr>
<td>Clayey silt ( statically compacted)</td>
<td>29</td>
<td>7.8</td>
<td>PI = 23%; γ = 14.8 kN/m³; w = 22.2% (Dhanauri clay) ;</td>
</tr>
<tr>
<td>Glacial till (compacted clayey sandy silt)</td>
<td>25.5</td>
<td>44.4</td>
<td>LL = 36%; PI = 19%; γ’ = 18 kN/m³; w = 16%; e = 0.51 to 0.77;</td>
</tr>
<tr>
<td>Clayey sand (undisturbed colluvium)</td>
<td>29</td>
<td>0</td>
<td>γ = 16 kN/m³; w = 15.8%; e = 0.99; LL = 28%, triaxial CD tests</td>
</tr>
<tr>
<td>Clayey sand (undisturbed colluvium)</td>
<td>31.2</td>
<td>10.5</td>
<td>γ = 17.1 kN/m³; w = 16.4%; e = 0.88; LL = 31%, triaxial CD tests</td>
</tr>
<tr>
<td>Clayey sand (undisturbed residual soil from sandstone)</td>
<td>26.4</td>
<td>28.3</td>
<td>γ = 18.8 kN/m³; w = 16.7%; e = 0.68; LL = 28%, triaxial CD tests</td>
</tr>
<tr>
<td>Yellow clayey sand (undisturbed colluvium)</td>
<td>26.4</td>
<td>0</td>
<td>LL = 46%, PI = 23%; γ = 14.9 to 15.7 kN/m³; w = 23.2 to 24.8%; e = 1.1 to 1.3. CD direct shear</td>
</tr>
<tr>
<td>Clayey sand (undisturbed mature residual soil)</td>
<td>28.7</td>
<td>13.7</td>
<td>LL = 51%, PI = 18%; γ = 15.6 to 17.1 kN/m³; w = 16.4 to 17.7%; e = 0.9 to 1.1. CD direct shear</td>
</tr>
<tr>
<td>Expansive clayey silt ( statically compacted)</td>
<td>21.2</td>
<td>35</td>
<td>LL = 48%; PI = 25%; γ’ = 15 kN/m³; w = 17%</td>
</tr>
<tr>
<td>Glacial till (compacted)</td>
<td>23</td>
<td>4 to 15</td>
<td>Specimens with δ = 17.3 kN/m³; w = 13%; CD direct shear</td>
</tr>
<tr>
<td>Decomposed fine ash tuff (silty coarse sand)</td>
<td>40</td>
<td>0</td>
<td>Multistage CD direct shear; samples US-5 and US-3</td>
</tr>
<tr>
<td>Clayey sand (undisturbed colluvium)</td>
<td>40</td>
<td>0</td>
<td>LL = 39%; PI = 14%; γ’ = 13.3 kN/m³</td>
</tr>
<tr>
<td>Madrid gray clay ( statically compacted)</td>
<td>25.2</td>
<td>5.4</td>
<td>Specimens with γ = 13.3 kN/m³; w = 29%; CD direct shear</td>
</tr>
<tr>
<td>Clayey sand (colluvium, undisturbed)</td>
<td>25.2</td>
<td>5.4</td>
<td>LL = 71%; PI = 35%; Standard Proctor δ_w = 13.3 kN/m³; w = 33.7%</td>
</tr>
<tr>
<td>Residual soil from gneiss ( Silty clayey sand, compacted)</td>
<td>32</td>
<td>10.3 (w_o)</td>
<td>Specimens with γ = 15 kN/m³; w = 25% Unconfined compression tests with suction measurements (high capacity tensiometer);</td>
</tr>
<tr>
<td>Sandy clay (colluvium, undisturbed)</td>
<td>24.5</td>
<td>10.2</td>
<td>LL = 47%; PI = 13%; Standard Proctor γ’ = 15 kN/m³; w = 25% Unconfined compression tests with suction measurements (high capacity tensiometer);</td>
</tr>
</tbody>
</table>

LL - liquid limit; PI - plasticity index; γ - unit weight; γ’ - dry unit weight; w-moisture content; w_o - optimum moisture content; e - void ratio; CD - consolidated-drained triaxial compression test.
Thus, if the effective shear strength parameters at saturation and the ultimate shear strength at the residual moisture content are measured, both parameters $a$ and $b$ can be obtained and the unsaturated shear strength can then be predicted based on the assumption that the relationship between suction and shear strength follows the general form of Eq. (6). Two alternatives of practical interest have been devised to deal with such question.

### 3.1. Alternative 1

As the residual water content is approached it is very difficult for liquid water to migrate. Water movement is primarily commanded by vapor flow at low flow rates and is reasonable to assume that in a specimen under these conditions the matric suction variation during shearing will not produce any significant change in mechanical properties, such as the shear strength. Since the procedure can be implemented considering only the unsaturated residual cohesion (through Eq. (9)) without reference to associated matric suction, it is proposed that testing of air dried specimens could be used to establish the residual cohesion. So a simplified testing procedure could be followed, performing, for instance, constant moisture tests drained to the air in order to approach the usual condition of drained tests and to avoid the complex procedures of suction controlled tests.

This proposition will be checked using data from Futai (2002), Reis (2004) and Escario (1988). Futai (2002) and Reis (2004) followed the previous condition of tests and performed suction controlled drained (CD) tests and constant moisture (CW) tests with air dried samples from two horizons of residual soils. Escario (1988) performed suction controlled direct shear tests up to large values of suction, probably beyond the residual suction of the soils studied, the Madrid gray clay and the Guadalix red clay.
The characteristics of these soils are presented in Table 3, together with parameters for the proposed model.

Futai (2002) tested a mature and a young residual soil from gneiss. These two materials showed angles of internal friction varying with the suction and to predict the shear strength envelope, the cohesion intercepts were directly taken from the envelopes without any arrangement regarding the variation of internal friction. The air dried samples presented a cohesion intercept of 125 kPa for the mature soil and of 77.5 kPa for the young residual soil. These values, together with the cohesion of the saturated sample allow to calculate $b$ using Eq. (9) and to predict the envelope, which matches fairly well the experimental results, as shown in Figs. 3(a) and 3(b).

The prevision considering the ultimate or residual strength applied to the data of Reis (2004) is shown in Fig. 4(b). In this case, the air dried samples of young residual soil presented a cohesion intercept of 115 kPa, while the mature soil showed cohesion of 215 kPa, considering an adjusted envelope with friction angle of 31°. It can be seen a good agreement between experimental and predicted data for the lower suction, but the predicted data become lower than the experimental ones for the larger suction. The deviation reaches a maximum, of about 20%, for the young soil, with calculated values lower than the measured ones. For this soil, the method was not so precise to predict the shear strength for the entire range of suction. However, it is worth to say that the method is still interesting as it is preferable a conservative than an optimistic prediction for preliminary studies.

As far as the data of Escario (1988) is concerned, Fig. 5 shows that the predicting equation nicely fits the experimental data and also confirms the good performance of the procedure. In this case, the value corresponding to the largest suction used in the tests was assumed as the value at the residual condition.

![Figure 2](image-url) - (a) Soil water retention curve and typical elements; (b) Hyperbolic function and assumed conditions to derive $a$ and $b$ parameters.

### Table 3 - Characteristics of the soils tested by Futai (2002), Reis (2004) and Escario (1988).

<table>
<thead>
<tr>
<th>Soil</th>
<th>Soil types</th>
<th>$\phi$ (°)</th>
<th>$c'$ (kPa)</th>
<th>$c_{ult}$ (kPa)</th>
<th>$a$</th>
<th>$b$</th>
<th>$R^2$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>Sandy clay (undisturbed) 1 m depth</td>
<td>27.3</td>
<td>17</td>
<td>125</td>
<td>1.94</td>
<td>0.0093</td>
<td>0.99</td>
<td>$w_L = 57%$; PI = 29%; $\gamma = 15$ kN/m$^2$; $w = 30%$; $e = 1.1-1.2$; triaxial CD tests [Futai, 2002]</td>
</tr>
<tr>
<td>19</td>
<td>Young residual soil from gneiss (undisturbed sandy silt)</td>
<td>26.4</td>
<td>33.5</td>
<td>77.5</td>
<td>2.016</td>
<td>0.0227</td>
<td>0.99</td>
<td>$w_L = 42%$; PI = 19%; $\gamma = 17$ kN/m$^2$; $w = 25%$; $e = 0.8-0.9$; triaxial CD tests [Futai, 2002]</td>
</tr>
<tr>
<td>20</td>
<td>Young residual soil from gneiss (silty sand, undisturbed)</td>
<td>28°</td>
<td>24.0</td>
<td>115</td>
<td>1.88</td>
<td>0.011</td>
<td>0.98</td>
<td>$w_L = 38%$; PI = 15%; $\gamma = 18$ kN/m$^2$; $w = 17.5%$; $e = 0.75$; triaxial CD tests [Reis, 2004]</td>
</tr>
<tr>
<td>21</td>
<td>Mature residual soil from gneiss (sandy silt clay, undisturbed)</td>
<td>31</td>
<td>19.2</td>
<td>215</td>
<td>1.665</td>
<td>0.0051</td>
<td>0.98</td>
<td>$w_L = 48%$; PI = 17%; $\gamma = 17$ kN/m$^2$; $w = 26%$; $e = 0.9$; triaxial CD tests [Reis, 2004]</td>
</tr>
<tr>
<td>22</td>
<td>Madrid gray clay (statically compacted)</td>
<td>25.2</td>
<td>170</td>
<td>580</td>
<td>2.126</td>
<td>0.0024</td>
<td>0.98</td>
<td>LL = 71%, PI = 35%; Standard Proctor $\lambda_{max} = 13.3kN/m^2$; $w_p = 33.7%$; Specimens molded $\gamma = 13.3kN/m^2$; $w = 29%$; CD direct shear Escario (1988)</td>
</tr>
<tr>
<td>23</td>
<td>Guadalix de la Sierra red clay (statically compacted)</td>
<td>32.5</td>
<td>93</td>
<td>650</td>
<td>1.570</td>
<td>0.018</td>
<td>0.98</td>
<td>LL = 33%, PI = 14%; Standard Proctor $\lambda_{max} = 18kN/m^2$; $w_p = 17%$; Specimens molded $\lambda = 18kN/m^2$; $w = 13.6%$; CD direct shear Escario (1988)</td>
</tr>
</tbody>
</table>
3.2. Alternative 2

Most test results presented in the literature are from soils that were tested under some limited value of soil suction and still show a tendency of increase shear strength. Consequently, it is probable that the residual shear strength was not reached. This feature can be accommodated by changing the way parameter $b$ is obtained. The point corresponding to the maximum test suction belongs to the curve
that represents shear strength envelope. So, calling \( c_m \) the maximum measured cohesion (or \( \tau_m \) the maximum shear strength) at the maximum value of matric suction, \( \psi_m \), it can be easily shown that

\[
b = \frac{1}{c_m - c} \frac{a}{\psi_m}
\]

substituting for \( b \)

\[
b = \frac{1}{c_m - c} \frac{1}{\tan \phi'}
\]

The use of Eqs. (6), (7) and (11) will be illustrated considering the data shown in Table 1. Table 4 shows some additional data for soils listed in Table 1 along with the derived soil parameters obtained and the coefficient of determination for the experimental and predicted data.

Figure 6 shows a comparison between the predicted function and experimental results. As it can be seen, there is a good agreement between them with coefficients of determination (\( R^2 \)) being larger than 0.95 for most of the data tested.

The performance of both alternatives of the proposed method was very good. Results from many types of soils and different test conditions were reproduced quite well, making the method a practical and reliable tool to calculate the shear strength of unsaturated soils. The method was kept as simple as possible and demands the use of saturated effective shear strength parameters and only one set of tests on air dried samples or at a known suction. The option of testing air dried samples can speed up the preliminary evalu-

| Table 4 - Parameters used to validate the alternative procedure of predicting unsaturated shear strength. |
|-----------------|-----------------|-----------------|-----------------|
| Soil | \( \phi' \) (°) | \( c' \) (kPa) | \( c_m \) (kPa) | \( \psi_m \) (kPa) |
| 1 | 30 | 30 | 103.8 | 200 |
| 2 | 38.7 to 40.1 | 9.2 | 62 | 150 |
| 3 | 29 | 7.8 | 104* | 394 |
| 4 | 25.5 | 44.4 | 173* | 498 |
| 5 | 29 | 0 | 35 | 160 |
| 6 | 31.2 | 10.5 | 36 | 160 |
| 7 | 26.4 | 28.3 | 60 | 160 |
| 8** | 26.4 | 28 | 47 | 206 |
| 9** | 28.7 | 28.3 | 47 | 208 |
| 10 | 21.2 | 35 | 92.3 | 200 |
| 11** | \( \sigma_u = 25 \text{ kPa} \) | 10 | 73 |
| 12** | \( \sigma_u = 100 \text{ kPa} \) | 23 | 38 | 113 | 500 |
| 13 | \( \sigma_u = 200 \text{ kPa} \) | 84 | 164 |
| 14** | \( \sigma_u = 100 \text{ kPa} \) | 40 | 150 | 183.7 | 307 |
| 16 | \( \sigma_u = 20 \text{ kPa} \) | 53 | 68 | 330 |
| 17 | \( \sigma_u = 120 \text{ kPa} \) | 40 | 0 | 17.5 | 60 |
| 18 | \( \sigma_u = 300 \text{ kPa} \) | 74.1 | 238.5 |
| 19 | \( \sigma_u = 450 \text{ kPa} \) | 163 | 363 |
| 20 | \( \sigma_u = 600 \text{ kPa} \) | 25.2 | 229.6 | 459.3 | 1000 |
| 21 | \( \sigma_u = 750 \text{ kPa} \) | 311.1 | 555.6 |
| 22 | \( \sigma_u = 120 \text{ kPa} \) | 377.8 | 651.9 |
| 23 | wot | 30.6 | 10.3 | 115 | 285 |
| 24 | dry | 29.2 | 6.4 | 67.1 | 275 |
| 25 | \( \sigma_u = 100 \text{ kPa} \) | 25.2 | 229.6 | 459.3 | 1000 |
| 26 | \( \sigma_u = 200 \text{ kPa} \) | 311.1 | 555.6 |
| 27 | \( \sigma_u = 300 \text{ kPa} \) | 377.8 | 651.9 |
| 28 | \( \sigma_u = 450 \text{ kPa} \) | 25.2 | 229.6 | 459.3 | 1000 |
| 29 | \( \sigma_u = 600 \text{ kPa} \) | 311.1 | 555.6 |
| 30 | \( \sigma_u = 750 \text{ kPa} \) | 377.8 | 651.9 |

* average value near the maximum suction.
** \( c' \) and \( c_m \) are the whole shear strength (cohesion intercept plus friction component).
Evaluation of soil parameters and avoid the use of complex test arrangements usually demanded by tests on unsaturated soil.

3.3. Limitations of the proposed procedure

Figure 7 shows a typical set of data which shows a decrease in shear strength after a maximum. The soil is a coarse silty sand and the decrease is more noticeable for the lower net normal stress used in the tests, when the soil showed a dilating behavior. In predominantly granular soils it is expected that the main contribution to soil suction is that from capillarity as the effect of adsorptive forces will be less pronounced in these soils. So it is reasonable to admit that the effect of suction on shear strength will reach a maximum and will reduce as strain and dilation induce a perturbation on capillary meniscus, thus causing a reduction on shear strength and other mechanical properties that depend on suction.

In this case, depending on the largest value of suction used in the tests, predicted values can be lower than the measured ones. This will take place when this suction is larger than the suction related to peak value of shear strength. As the model considers that shear strength associated to the largest suction used in the tests is the maximum, the difference between measured and predicted values will increase the larger is the decrease of shear strength past a maximum. In the case of the test results of Fig. 7, the coefficient of determination is still high but one must be aware that in granular soils the model can yield conservative values.

Equation (12) can be formulated as an alternative to fit test results of soils whose shear strength rise and then fall with increasing suction.

\[ c = a + b_1 \psi \]

This equation needs three parameters, \(a\), \(b_1\) and \(\lambda\). The parameter \(a\) is the same in both Eqs. (6) and (12) and can be obtained following Eq. (7). However, parameters \(b_1\) and \(\lambda\) will need two additional tests at different suctions to be determined. In addition it can not be assigned to them a physical meaning as done with the \(a\) and \(b\) parameters of Eq. (6).

---

**Figure 6a-d** - Experimental and predicted shear strength of soils from Table 1, considering the effective shear strength parameters of saturated soil and the shear strength at the largest suction used in the tests.
Figure 7 shows the data of Gan & Fredlund (1996) fitted with Eq. (12), together with the curve yielded by Eq. (6). The adjustment of Eq. (11) to the experimental data was obtained from best fitting analysis, for parameters $b_1$ and $\lambda$, since parameter $a$ is 1.913 from Eq. (7). For the test US3, the $R^2$ obtained was 0.96, $b_1 = 0.00043$ and $\lambda = 1.83$, while for the test US5, the correspondent values were 0.98; 0.002 and 1.456. It can be seen that, in these cases, Eq. (12) is able to fit experimental data fairly well, yielding best results than Eq. (4). However, it must be emphasized that the example of use of Eq. (12) rests on best fit analysis as the parameters cannot be determined following an easily and straightforward procedure as is the case of the method here proposed. So the major interest in using Eq. (12) is as a mathematical function that can be useful to fit test results from soils with a behavior similar as the one shown in Fig. 7, but not as an equation easily used in a prevision procedure.

Contrary to the available theory of unsaturated soil, some soils have shown a large increase in shear strength for low values of suction. In this range of suction, especially beneath the air entry value of the soil, it should be expected that the angle $\phi''$ would reach as much as the value of $\phi'$, however values of $\phi''$ larger than $\phi'$ have been reported by many authors (Rohm & Vilar, 1995; Abramento & Carvalho, 1989; Soares & de Campos, 2005). In these cases the proposed procedure will reproduce the experimental data in a conservative way or even fail. Figure 8(a) illustrates the use of the proposed method to the data of Rohm & Vilar (1995). It can be seen that for the lower net normal stress that the shear strength is underestimated as $\phi''$ is larger than $\phi'$. In extreme cases, such as in the soil tested by Soares &
de Campos (2005), the procedure will fail, as can be seen in Fig. 8(b). It is not known the mechanism that leads these soils to this behavior. In common they are of lateritic nature, their air entry value is almost zero and they reach the residual condition at relatively low values of suction. Laterites are known to harden after wetting and drying. Cut slopes in lateritic soils begin to develop a hard crust on its surface after few days of exposure in a process commanded by evaporation and silica deposition (Vilar et al., 1986). Thus the development of any kind of incipient cementation during the process of suction installation (especially when the specimen is wetted and then drained) should not be excluded as one of the possible reasons that justify the behavior shown for these lateritic soils.

When applying the procedure, caution should be exercised when the product of the parameter \( b \) and matric suction is negligible when compared to \( a \) as the relationship between shear strength and suction will be almost linear and represented by

\[
c = c' + \psi \tan \phi'
\]  

(13)

Thus the prevision will yield values corresponding to the effective shear strength and in this situation the effect of matric suction will be similar to the confining effective stress.

The prevision will fail and should not be used when the value of \( b \) parameter, as calculated through Eq. (11), is negative as is the case of the soil of Fig. 8(b).

The adopted mathematical expression and the test results used to check the proposed method suggest that the predicted values are always underestimated. No reasons for overestimated values have been devised up to now.

The limitations noted so far and others that may arise as more data become available should be seen as a common feature of all the empirical methods.

4. Conclusion

An expedite procedure to predict the shear strength envelope of unsaturated soils was developed and tested against many soils of different origins, showing a good agreement between experimental and calculated values. The method requires effective stress parameters from saturated samples and results of only one set of tests performed on air dried specimens or, alternatively, on specimens tested under a controlled suction, larger than the maximum suction expected in the problem under analysis. The use of air dried samples may be a promising option and could replace the more sophisticated suction controlled tests, considering the good agreement between the limited test data available and the values predicted by the procedure here presented. The procedure has some limitations, as any empirical method, and is intended to be a tool to estimate the shear strength parameters of unsaturated soils for preliminary purposes, not to replace a more complete characterization of soil properties.

References


