Estimating Young Moduli in Sands from the Normalized N_{60} Blow Count

A. Conde de Freitas, M. Pacheco, B.R. Danziger

Abstract. This paper presents statistical correlations between the static Young modulus E_s and the normalized SPT penetration resistance N_{60} (corresponding to 60% of the theoretical energy of the SPT test) for preliminary estimation of settlements in sedimentary sands. The correlations have been established through the statistical interpretation of many experimental studies available in the literature, based on power regression analyses. Equations and charts are presented to estimate the mean value E_s and related statistical limits of E_s as a function of N_{60} . The obtained correlations have been used to compare the calculated settlements of a rigid raft bearing on a multi-layered sand deposit (comprising distinct relative densities) with the measured settlements in eight points underneath the raft.

Keywords: sand compressibility, normalized SPT, Young modulus, correlations.

1. Introduction

The use of seismic tests to obtain correlations between the shear wave velocity V_s and N_{SPT} is widely used in foundation engineering. However, most of the published correlations do not provide the statistical inferences used to support the published results. Also, many of these correlations do not take into consideration normalized N_{60} blow counts, causing a large scatter of the data compiled from different countries.

Hanumantharao & Ramana (2008) compiled several correlations between V_s and N_{SPT} (uncorrected) worldwide for different soil types. It is well known that the low amplitude shear modulus (G_{max}) is directly related to the shear wave velocity and to the density of the soil through which the wave travels (Richart *et al.*, 1970). Thus, G_{max} is estimated by measuring the shear wave velocity and the soil density to predict the stress-strain behavior of soils under low amplitude dynamic excitation. Unfortunately the use of seismic tests is not yet widespread in Brazil, while the SPT is still the most used technique for site investigation there and in many other countries, what supports the development of several correlations between V_s and N_{SPT} worldwide. Anbazhagan & Sitharam (2010) developed recent relationships between V_s and N_{SPT} and pointed out that better correlations could be obtained by correlating N_{SPT} and G_{max} also to the overburden stress.

The most common relationship between V_s to N_{SPT} is of the type given by Eq. 1, although Eq. 2 is also commonly used. In both equations A, B and C are regression constants and N may represent either N_{SPT} or N_{60} .

$$V_{s} = A \cdot N^{B} \tag{1}$$

$$V_s = C + A \cdot N^B \tag{2}$$

The correlations compiled by Hanumantharao & Ramana (2008) for sands have been used to form the database of the present paper, along with further correlations by Ohsaki & Iwasaki (1973), Jafari *et al.* (2002), Athanasopoulos (1995), Wride & Robertson (1997), Wride *et al.* (2000) and Robertson *et al.* (2000). In this paper, a power regression analysis (Eq. 1) is used to relate statistically V_s to N_{60} . It is worth noting that the velocity of the shear wave can be determined reliably by seismic field tests such as crosshole, downhole, uphole, seismic piezocone, seismic refraction, among others. The velocity V_s and all parameters related to V_s are limited to small strains, of the order of $10^4\%$ (Barros, 1997).

Hanumantharao & Ramana (2008) mention that the measured shear wave velocities obtained from different field methods differ only by the order of 10 to 15%. This may be regarded as negligible when compared to the uncertainties to estimate dynamic moduli and make feasible the use of correlations between V_s and N_{60} , even when V_s is determined by different field techniques. Giachetti *et al.* (2006) and Moura (2007) reported a difference of only 6.7% in the average shear wave velocity determined by downhole and crosshole tests in a fine clayey sand from the region of Bauru (State of São Paulo, Brazil).

Starting from published correlations between V_s and N_{SPT} , regression equations relating G_{max} and E_{max} (small strain dynamic Young modulus) to V_s are readily obtained by well known equations of the classical theory of elasticity. Further correlations between E_{max} and the static Young modulus E_s , according to Buzdugan (1972), enable the final correlations

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between the static Young modulus E_s and the corrected N_{60} blow count for sands, as proposed in this paper.

2. Regression Equations Relating E_{max} to N_{60}

Hanumantharao & Ramana (2008) compiled several correlations between V_s and N_{spt} for clayey and sandy soils. In the present paper, however, emphasis is given only to correlations in sands, according to Table 1. Wride *et al.* (2000) and Robertson *et al.* (2000) summarized the Canlex research project (Canadian Liquefaction Experiment) whose main objective was the study of soil liquefaction in saturated sandy soil. From the Canlex database some correlations between shear wave velocity V_s and N_{60} have been established and presented in the research report from different sites, also included in Table 1. The fourth column in Table 1 refers to the range of N_{spt} corresponding to the sands investigated by each author. The fifth column refers to the range of N_{60} for the correlations established in the Canlex research project.

Selection of appropriate correction factors are required to convert N_{SPT} into N_{60} to normalize the correlations in Table 1 according to the actual energy delivered during the SPT test in each country. Several authors have proposed correction factors to account for the actual energy transmitted to the rods in the SPT test (De Mello, 1971; Kovacs *et al.*, 1977; Palacios, 1977; Schmertmann & Palacios, 1979; Belincanta, 1985; Danziger *et al.*, 2008, among others). The ISSMFE (1989) established 60% of the theoretical potential energy as the international reference. Therefore, N_{SPT} should be converted to N_{60} by the expression:

$$N_{60} = N_{SPT} \frac{E}{E_{60}}$$
(3)

In Eq. 3 *E* represents the actual energy delivered to the rods in the SPT test and E_{60} refers to 60% of the theoreti-

cal potential energy of the SPT hammer. If the energy *E* is measured, the above expression should be used. Otherwise, an estimated value for *E* based on past experience is required. Therefore, correcting N_{SPT} is essential to compare correlations from different countries. Décourt (1989) pointed out that the energy just below the anvil (*E*) can be obtained as:

$$E = e_1 \ e_2 \ e_3 \tag{4}$$

where e_1 , e_2 , e_3 , are the efficiencies (or correction factors). The efficiency factor e_1 relates the kinetic energy just before the impact, being mainly dependent on the way the hammer is lifted and released. Values of e_1 suggested by Décourt (1989) are shown in Fig. 1.

The factor e_2 is the ratio between the energy just below the anvil and the kinetic energy just before the impact and it is dependent on the anvil mass (Skempton, 1986). Figure 2 summarizes the main results (Décourt, 1989).



Figure 1 - Efficiency factor e_1 (adapted by Décourt, 1989, from Skempton, 1986).

Table 1	- Correlations betwee	V_{L} and N_{SPT} in sands	s (adapted from Har	numantharao & Ramana,	2008), including other data
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Author	V_{s} (m/s)	Country	$N_{\rm SPT}$ range	N_{60} range
Ohta <i>et al.</i> (1972) ^(*a)	$87.0N_{SPT}^{0.36}$	Japan	1 to 50	
Ohsaki & Iwasaki (1973)	59.0N ^{0.47} _{SPT}	Japan	1 to 50	
JRA (1980) ^(*b)	$80.0N_{SPT}^{0.33}$	Japan	1 to 50	
Seed et al. (1983)	56.4N ^{0.50} _{SPT}	USA	1 to 50	
Sykora & Stokoe (1983)	$106.7N_{SPT}^{0.27}$	USA	1 to 50	
Lee (1990)	57.0N ^{0.49} _{SPT}	USA	1 to 50	
Massey – Fraser River Delta(* ^c)	$92.9N_{60}^{0.25}$	Canada		9 to 21
Kidd – Fraser River Delta(*°)	$83.7N_{60}^{0.25}$	Canada		13 to 43
J-Pit – Syncrude(*°)	$92.8N_{60}^{0.25}$	Canada		2 to 29
LL Dam – HVC Mine(*°)	$102.9N_{60}^{0.25}$	Canada		2 to 7
Highmont Dam – HVC Mine(*°)	$95.4N_{60}^{0.25}$	Canada		3 to 13

(*a) from Ohsaki & Iwasaki (1973). (*b) from Jafari et al. (2002). (*c) CANLEX Research.

1989).



Figure 2 - Efficiency factor e_2 as a function of the anvil mass (Décourt, 1989).

The efficiency factor e_3 is due to the rod length. According to Schmertmann & Palacios (1979) the driving energy would only be fully transmitted to the rods if they had a minimum critical length. This would occur because in most cases the first compression wave pulse is reflected in the lower end of the sampler as a tension wave. Therefore the tension wave induces a separation between the hammer and the rods, preventing further transfer of energy. Recent researches, however, have shown that the subsequent (secondary) impacts in the same blow contribute to full energy transmission, indicating that the energy just below the anvil is independent of the length of the rod stem, and thus the factor e_3 should be taken as unity.

Odebrecht (2003) and Odebrecht *et al.* (2005) have shown that the potential energy resulting from the penetration of the sampler should be added to the nominal potential energy, which is significant mostly in case of small rod lengths in soft clays and loose sands.

Aoki & Cintra (2000) pointed out that the energy producing the sampler penetration (which is associated to the N value) is the one that reaches the sampler, and not the one below the anvil. Thus, the corresponding energy loss over the rods should also be taken into account. Thus another factor e_4 that quantifies the energy loss over the rods should also be included in Eq. 4. Unfortunately, very few data is available about this factor (*e.g.*, Cavalcante, 2002; Odebrecht, 2003; Johnsen & Jagello, 2007).

It turns out that the only reliable way to quantify the SPT energy losses is by proper measurement of the actual energy delivered to the sampler. In this paper, N_{60} values are assigned according to average correction factors *C* reported by Décourt *et al.* (1989), as shown in Table 2. For example, the value C = 1.05 (USA) in Table 2 refers to the average value for 0.75 (donut/rope-cathead); 1.00 (safety/hope/cathead) and 1.40 (safety/free fall). The correction factor *C* is given by:

$$C = \frac{E}{E_{60}} \tag{5}$$

$$N_{60} = C \cdot N_{SPT} \tag{6}$$

Country С 0.75 Argentina Brazil 1.20 China 1.00 Colombia 0.83 Japan 1.27 Paraguay 1.20 U.K. 0.92 U.S.A. 1.05 Venezuela 0.72

Table 2 - Correction factors C - (adapted from Décourt et al.,

For the Brazilian SPT, Décourt (Table 2) proposed:

$$N_{60} = 1.20 \cdot N_{SPT} \tag{7}$$

Further research based on actual energy measurements on the SPT hammers mostly used in Brazil (*e.g.*, Belincanta, 1985, 1998; Cavalcante, 2002; Odebrecht, 2003) has indicated:

$$N_{60} = 1.37 \cdot N_{SPT}$$
 (8)

Equations 7 and 8 are used in Session 3 to enable correlations for practical application in Brazil. Hanumantharao & Ramana (2008) pointed out that most correlations may produce acceptable V_s predictions for N_{spr} values up to 40. According to the Authors experience in settlement predictions based on N_{spr} , the correlations above require good engineering judgment for $N_{spr} > 30$ and should be avoided for N_{spr} above 50 or below 4.

To relate E_{max} to N_{60} , the following equations of the classical theory of elasticity are used:

$$E_{\max} = 2 \cdot G_{\max} \cdot (1 + \nu) \tag{9}$$

$$G_{\max} = V_s^2 \cdot \rho \tag{10}$$

From Eqs. 9 and 10:

$$E_{\max} = 2 \cdot \left(V_s^2 \cdot \frac{\gamma}{g} \right) \cdot (1 + \nu)$$
(11)

In the equations above v is the Poisson ratio, ρ the soil unit mass, γ the soil unit weight and g the acceleration of gravity. Equation 11 enables to relate E_{max} to N_{60} by assigning a power regression curve to V_s (Eq. 1). Therefore, each pair N_{60} x E_{max} (for each regression equation in Table 1) is plotted in Fig. 3 for all integer N_{SPT} and N_{60} values according to the ranges given by the two last column of Table 1. N_{SPT} values are corrected to N_{60} according to Table 2. The points in Fig. 3 have been plotted by assigning v = 0.3, γ = 18 kN/m³ and g = 9.8 m/s² in Eq. 11.

According to Conde de Freitas (2010) the soil specific weight increases only moderately with N_{60} , with minor impact on Eqs. 9, 10 and 11. Therefore, a constant value $\gamma = 18 \text{ kN/m}^3$ was assigned throughout this work.

Attention should be pointed out that Eqs. 9 and 10 have been derived from the classical theory of elasticity that assumes a constant modulus for soil mass. Such a simplification is only justified in the proposed correlations as long as its use has been conceived as preliminary settlement estimations.

To fit a regression power curve through the 357 points in Fig. 3a, a transformed linear regression analysis is carried out by plotting $\log(N_{60}) \ge \log(E_{\max})$, as in Fig. 3b (Conde de Freitas, 2010). The desired regression power curve in Fig. 3a is readily determined by mapping back the linear regression curve (Fig. 3b) into the original plot ($N_{60} \ge E_{\max}$), producing:

$$E_{\rm max} = 24975 \cdot N_{60}^{0.75} \, (\rm kPa) \tag{12}$$

For comparison, an alternative relationship between N_{SFT} (uncorrected) and E_{max} is readily obtained from Eq. 13 below, widely used in the oil industry in Brazil for design of foundations for machinery (Petrobras, 2008; Machado, 2010):

$$G_{\rm max} = 12000 \cdot N_{SPT}^{0.80} \, (\rm kPa) \tag{13}$$

From Eqs. 9 and 13:

$$E_{\max} = 31200 \cdot N_{SPT}^{0.80} \text{ (kPa)}$$
(14)

Equations 12 and 14 are compared in Session 3 to estimate the static Young modulus E_s , the main objective of this paper. Experimental data by Machado (2010) has confirmed the adequacy of Eq. 13 to estimate the shear modulus G_{max} for design of foundations for machinery.



Figure 3 - (a). N_{60} vs. E_{max} points for sands and corresponding power regression curve and (b). Transformed linear regression $\log(N_{60})$ vs. $\log(E_{max})$ (Conde de Freitas, 2010).

Statistical limits for the points in Fig. 3a have been determined in Fig. 4 (Conde de Freitas, 2010) according to Neter *et al.* (1982); Pacheco & Lima (1996), for *n* standard deviations about the mean regression curve (*n* ranging from 0.5 to 2).

3. Static Young Moduli

Equations 12 and 14 are applicable to engineering problems related to small strains, as in dynamic analyses of foundations for machinery. For static problems, however, a reduction factor is to be applied to the dynamic modulus E_{max} to estimate the corresponding static Young modulus E_s . Laboratory tests on reconstituted samples in sands indicate that the dynamic shear modulus may be reduced more than tenfold for shear deformations of $10^{-3}\%$ to 1% (Barros, 1997; Moura, 2007). According to Kulhawy & Mayne (1990); Moura (2007), the shear modulus for static loads is about 5 to 10% G_{max} . A similar reduction is also reported by Sitharam *et al.* (2004), Fig. 5. Therefore it is important to keep in mind that estimates of E_s (or G_s) should be made according to the range of shear deformation expected for each problem under consideration (Silveira *et al.*, 2006).



Figure 4 - Statistical limits for E_{max} from the points in Fig. 3a (adapted from Conde de Freitas, 2010).



Figure 5 - Shear modulus *vs.* shear deformation for Ahmedabad sand for two relative densities (Sitharam *et al.*, 2004).

Buzdugan (1972) summarized the range of variation of dynamic (E_{max}) and static (E_s) Young moduli (Table 3) for foundation design. An average ratio $E_{max}/E_s = 3$ is inferred from Table 3 to estimate E_s from Eqs. 12 and 14 in sands. This ratio is recommended only to well-designed foundations whose applied pressure is far from failure, as expected in large dimension foundations like rafts, tanks and silos. The ratio E_{max}/E_s is expected to increase for smaller safety factors (larger shearing strains) and therefore good engineering judgment and experience are required for practical use of the correlations proposed herein. Furthermore, the ratio E_{max}/E_s increases for clayey soils (Buzdugan, 1972). Therefore, the ratio $E_{max}/E_s = 3$ as well the correlations presented in this paper should also be restricted to sedimentary pure sands with negligible amount of fines, considering that even small percentages of fines are likely to produce higher $E_{\rm max}/E_{\rm s}$ ratios.

The regression curves in Fig. 4 ($N_{60} \ge E_{max}$) are converted into $N_{60} \ge E_s$ in Fig. 6 for $E_{max}/E_s = 3$. The corresponding mean power curve in Fig. 6 (determined from Eq. 12) is given by:

$$E_{s} = 8325 \cdot N_{60}^{0.75} \text{ (kPa)} \tag{15}$$

For comparison, the static modulus E_s estimated from Eq. 14 for $E_{max}/E_s = 3$ is:

$$E_{s} = 10400 \cdot N_{SPT}^{0.80} \text{ (kPa)}$$
(16)

For practical applications using the Brazilian SPT, curve (a) in Fig. 6 is obtained from Eq. 16 for C = 1.20 (Eq. 7), as:

$$E_{s} = 8988 \cdot N_{60}^{0.80} \text{ (kPa)} \tag{17}$$

Alternatively, curve (b) in Fig. 6 is obtained from Eq. 16 for C = 1.37 (Eq. 7), as:

$$E_{s} = 8084 \cdot N_{60}^{0.80} \text{ (kPa)} \tag{18}$$

Curve (a) in Fig. 6 is very close to the curve corresponding to 1.0 standard deviations above the mean curve, whereas curve (b) nearly coincides with the curve corresponding to 0.5 standard deviations above the mean curve. The good agreement between curves (a), (b) and the mean curve given by Eq. 15 seems to indicate that the regression curves in Fig. 6 provide a simple and reasonable statistical procedure to estimate E_s , in addition to providing a simple way to account for the uncertainty in the predictions (varia-

Table 3 - Ranges of static and dynamic moduli for sands (adapted from Buzdugan, 1972).

Soil type	Young Modulus (x10 ⁵ N/m ²)				E _{max}	$E_{\rm max}$ / E_s	
	E_s (s	tatic)	$E_{\rm max}$ (dy	/namic)	Lower range	Upper range	
(1) Loose sand (rounded grains)	400	800	1500	3000	3.75	3.75	
(2) Loose sand (angular grains)	500	800	1500	3000	3.00	3.75	
(3) Medium dense sand (rounded grains)	800	1600	2000	5000	2.50	3.13	
(4) Medium dense sand (angular grains)	1000	2000	2000	5000	2.00	2.50	
(5) Clean gravel (no sand fraction)	1000	2000	3000	8000	3.00	4.00	
(6) Gravel (angular grains)	1500	3000	3000	8000	2.00	2.67	
Lower and upper mean E_{max}/E_s	2.71	3.30					
Global mean E_{m}/E_{c}	3.00						



Figure 6 - Static moduli E_s for sands $(E_{max}/E_s = 3)$, adapted from Conde de Freitas (2010).

tions of *n* standard deviations about the mean). Equations 15, 17 and 18 indicate that the static Young modulus E_s can be expressed in round numbers by:

$$E_s = 8000 \cdot N_{60}^{0.80} \, (\text{kPa}) \tag{19}$$

It should be pointed out that the Young modulus E_s increases with depth in nearly normally consolidated sand deposits and this can be easily accounted for in FE modeling. In the present study, however, the variation of E_s with depth was modeled by Eq. 19 after subdividing the sand deposit into several thin layers represented by individual average N_{60} values which account indirectly for the overconsolidation condition.

4. Settlement Analysis of a Raft on Multilayered Sand Deposit

The following is an application of Eq. 19 to predict the settlements of an instrumented nearly rigid raft bearing on a sedimentary sand deposit investigated by Lopes *et al.* (1994), and Lopes (2000). Those authors describe the measured settlement of a raft foundation during a period up to 3 years after completion of the structure. The raft supports the building housing a diesel power generator and applies to the soil a net uniform pressure $\Delta p = 123$ kN/m² (building + generator). The raft dimensions (16.6 m x 27.0 m) and the instrumentation points are shown in Fig. 7 (Lopes *et al.*, 1994).

The sand deposit has been subdivided into six layers whose deformation parameters have been assigned in Table 4 according to the normalized N_{60} blow count obtained from Fig. 8.

The soil profile is subdivided into six moderately inclined sand layers interpreted from the SPT boring logs, as depicted in Fig. 9 (input geometry to the finite element program Plaxis 3-D Foundation). The raft bears on top of the sand deposit above which was placed a 4-m high fill. The water table is assumed to coincide with the base of the raft (4 m below the fill surface). The Young modulus for each sand layer is estimated by Eq. 19 taking the average normalized N_{60} blow counts in Table 4. Other input parameters to the numerical analysis are listed in Table 5.



Figure 7 - Raft dimensions and location of the instrumentation points.



Figure 8 - Soil profile, Lopes et al. (1994).

Table 4 - Deformation parameters for the sand deposit.

Layer	Average thickness (m)	Normalized blow count N_{60}	Young modulus E_s (kN/m ²)
Sand 1	3	22	95000
Sand 2	2	18	80000
Sand 3	2	10	50000
Sand 4	2	4	24000
Sand 5	2	6	34000
Sand 6	1	17	77000



Figure 9 - Raft bearing on multilayered sand deposit.

In Table 5, γ_t is the total (saturated) unit weight, γ_h the natural (non saturated) unit weight, v the Poisson ratio, *c* the cohesion and ϕ the friction angle.

Figure 10 shows the displacement contours at a longitudinal cross section passing through the center of the raft. The measured and the calculated displacements at the instrumentation points are shown in Table 6. It is seen that the calculated values are in reasonably good agreement with

Layer	$\gamma_t (kN/m^3)$	$\gamma_h (kN/m^3)$	E_s (kN/m ²)	ν	$c (\text{kN/m}^2)$	φ (°)
Fill	-	18	20000	0.30	5	30
Sand 1	20	-	95000	0.30	1	37
Sand 2	20	-	80000	0.30	1	35
Sand 3	20	-	50000	0.30	1	35
Sand 4	19	-	24000	0.35	1	30
Sand 5	19	-	34000	0.35	1	30
Sand 6	20	-	77000	0.30	1	35
Residual	18	-	200000	0.30	20	37
Raft (concrete)	25	-	2.5 x 10 ⁷	0.20	linear e	lastic

 Table 5 - Input parameters for the FEM analysis (program Plaxis 3-D Foundation).



Figure 10 - Displacement contours, longitudinal cross section through the center of the raft.

the measured results at the instrumentation points, whereas the average settlement (15.06 mm) coincide.

Equation 19 may also be useful to estimate the oedometric modulus E_0 in sands. From the theory of elasticity:

$$E_{0} = \frac{E_{s}(1-\nu)}{(1-2\nu)(1+\nu)}$$
(20)

Combining Eqs. 19 and 20 for v = 0,3:

$$E_0 \cong 1.35 \cdot E_s = 10800 \cdot N_{60}^{0.80} \tag{21}$$

Equation 22 below and Eq. 21 are used to estimate the one-dimensional settlement (*r*) of the 6-layer sand in Fig. 9 under the raft pressure $\Delta p = 123 \text{ kN/m}^2$.

$$r = \sum_{i=1}^{6} \frac{\Delta p \cdot h_i}{E_{0i}} \tag{22}$$

The thicknesses h_r the oedometric moduli E_{0i} and the settlement r_i for each sand layer are shown in Table 7. The estimated total settlement is r = 22.87 mm. This is higher than the average measured settlement of 15.06 mm due to the assumption of infinite loading and also because Eq. 22 does not consider the confinement provided by the 4-m

Table 6 - Measured vs. calculated settlements.

	Settlement (mm)				
Point	Measured	Calculated (3-D FEM analysis)	Deviation from mean settlement (%)		
А	19.5	14.5	33.20		
В	17.5	15.5	13.28		
С	12.5	14.0	-9.96		
D	9.8	12.0	-14.61		
Е	20.0	16.5	23.24		
F	17.5	18.0	-3.32		
G	13.7	17.0	-21.91		
Н	10.0	13.0	-19.92		
Average	15.06	15.06			

high fill and the stiffness of the raft (both accounted for in the FEM analysis).

To account for those effects, the following equation from the theory of elasticity modified by Barata (1962, 1984, 1986) is used:

$$r = \frac{\lambda \cdot c_{\Delta} \cdot \Delta p \cdot B \cdot (1 - v^2) \cdot I}{E_s}$$
(23)

The following values are plugged in Eq. 23 (Barata, 1962, 1984, 1986); $\lambda = 0.96$ (Mindlin's coefficient); $c_{\Delta} = 1.09$ (shape factor, rigid foundation); $\Delta p = 123$ kN/m²; B = 16.6 m (foundation width); v = 0.3 (Poisson ratio); I = 0.63 (influence factor accounting for a rigid boundary about 12 m below the raft); $E_s = 61500$ kN/m² (weighted mean static Young modulus, according to Table 4). The settlement estimated by Eq. 23 is r = 19.9 mm. This is about 33% higher than the average FEM predictions (which coincided with the average measured settlement) and 15% lower than the estimated 1-D settlement.

The settlements estimated by the three models above are superimposed on Burland and Burbidge chart in Fig. 11

Layer	Average thickness (m)	Normalized blow count N_{60}	Oedometric modulus E_0 (kN/m ²)	1-D settlement (mm) (*)
Sand 1	3	22	128045	2.88
Sand 2	2	18	109054	2.26
Sand 3	2	10	68143	3.61
Sand 4	2	4	32739	7.51
Sand 5	2	6	45284	5.43
Sand 6	1	17	104180	1.18
			Σ	22.87

Table 7 - Deformation parameters for the sand deposit.

(*) Eqs. (21), (22).

(Burland & Burbidge, 1985; Sivakugan & Pacheco, 2011). Considering that Burland and Burbidge chart provides an upper bound for the expected settlements, the results shown in Fig. 11 seem to indicate the adequacy of the estimated Young moduli according to Eq. 19 to calculate the raft settlements.

5. Conclusions

This paper presents correlations between the static Young modulus E_s and the normalized penetration resistance N_{60} for pure sands, aiming at preliminary settlement predictions. The proposed correlations can also be extended to preliminary estimates of the oedometric modulus E_o in pure sands.

The correlations in this paper are limited to the statistical interpretation of several results published in the literature. Therefore, practical application of equations and charts presented in this paper should be supported further by load tests and other *in-situ* tests. Eq. 19 is generally suited to applied pressures sufficiently far from failure, as expected in well-designed rafts and foundations for tanks and silos where reasonably low static shearing strains result. For higher shearing strains, it is recommended to select E_s values below the mean trend equation in Fig. 6. In



Figure 11 - Burland and Burbidge chart: upper limits of settlement per unit pressure (Burland & Burbidge, 1985; Sivakugan & Pacheco, 2011).

contrast, E_s values above the mean trend curve should be used with increased caution and only when supported by load tests.

The proposed correlations have been used to predict satisfactorily the settlements of a nearly rigid raft supporting the structure of a diesel power generator on sedimentary sand. Good predictions have been obtained by 3-D FEM analysis taking E_s values from Eq. 19. Reasonable predictions have also been achieved by one-dimensional calculations using the oedometric modulus E_o (Eqs. 21 and 22) and the elasticity equation modified by Barata (Eq. 23). The results coming from the three models plotted satisfactorily on Burland and Burbidge chart, indicating that the proposed correlations are useful for preliminary estimation of settlements in sedimentary sands.

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