Considerations on the Probability of Failure of Mine Slopes

A.S.F.J. Sayão, S.S. Sandroni, S.A.B. Fontoura, R.C.H. Ribeiro

Abstract. Probabilistic and deterministic stability analyses of the progress of a large mine pit excavation in Brazil are presented herein. A simple method of reliability analysis for quantifying the probability of failure of slopes has been considered and its advantages and limitations are briefly discussed. The variance of the factor of safety is computed for several stages of the mine excavation. It is shown that, depending on the slope height, either the friction angle or the effective cohesion may be the most important variable controlling stability. In the case of mine pit excavations ranging from 50 to 400 m in height, pore pressures are of lesser relative importance. Consequently, increasing the capacity of the horizontal drainage system may be of limited efficiency in stabilizing potentially unsafe mine slopes. In addition, variables with no significant effect on the stability, such as the apparent specific gravity of the slope material, may be simply considered as deterministic parameters.

Keywords: probability of failure, reliability analysis, slope stability, factor of safety.

1. Introduction

In open pit mine slopes, adequate safety and serviceability must be ensured with maximum economy. Deterministic stability analysis for a specific slope configuration involves assigning an average value to each variable considered in calculating the Factor of Safety (*FS*), which is usually defined as the ratio between strength R and solicitation S. The minimum acceptable value of *FS* in slope studies is usually defined on the basis of the designer's previous experience and on the predicted consequences of a potential failure. Uncertainties present in the definition of geomaterial parameters are not taken into consideration in deterministic computations.

However, each variable has a distribution of probable values, from which a mean value and a standard deviation can be defined. A probabilistic analysis considers the distributions of random variables in *R* and *S* for obtaining the distribution of *FS* values. Acceptable risk in mine pit slope design varies from one situation to another. It is not rare to consider a high probability of failure to be tolerable in situations where the cost of slope stabilization is higher than the costs of cleaning up or mining to flatter angles (Barnett *et al.*, 2001).

Different designers will usually accept or assume different safety degrees for a given slope situation. As a consequence, diverse values of calculated risk will be inherently adopted (Sandroni & Sayão, 1992). Hence, two questions may be raised: (1) How reliable is the adopted slope design? (2) How to compute the calculated risk of a slope failure in a simple, practical method?

In trying to answer these questions, probabilistic and statistic tools should be used in a rational reliability procedure, thus providing a means for evaluating the combined effects of uncertainties (Duncan, 2000). Two types of reliability analysis may be considered. The first, denoted as Relative Reliability, consists on the evaluation of the slope safety by a reliability index (β). Its use has become progressively common in geotechnical practice. The second type of analysis, defined as Global Reliability, describes the slope risk by taking into consideration all random variables involved. Although this global risk analysis may be considered as more accurate, it is very difficult to be implemented in practice (Mostyn & Li, 1993).

This paper presents a simplified procedure for quantifying the relative reliability of mine slopes, with comments on the advantages and limitations of the probabilistic approach. The examples herein reported are based on the First Order Second Moment (FOSM) method (Christian *et al.*, 1992).

Other probabilistic techniques, like the Monte Carlo and the Point Estimate methods, have also been used in Geotechnical Engineering. Details on these methods are provided by El-Ramly *et al.* (2002) and Baecher & Christian (2003). More innovative probabilistic analysis procedures, like the random finite-element method (RFEM), are also becoming available. This method employs elasto-plastic relations in a finite-element model combined with a random field theory in a Monte Carlo or a Point Estimate framework (Hammah *et al.*, 2009; Griffiths *et al.*, 2009). The RFEM was shown to correctly search for the weakest

A.S.F.J. Sayão, Associate Professor, Departamento de Engenharia Civil, Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro, RJ, Brazil. E-mail: sayao@puc-rio.br.
S.S. Sandroni, Principal, Geoprojetos Eng. Ltd, and Visiting Professor at Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro, RJ, Brazil. E-mail: sandro@geoprojetos.com.br.
S.A.B. Fontoura, Associate Professor, Departamento de Engenharia Civil, Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro, RJ, Brazil. E-mail: fontoura@puc-rio.br.
R.C.H. Ribeiro, Associate Professor, Departamento de Engenharia Civil, Universidade Federal do Espírito Santo, Vitória, ES, Brazil. E-mail: romulocastello@yahoo.com.br.
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failure path through heterogeneous materials, leading to probabilities of failure higher than would be predicted by disregarding spatial variations. Although promising, this advanced numerical technique is not yet readily available to geotechnical practice.

2. Reliability Index

The reliability index concept comprises the behaviour of a performance function defined by $G = R - S$, which describes the safety condition of a slope and may be denoted by $G(X)$. In this function, X is an array of the random input parameters or variables. A safe condition is defined by $G(X) > 0$. An unstable domain is defined by $G(X) < 0$, characterizing the slope failure. The boundary value, $G(X) = 0$, is generally referred as the limit state boundary.

In a slope stability problem, considering that the random variables have a normal (Gaussian) distribution, the margin of safety may be expressed by the reliability index β, as proposed by Hasofer & Lind (1974):

$$\beta = \frac{\overline{FS} - 1.0}{\sigma[FS]} \tag{1}$$

In Eq. 1, $\overline{FS}$ denotes the mean value of the factor of safety distribution and 1.0 is the value corresponding to failure, while $\sigma[FS]$ is the standard deviation of *FS*. The index β is therefore equivalent to the number of standard deviations that separates the computed safety factor from the failure value. It is important to note that the use of non-Gaussian distributions in Eq. 1 may yield to inaccurate results. Similar considerations in slope safety applications have been reported by Chowdhurry *et al.* (1987) and Christian *et al.* (1994). Probabilistic approaches for various design applications have been presented: Fenton at al (2005) and Ribeiro *et al.* (2008) reported on slope stabilization walls while Aoki & Tsuha (2010) described the use of reliability on pile design.

3. Probability of Failure

The risk associated to the collapse of a slope is directly related to its relative probability of failure P_f, which is the probability of $G(X)$ being smaller than zero. The relationship between the reliability index β and the probability of failure P_f is given by the following expression (Catalán & Cornell, 1976):

$$P_f = \Phi(-\beta) = 1 - \Phi(\beta) \tag{2}$$

In this equation, $\Phi(\beta)$ is the cumulative distribution function of normalized $G(X)$ with reference to β. Equation 2 is applicable to any distribution of *FS*, depending on the value of the reliability index β. Lee *et al.* (1983) and Whitman (1984) present typical values of the reliability index for non-Gaussian distributions.

Dell Avanzi (1995) proposed an indirect relation between Eq. 1 and other non-Gaussian distributions. In this case, the relationship between β and P_f is not unique and depends on the standard deviation $\sigma[FS]$.

Figure 1 compares the β x P_f relations for normal and lognormal densities. For any given *FS* distribution, it is seen that an increasing reliability index β corresponds to a decreasing probability of failure P_f. For $\beta < 0.8$, the value of P_f is nearly independent of the assumed distribution of *FS* function. For $\beta > 0.8$, the normal (Gaussian) distribution gives the highest value of P_f. Hence, it may be considered as a conservative assumption.

The value of maximum P_f (or minimum β) to be recommended for a slope condition will depend on the consequences of a potential failure and on the uncertainties related to the project. Whitman (1984) and Harr (1987) recommended typical values of relative probability of failure for various geotechnical design situations in practice. Table 1 presents a summary of these recommendations. It may be noted that the reliability of open pit mine slopes is usually the lowest within geotechnical problems. This suggests that mining engineers usually work with lower margins of safety, or higher values of relative probability of failure. This may be explained by the need to minimize volumes of

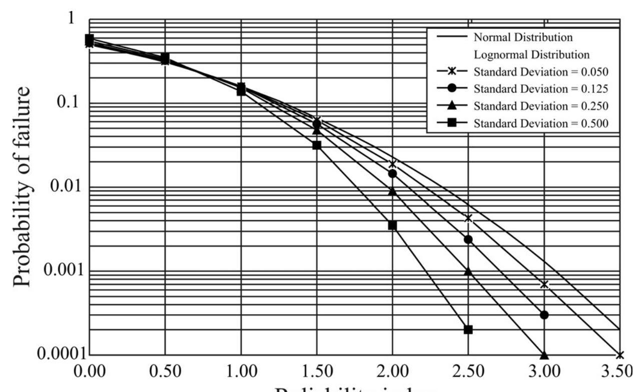

Figure 1 - Comparison of β *vs.* P_f relationships for normal and lognormal distributions of *FS*.

Table 1 - Typical values of reliability index and relative probability of failure in geotechnical practice.

Application	Reliability index β	Probability of failure P_f
Mine pit slopes	1.0 to 2.3	2×10^{-1} to 10^{-2}
Foundations and retaining structures	2.0 to 3.0	2×10^{-2} to 10^{-3}
Earth dams	3.5 to 5.0	2×10^{-3} to 10^{-5}

waste excavation. Moreover, an eventual slope failure may have only limited consequences.

4. Procedure for Reliability Analysis

The technique considered for reliability analyses is called FOSM – First Order Second Moment method (Christian *et al.*, 1992) and may be summarized in five simple steps:

Step 1 - Evaluation of the mean and standard deviation values for all variables

Geotechnical parameters (friction angle, cohesion, unit weight, and pore pressure) and geometrical factors (slope height, and inclination) may be defined as random variables. The cost for obtaining meaningful experimental data is an important consideration in this step. Lee *et al.* (1983) and Ribeiro (2008) listed typical values of coefficient of variation ($CV = \sigma[FS]/\overline{FS}$) for different soil parameters. These typical values are summarised in Table 2 and may be helpful in preliminary reliability estimates.

Step 2 - Search for the critical slip surface and $\overline{FS}$

In this step, deterministic stability analyses are accomplished with one of the well known limit equilibrium methods (*i.e.*, Bishop, 1955; Janbu, 1957; Morgenstern & Price, 1965). The mean values of the random variables are used in these analyses. Two assumptions are implicitly related to the critical slip surface (Sandroni & Sayão, 1992): the computed factor of safety is the mean value of the *FS* distribution, and the corresponding β is the lowest value of the reliability index. The errors associated to these assumptions were reported to be negligible for earth dam and mine slopes (Dell Avanzi, 1995, and Guedes, 1997, respectively). However, Li & Lumb (1987) and Assis *et al.* (1997) suggested that critical surfaces for minimum *FS* and minimum β should be always investigated.

Step 3 - Evaluation of the partial derivatives of the *FS* function

These derivatives are estimated by divided differences. A small increment is separately imposed to each variable x_i, yielding the corresponding variation of the factor of safety (dFS). The ratio dFS/dx_i may be approximated to the partial derivative of the performance function for the variable x_i (Christian *et al.*, 1992). The magnitude of dx_i needs to be small enough to validate the partial derivative approximation, but large enough to yield a meaningful value of dFS. Dell Avanzi (1995) reported that increments of 10% of the mean value of the selected variable may be acceptable for slope safety calculations. However, in other practical situations, like shallow or deep foundations, the increments dx_i may need to be smaller (Ribeiro, 2008).

Step 4 - Evaluation of the standard deviation of *FS* distribution

The variance of the *FS* function has been defined by (Harr, 1987):

$$V[FS] = \sum_{i=1}^{n} \left(\frac{\partial FS}{\partial x_i} \right)^2 \cdot V[x_i] \qquad (3)$$

where $V[x_i]$ is the variance of variable x_i. By definition, the variance of x_i is the square of its standard deviation ($\sigma[x_i]$). Likewise, the standard deviation $\sigma[FS]$ is obtained by computing the square root of the variance $V[FS]$. The use of Eq. 3 is appropriate only to statistically independent random variables. Its application to dependent variables may induce significant errors, depending on the covariance of each pair of variables considered in the analysis. In the applications herein described, the assumption of statistical independence is adequate.

Table 2 - Typical values of the coefficient of variation from geotechnical literature.

Geotechnical parameter	Coefficient of variation (%)	
	Minimum	Maximum
Unit weight of saturated soils (γ_{sat})	3	7
Coefficient of permeability of saturated clays (k)	68	90
Coefficient of consolidation of saturated clays (c_v)	33	68
Compression index of saturated clays (c_c)	10	37
Undrained strength of saturated clays (S_u)	13	40
Effective friction angle of saturated clays (ϕ')	2	13
Tangent of effective friction angle of gneissic residual soils (tan ϕ')	2,4	16,1
Effective cohesion of gneissic residual soils (c')	13,4	18,4
Standard penetration number (N_{SPT})	15	45
Piezocone tip resistance (q_c)	5	15

Step 5 - Evaluation of the probability of failure

After computing the reliability index β from Eq. 1, the value of P_f may be obtained from Fig. 1, considering the assumed distribution of *FS*.

5. Example of Probabilistic Analysis of a Mine Slope

The FOSM method described in the previous section allows identifying the relative contributions of each random variable in the uncertainty associated to the factor of safety. This may be very useful for the designer, when deciding upon the most adequate method for stabilising a potentially unstable slope (Sandroni & Sayão, 1992; Dell Avanzi & Sayão, 1998).

An example illustrating this aspect is the following analysis of relative probability of failure for a large open pit mine in Brazil. The slope consists predominantly of residual soil from a schist rock, with high content of quartz and mica, remaining from exploration of iron ore. Fig. 2 shows a schematic cross section of the slope. Average inclination was 34° and the height ranged from H = 30 to 400 m, as mining operations progressed.

Piezometric conditions were defined from comprehensive long-term field instrumentation and may be reasonably represented by two linear phreatic segments. At the top portion of the slope, the water table was horizontal and 80 m deep. As the mine pit was deepened, horizontal drainage holes were progressively installed to avoid the pit bottom area to becoming submerged. Therefore, the phreatic line could be taken as an inclined line ending at the foot of the slope.

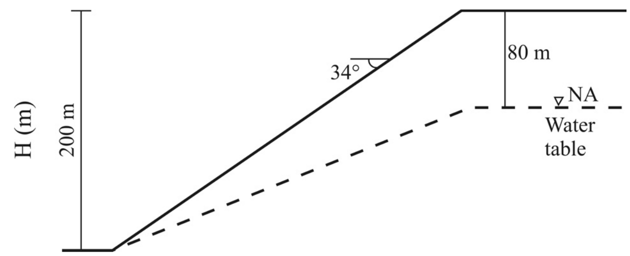

Figure 2 - Cross-section of a large mine slope in Brazil.

Average and standard deviation values of geotechnical and piezometric parameters, which were considered as variables, are presented in Table 3. Geotechnical data was obtained from about 50 direct shear tests on undisturbed specimens of the residual soil. For convenience, tan ϕ' was considered as the friction random variable, instead of ϕ'. Farias & Assis (1998) reported that either ϕ' or tan ϕ' may be used, with no differences in the computed value of P_f. In this case, geometric factors (slope height and inclination) have been considered as deterministic variables.

Conventional limit equilibrium slope stability analyses, with Janbu's method and average values from Table 3, indicated a safety factor $\overline{FS}$ = 1.34 for a 200 m high slope. Computation of the variance of the safety factor for this case is detailed in Table 4, which summarizes steps 3 and 4 of the procedure for reliability analysis. The computed variance of the safety factor is $V[FS]$ = 0.0259, which corresponds to a standard deviation $\sigma[FS]$ = 0.161.

From Eq. 1, the reliability index β = 2.12 is computed. For estimating the relative probability of failure (step 5), an assumption is required on the *FS* distribution. From Fig. 1,

Table 3 - Variable parameters for the iron ore mine slope analysis.

Variable x_i	Symbol	Average $\bar{x}_i$	Standard deviation σ $[x_i]$	Variance $V[x_i]$
Effective friction	tan ϕ'	0.781	0.085	0.0072
Effective cohesion	c' (kPa)	25.0	24.3	590.0
Unit weight (natural)	γ_{nat} (kN/m^3)	28.3	1.4	1.96
Unit weight (saturated)	γ_{sat} (kN/m^3)	29.0	1.4	1.96

Table 4 - FOSM method: probabilistic analysis of a mine slope with $\overline{FS}$ = 1.34.

Variable $\bar{x}_i$	$V[x_i]$	dx_i	dFS_i	dFS_i/dx_i	(dFS_i/dx_i)^2.$V[x_i]$
tan ϕ' = 0.781	0.0072	0.11	+0.188	+1.6682	0.020037 (77.1%)
c' = 25.0 kPa	590.0	2.50	+0.004	+0.0016	0.001510 (5.8%)
γ_{nat} = 28.3 kN/m^3	1.96	2.83	-0.004	-0.0014	0.000004 (0.0%)
γ_{sat} = 29.0 kN/m^3	1.96	2.90	+0.022	+0.0078	0.000119 (0.4%)
z_p = 80 m	400.0	10.0	-0.033	-0.0033	0.004356 (16.7%)
					Total V [FS] = 0.026026 (100%)

$\overline{FS}$ = 1.34, $\sigma[FS]$ = 0. 161, β = 2.12, P_f = 0.017.

and assuming a normal distribution of FS, the value P_f = 0.017 (or P_f = 1:60) may be obtained for the 200 m high slope.

An important aspect to be noted in Table 4 refers to the relative significance of each variable in the stability calculations. Friction angle is by far the most relevant variable in this case, contributing to about 77% of the computed variance of FS. On the other hand, uncertainty in cohesion has little effect, contributing to less than 6% of $V[FS]$. The piezometric head, given by the phreatic surface, is of limited importance (about 17% of $V[FS]$). This implies that a more intense drainage system could be of limited efficiency for increasing the safety in this 200 m high mine slope.

For the same 200 m high mine slope, an attempt was made for obtaining the value of P_f from Monte Carlo technique. Computations with 10^5 iterations made use of Geo-Slope software adopting Janbu method. The random field piezometric conditions could not be correctly simulated in this procedure, because the inclined phreatic segment would not end at the slope's base when the standard deviation was automatically taken into account. Hence, deterministic piezometric conditions were considered, with the top horizontal phreatic level at the minimum (z_p = 60 m), intermediate (z_p = 80 m) or maximum (z_p = 100 m) depths and the inclined phreatic line passing through the foot of the slope. Strength parameters and unit weights were the only random variables, with average and deviation values listed in Table 3. The results of Monte Carlo simulations are shown in Table 5.

The worst scenario in Monte Carlo analyses (60 m deep deterministic phreatic level) leads to a P_f value similar to the one given by FOSM method (P_f = 0.017). Disregarding the field variation of the phreatic level about its average 80 m depth was highly non-conservative, for it resulted in a much lower probability of failure. Considering this limitation of the Monte Carlo method in simulating the field phreatic variations in this case, further probabilistic analyses were carried out with the FOSM technique.

6. Probabilistic Analyses of a Progressing Excavation

Several configurations of the same iron ore mine slope have then been analyzed, with heights ranging from 30 to 400 m. Geotechnical variables were presented in Table 3. Figure 3 shows schematically the slope cross-sections, with the water level being considered to pass through the slope foot, in slopes with H > 80 m. For smaller slopes, pore pressures were considered to be insignificant.

Table 5 - Monte Carlo method: probabilistic analysis of a mine slope with $\overline{FS}$ = 1.34.

z_p (m)	FS	σ [FS]	β	P_f
60	1.28	0.14	1.98	0.0166
80	1.37	0.15	2.47	0.0037
100	1.45	0.16	2.85	0.0017

Figure 4 presents a summary of the stability studies, with both $\overline{FS}$ and P_f, plotted as a function of slope height. In these analyses, safety factors were computed by the Simplified Bishop procedure (Bishop, 1955).

It may be noted that the magnitude of the mean factor of safety ($\overline{FS}$) is continuously reduced with the progress of mine excavation (or with increasing slope height). As a consequence, a continuous increase in the probability of failure with increasing height could be expected. However, for H values up to 150 m, P_f exhibits no increase with increasing slope height. On the contrary, an initial slight reduction of P_f may be observed. This may be explained by a stronger contribution of cohesion to the variance of FS, when slope height is still less than 150 m. A high variability of c', as represented by its high value of standard deviation, has been commonly reported for unsaturated residual soils.

An investigation of the relative contributions of all variables to the computed values of $V[FS]$ is summarized in Table 6. The relevance of c' is noted to decrease significantly with increasing height. Cohesion is shown to be the most important variable only for mine slope heights inferior to 100 m. For higher values of H, normal stresses at the potential slip surface become large enough to cause friction

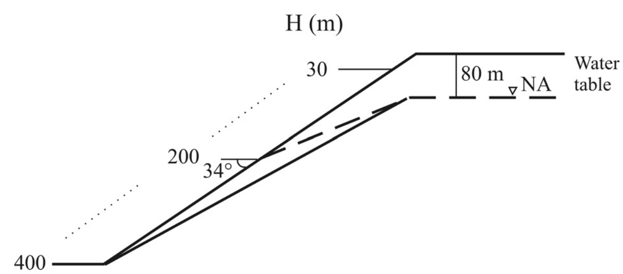

Figure 3 - Geometry of mine slopes considered in the reliability studies of stability.

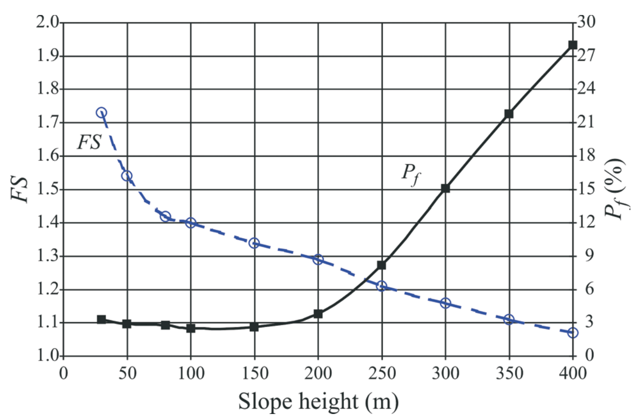

Figure 4 - Effect of height in the stability studies of the iron ore mine slope.

Table 6 - Relative contribution of random variables to $V[FS]$ for various slope heights.

Height (m)	Relative contribution (%)				
	c'	$\tan\phi'$	γ_{nat}	γ_{sat}	Piezometric head (m)
30	86.3	13.4	0.18	0.00	0.00
50	76.8	22.9	0.20	0.00	0.00
80	66.0	33.8	0.15	0.00	0.00
100	58.4	41.2	0.15	0.00	0.15
150	43.9	55.1	0.76	0.01	0.12
200	9.2	69.2	0.02	0.07	21.4
250	10.1	68.5	0.00	0.20	21.1
300	7.4	72.0	0.00	0.29	20.1
350	7.9	72.8	0.02	0.35	18.8
400	2.2	76.9	0.02	0.36	20.4

to be the most significant variable in the stability of this mine slope.

Table 6 also shows that natural or saturated unit weights always have a very small contribution to the uncertainty of *FS*. Therefore, γ_{nat} or γ_{sat} could have been taken as deterministic variables in this case, with very little effect in the computed values of β and P_f.

A relevant practical conclusion may also be taken from Table 6, regarding the role of pore pressures in the stability of these mine slopes. For all excavation depths ranging from 80 to 400 m, the relative contribution of the piezometric head ranges from 15 to 20% of $V[FS]$. This suggests that the drainage system was well dimensioned and further drainage would be of limited consequence to the stability of this mine slope.

An additional aspect to be noted is the marked influence of the limit equilibrium stability method on the relative probability of failure. For H = 200 m, a value of P_f = 0.038 (or P_f = 1:26), based on the Simplified Bishop's method, is obtained from Fig. 4. With Janbu's method, however, a lower probability of failure value of P_f = 0.017 (or P_f = 1:60) has been computed, as reported in section 5.

Similar conclusions have also been warned by Dell Avanzi & Sayão (1998) and Farias & Assis (1998). It is therefore essential to always consider the same limit equilibrium method when comparisons are to be made in probabilistic analyses of slope safety.

7. Conclusions

A comprehensive investigation on the reliability of a mine slope has been reported. Several situations of the same mine, as excavation proceeded with slope heights from 30 to 400 m, have then been analysed. Practical conclusions on the advantages and limitations of the probabilistic approach in assessing the stability of the slope mine have been presented. Reliability analyses based on the FOSM (First Order Second Moment) method proved to be very simple and practical.

The value of mean factor of safety ($\overline{FS}$) was shown to decrease continuously with the progress of mine excavations. However, the relative probability of failure (P_f) was noted to increase only for slope heights H greater than 150 m.

The relative importance of each variable in the stability of the mine slope is a relevant information resulting from FOSM computations. The contributions of natural and saturated unit weights to the uncertainty of *FS* were negligible. Therefore, γ_{nat} or γ_{sat} could be taken as deterministic variables in this stability assessment. As the mine excavation progressed, the most important parameter was gradually changed from effective cohesion to friction angle. Moreover, for excavation heights smaller than 150 m, pore pressures were shown to be insignificant to these reliability analyses. For higher slopes, pore pressure was still of secondary relevance in comparison to the friction angle. As a consequence, further drainage of the mine slope would have a limited impact on the stability of the excavation.

For the mine slope herein described, the use of Monte Carlo method was not suitable for incorporating the field random variation of the piezometric head (or pore pressure). Horizontal drains ensured the phreatic line was fixed at the foot of the excavation, but it was variable at the top. This condition has been be easily incorporated in the FOSM analyses, but is difficult to be duplicated in the Geo-Slope program. Yet, for comparison, Monte Carlo computations were carried out considering the pore pressure as a deterministic variable. For a 200 m high excavation, the value of P_f obtained from FOSM was close to P_f from Monte Carlo with the top phreatic line at its maximum level (worst condition).

The marked influence of the limit equilibrium stability method on the relative value of P_f was demonstrated. It is thus essential to always consider a single limit equilibrium method in probabilistic analyses of slope safety.

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References

Aoki, N. & Tsuha, C.H.C (2010) Use of Characteristic Value in the analysis of safety and confiability of structures and foundations (in Portuguese). Proc. 15° Brazilian Conference on Soil Mechanics and Foundation Engineering, COBRAMSEG, Gramado, v. 1, ID467.

Assis, A.P.; Espósito, T. & Almeida, M. (1997) Comparison between two probabilistic methods in the stability analysis of a mine slope (in Portuguese). Proc. 2nd Pan-American Symposium on Landslides, PSL, Rio de Janeiro, v. 1, p. 347-352.

Baecher, G. & Christian, J.T. (2003) Reliability and Statistics in Geotechnical Engineering. John Wiley and Sons Ltd, 618 pp.

Barnett, W.; Guest, A.; Terbrugge, P. & Walker, D. (2001) Probabilistic pit slope design in the Limpopo metamorphic rocks at Venetia Mine. The Journal of the South African Institute of Mining and Metallurgy, v. 101:8, p. 381-392.

Bishop, A.W. (1955) The use of the slip circle in the stability analysis of slope. Géotechnique, v. 5:1, p. 7-17.

Catalán, J.M. & Cornell, C.A. (1976) Earth slope reliability by a level crossing method. Journal of Geotechnical Engineering Division, ASCE, v. 102:GT6, p. 591-604.

Chowdhurry, R.N.; Tang, W. & Sidi, I. (1987) Reliability model of progressive slope failure. Géotechnique, v. 37:4, p. 467-481.

Christian, J.T.; Ladd, C.C. & Baecher, G. (1992) Reliability and probability in stability analysis. Journal of Geotechnical Engineering Division, ASCE, v. 120:2, p. 1071-111.

Christian, J.T.; Ladd, C.C. & Baecher, G. (1994) Reliability applied to slope stability analysis. Journal of Geotechnical Engineering Division, ASCE, v. 120:12, p. 2180-2207.

Dell Avanzi, E. (1995) Confiability and Probability in Slope Stability Analyses (in Portuguese). M.Sc. Thesis, Civil Engineering Department, Pontifical Catholic University of Rio de Janeiro, 125 pp.

Dell Avanzi, E. & Sayão, A.S.F.J. (1998) Evaluation of the probability of failure of slopes (in Portuguese). Proc. 11th Brazilian Conference on Soil Mechanics and Geotechnical Engineering, COBRAMSEG, Brasília, v. 2, p. 1289-1296.

Duncan, J.M. (2000) Factors of safety and reliability in geotechnical engineering. Journal of Geotechnical and GeoEnvironmental Engineering, ASCE, v. 126:4, p. 307-316.

El-Ramly, H.; Morgenstern, N.R. & Cruden D.M. (2002) Probabilistic slope stability analysis for practice. Canadian Geotechnical Journal, v. 39:3, p. 665-683.

Farias, M.M. & Assis, A.P. (1998) A Comparison of Probabilistic Methods Applied to the Stability of Slopes (in Portuguese). Proc. 11th Brazilian Conference on Soil Mechanics and Geotechnical Engineering, COBRAMSEG, Brasília, v. 2, pp. 9.

Fenton, G. A.; Griffiths, D.V. & Williams, M.B. (2005) Reliability of traditional retaining wall design. Géotechnique, v. 55:1, p. 55-62.

Griffiths, D.V.; Huang, J. & Fenton, G. A. (2009) Reliability influence of spatial variability on slope reliability using 2-D random fields. Journal of Geotechnical and Geo-Environmental Engineering, ASCE, v. 135:10, p. 1367-1378.

Guedes, M.C.S. (1997) Considerations on Probabilistic Analyses of Slope Stability (in Portuguese). M.Sc. Thesis, Civil Engineering Department, Pontifical Catholic University of Rio de Janeiro, 146 pp.

Hammah, R.E.; Yacoub, T.E. & Curran, J.H. (2009) Probabilistic slope analysis with the finite element method. Proc. 43rd US Rock Mechanics Symposium and 4th US-Canada Rock Mechanics Symposium, Asheville, v. 1, pp. 8.

Harr, M.E. (1987) Reliability Based Design in Civil Engineering. McGraw Hill Book Co., New York, 291 pp.

Hasofer, A.M. & Lind, N.C. (1974) Exact and invariant second-moment code format. Journal of the Engineering Mechanics Division, ASCE, v. 100:1, p. 111-121.

Janbu, N. (1957) Earth pressure and bearing capacity calculations by generalised procedure of slices. Proc. 4th International Conference on Soil Mechanics and Foundation Engineering, London, v. 2, pp. 207-212.

Lee, I.K.; Weeks, W. & Ingles, O. (1983) Geotechnical Engineering. Pitman Pub. Co., Marshfield, 508 pp.

Li, K.S. & Lumb, P. (1987) Probabilistic design of slopes. Canadian Geotechnical Journal, v. 24:4, p. 520-535.

Morgenstern, G.G. & Price, V.E. (1965) The analysis of the stability of general slip surfaces. Géotechnique, v. 15:1, p. 79-93.

Mostyn, G.R. & Li, K.S. (1993) Probabilistic Slope Analysis – State of Play. Li, K.S. & Lo, S-C.R. (eds) Probabilistic Methods in Geotechnical Engineering, A.A. Balkeema, The Netherlands, pp. 89-110.

Ribeiro, R.C.H.; Sayão, A.S.F.J. & Castello, R.R. (2008) Application of probabilistic methods to the design of retaining walls (in Portuguese). Proc. 11th National Geotechnical Congress, Portuguese Geotechnical Society, Coimbra, v. 2, pp. 283-288.

Ribeiro, R.C.H. (2008) Applications of Probability and Statistics in Geotechnical Analyses (in Portuguese). Doctoral Thesis, Civil Engineering Department, Pontifical Catholic University of Rio de Janeiro, 161 pp.

Sandroni, S.S.; Santana, F.C.; Ramos, J.M. & Sayão, A.S.F.J. (1992) Failed slope in mica-schist saprolitic soil in Cauê Mine (in Portuguese). Proc. 1st Brazilian Conference on Slopes, COBRAE, Rio de Janeiro, v. 1, pp. 285-292.

Sandroni, S.S. & Sayão, A.S.F.J. (1992) Statistical evaluation of the factor of safety of slopes (in Portuguese). Proc. 1st Brazilian Conference on Slopes, COBRAE, Rio de Janeiro, v. 2, pp. 523-535.

Whitman, R.V. (1984). Evaluating calculated risk in geotechnical engineering. Journal of Geotechnical Engineering Division, ASCE, v. 110:2, p. 145-188.