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Requiem for risk classification matrices

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Article

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Abstract
Classification matrices are scrutinized for inconsistencies, errors and deficiencies in meaning. Proper definition, measurement and ranking of risks are demonstrated as compelling arguments whenever risk and reliability analyses of geotechnical structures, such as dams, are required.

1. Introduction: measurement scales

Classification is a most fundamental organizational activity. It may involve, for example, grouping, in classes or categories, objects which exhibit similar characteristics that distinguish them from others. For such a purpose, a nominal scale is enough. Figure 1 presents the example of a classification of a group of tailings dams exclusively in accordance with the construction procedure.

It should be quite clear that, even when numbers are used to identify different categories, none of the usual mathematical operations are valid on those numbers, because they just serve the purpose of nominating classes (thus *nominal* scale).

One could also *sort, order or rank* objects in accordance with a chosen criterion. Using a similar example, a relevant sort might be in order of increasing vulnerability (Fig. 2). The term risk is being purposely avoided at this point, while vulnerability is being temporarily proposed as a rather intuitive concept associated with the adopted construction procedure.

Figures 2 and 3 provide evidence of statements by Ackoff (1962) and other theoreticians of measurement (bold not in original paper):

> *"The use of a letter or a word is no less measurement than is the use of a number,* ***provided that we make explicit, as we must in the case of numbers as well, what operations may be performed on the symbols****.*
>
> *Measurement is a way of obtaining symbols to represent the properties of objects, events, or states, which symbols have the same relevant relationship to each other as do the things which are represented."*

It is indeed indifferent to name a certain dam either H or 4. No mathematical operation can or should be performed on those symbols. It will be shown, however, that those dams are sorted according to a measure of decreasing risk.

In its strict sense, measurement involves the use of a constant measurement unit. This unit can be arbitrarily es-

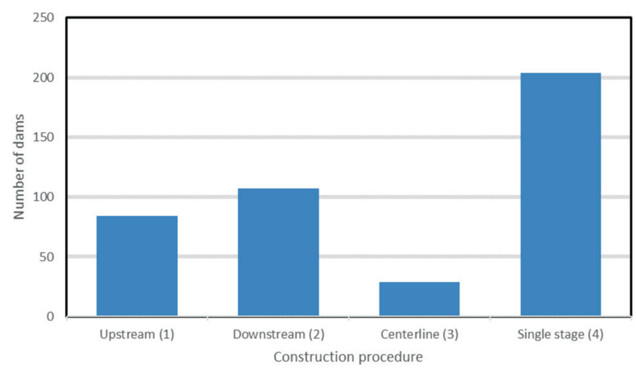

Figure 1. Example of classification of a group of tailings dams according to construction procedure.

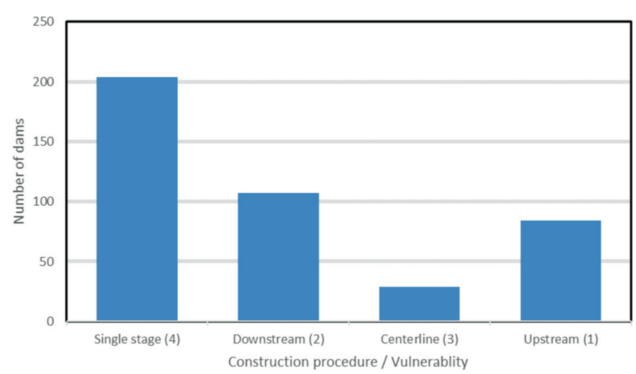

Figure 2. Example of classification of a group of tailings dams according to vulnerability derived from construction procedure.

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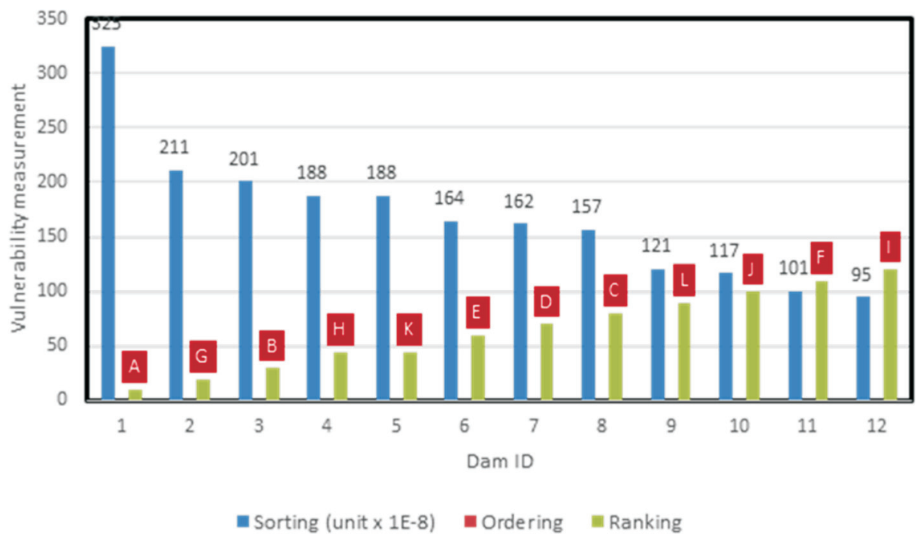

Figure 3. Example of sorting, ordering and ranking of a group of dams according to vulnerability.

tablished when there is no natural zero, such as in the case of the Celsius and Fahrenheit scales. In such cases mathematical operations can be performed on intervals, but not directly on the values themselves. Those are called *interval scales*.

When there is a natural zero, such as in the scales of length, weight, and so on, all usual mathematical operations are valid for the numbers that express the measurements and those scales are called *ratio or proportional scales*. 40 cm, for example, is twice 20 cm. One cannot say, however, that 40 degrees Celsius is twice 20 °C, while it is possible to say that the difference in temperature between 20 and 40 is equal to the difference between 40 and 60 °C.

A ratio scale is usually preferred over any of the others because it is more informative about the measured quantity. Given our interest in the risk associated with geotechnical structures such as fills, slopes, dams, the question is obvious: can a ratio scale be devised to appropriately measure risk?

2. Measurement scale for risk

The answer to that question must be based upon the definition of risk itself, as firmly established in the field of *Risk Analysis*: risk involves a combination (product) of probability of a certain action (or hazard) and the consequences thereof (it is worth noting that the insurance industry uses a different definition of risk).

Thus, Risk Analysis defines risk as the probability of an event, p, multiplied by its consequences, C^* (Fig. 4, Hachich, 2002). Consequences are seldom just economic. For the sake of conciseness, other types of consequences, such as social and environmental, which are obviously equally relevant from a practical standpoint, are not going to be explored in this paper, given that the fundamental flaw of risk matrices can be demonstrated on the basis of just one type of consequence (Pratt *et al.*, 1965).

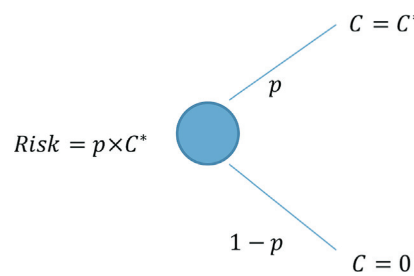

Figure 4. Risk as the product of uncertainty and consequences (Hachich, 2002).

As a matter of fact, the proper definition of risk and its use for classification of geotechnical structures is the crucial point of this paper.

When hazards present themselves at several levels, each of them associated with a certain probability and consequence, risk is computed as a weighted average of the consequences, having probabilities as weights (Fig. 5, Hachich, 2002). Risk is, therefore, the *expected value* of consequences. The risk associated with the circumstances represented by Fig. 6, for example, is quantified by the area below the dotted line.

The unduly and conceptually wrong use of matrices for risk classification has been criticized for almost 20

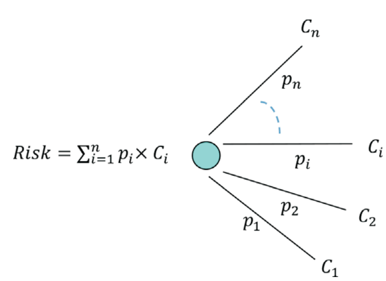

Figure 5. Risk as the expected consequence (Hachich, 2002).

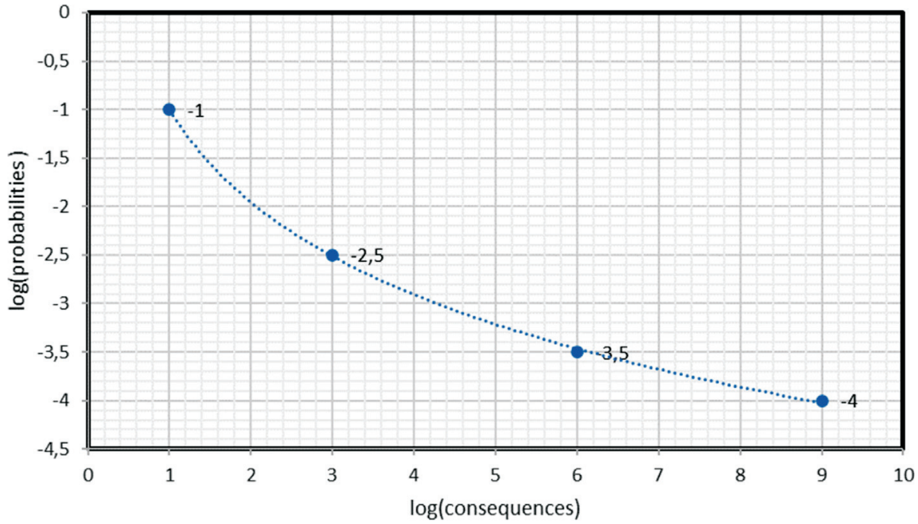

Figure 6. Example of graphical representation of risk on the probability-consequence space (adapted from Oboni, 1998).

years (Hachich, 2002). The final objective of risk evaluation is to provide guidance as to decisions that have to be made. It is therefore natural that risk be interpreted within the context of Decision Analysis (Raiffa, 1968) and *Utility Theory*.

As previously pointed out, the definition in Fig. 5 corresponds to the application of the expected value operator to the consequences. If one considers several different sets of circumstances, each with a graphical representation similar to that in Fig. 6, the values of the areas below the curves may be interpreted as a mapping on a scale of preferences, the case of smallest area being preferred over any of the others. As previously pointed out, those areas need not (or perhaps should not) be restricted to economic values: as a matter of fact, if *Utility Theory* is invoked to assign values of *utilities* to different combinations of economic, social and environmental consequences, Decision Analysis can be applied to more general situations (Keeney & Raiffa, 1976).

The preference for ratio scales has been previously stated. Probabilities are measured between zero and one in a ratio scale. Consequences are also measured in a ratio scale, and utilities can also be defined between zero and one. Given the definition of risk, there is no reason whatsoever why it should not be measured in a ratio scale.

Figure 3 presented the classification of a set of dams on the basis of risks posed by them. Classification must start, of course, with the evaluation of risks, and that is the only way of doing it correctly.

3. "Risk" classification matrices

Our interest is focused, of course, in those dams that pose higher risks: they should be the priority of mitigating actions. Given the definition of risk, its evaluation requires studies of some complexity performed by a team of engineers capable of evaluating probabilities of geotechnical, hydrological, hydraulic and many other engineering-related events, in addition to their consequences (and possibly utilities as well).

In some cases, it is known beforehand that risks are not small because of the construction procedure, the lack of information and contingency plans, faulty conservation and many other reasons. In such cases it is usual to see published tables such as Table 1, often based on a wrong definition of risk. The scale adopted for the table is obviously nominal, even if someone decides to exchange symbols for numbers in the cells, such as in Table 2. It follows that mathematical operations performed on those numbers are not acceptable.

The inconsistencies of such an approach are further explored in Hachich (2002). Re-stating Ackoff (1962): "used symbols, such as numbers, must have the same rele-

Table 1. Example of "risk" matrix with the usual type of symbol-based nominal scale (*ad hoc* chosen characters).

	Potential damage		
"Risk"	High	Medium	Low
High	A	B	C
Medium	B	C	D
Low	B	C	E

Table 2. Example of "risk" matrix with the usual type of symbol-based nominal scale (*ad hoc* chosen digits).

	Potential damage		
"Risk"	> 1000	1 to 1000	< 1
> 0.01	5	4	3
0.0001 to 0.01	4	3	2
< 0.0001	4	3	1

Table 3. Example of "risk" matrix with the usual type of symbol-based nominal scale (arbitrarily chosen description/classification and "corresponding" digits).

Seepage (e)	Displacements (f)	Flood return period (g)
Perfectly controlled (0)	No significant displacements (0)	< 500 (0)
Some small areas of leakage downstream but abutments in good condition (3)	Small cracks and settlements undergoing corrective measures (2)	500 (2)
Areas of leakage downstream, slopes and abutments lacking proper corrective measures (6)	Small cracks and settlements lacking proper corrective measures (5)	1000 (5)
Areas of leakage downstream, with increasing flow and material (10)	Cracks, settlements and local instabilities (10)	10000 (10)
EC = Σ (e to g)		

vant relationship to each other as do the things which are represented". In the present case, our interest in risks would require such numbers to be values measured in a ratio scale, so as to represent actually computed risks.

The possibility of "risk" scales such as those in Table 2 leading to decisions that reflect the decision maker's preferences is never demonstrated, while Decision Analysis and Utility Theory offer mathematical proof (Keeney & Raiffa, 1976). Surprisingly, however, arbitrarily chosen nominal scales are one of the most ubiquitous features of published papers on "risk" assessment. Table 3 is just one such example, borrowed from a real-world situation.

The scales in Table 3 are obviously nominal scales. It is indifferent to identify seepage control as "perfect" or to assign the symbol "0" to it. For this reason, the summation presented in the last line of the table has no meaning whatsoever. But supposing, just for the sake of the argument, that the numbers that appear in the cells of Table 3 would have been arrived at by correctly engineered evaluations, the summation would still be completely wrong: Probability Theory (*e.g.* Benjamin & Cornell, 1970) teaches us that the probability of a joint event is the **product** (not sum) of the individual probabilities, whenever the events can be assumed to be independent from each other, which is not necessarily true for some of the failure modes.

In the original source, however, Table 3 is presented as a table of "risk" classification. As previously discussed, those cell contents cannot be called risks for at least two reasons: their scale is just nominal, and consequences are not taken into account. As far as the latter, Table 4 presents an attempt at classification of at least part of the information that is relevant for the evaluation of the consequences of failure.

Once again, and for similar reasons, the summation presented in the last line of Table 4 is meaningless.

Table 4 naturally implies 4^5 categories (or classes), so that a number between one and 1024 can be assigned to technically classify a given dam. If two dams fall in the same class, they may be considered as "equal" from a technical point of view. When they fall in different classes, however, Table 4 is of no help for deciding which one poses the higher risk.

It is also possible to use just the cell positions to create a 5-digit code number (with a fixed digit position for each property) to identify each technical class. Code 23442, for example, would identify a dam with height between 15 m and 30 m, crest length between 200 m and 600 m, design flow lower than 500, upstream construction and monitoring

Table 4. Classification matrix with part of the information that is relevant for the evaluation of the consequences of failure (symbol-based nominal scale with arbitrarily chosen description/classification and "corresponding" digits).

Design criteria and maintenance				
Height (a)	Length (b)	Design flow PMF (c)	Construction procedure (d)	Monitoring (e)
≤ 15 m (0)	≤ 50 m (0)	10000 (0)	Single stage (0)	Monitoring instruments installed according to design (0)
15 to 30 m (1)	50 to 200 m (1)	1000 (2)	Downstream (2)	Monitoring instruments in the process of being installed (2)
30 to 60 m (4)	200 to 600 m (2)	500 (5)	Centerline (5)	Monitoring instruments do not follow the design (6)
> 60 m (7)	> 600 m (3)	< 500 (10)	Upstream (10)	No monitoring instruments (8)
CT = Σ (a to e)				

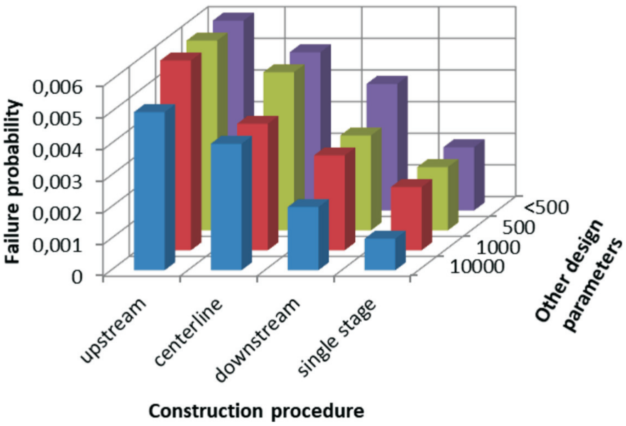

Figure 7. Example of quantitative results obtained from the elicitation of probabilities of failure of dams.

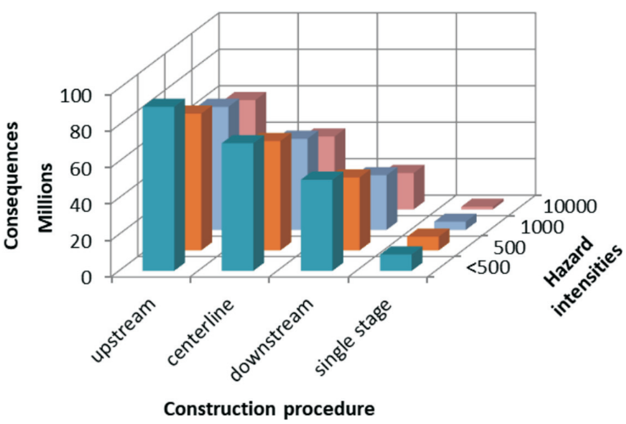

Figure 8. Example of quantitative results obtained from the evaluation of failure scenarios and their consequences.

instruments being installed. Neither this classification nor the approach based on the numeric symbols assigned to cells of Table 4 (the summation formula in particular) would support any decision regarding the relative risks of class 23442 versus, for example, class 32341.

4. Sorting a group of dams according to risk

The need to rank a group of dams according to the risks they pose is obviously desirable.

Despite having been often and extensively attempted, for the aforementioned conceptual reasons this objective cannot be correctly achieved by means of classification matrices such as Tables 3 and 4, let alone by their summaries of summation points.

Again Ackoff (1953) warns that:

> *"We must be careful not to impute automatically to numbers obtained by any process of assigning numbers to objects, events, or properties, the properties which these numbers have as numbers. We can add the numbers of two houses or of two car registrations, but the question is whether or not the sum has any meaning, and if so what."*

The desired result would be Fig. 3, with the y-axis representing risks associated to the series of blue columns, and risks computed according to the proper engineering definition (Fig. 5). Two activities are therefore required:

a. Engineering analysis for the quantitative elicitation of probabilities of failure of dams, usually complemented by extensive historical research in order to generate results which include and extend those in Fig. 7;
b. Preview and evaluate failure scenarios and their consequences, in order to generate quantitative results which include and extend those in Fig. 8.

5. Conclusions

Decision Analysis and Utility Theory (Pratt *et al.*, 1965) provide a sound theoretical basis for the definition, evaluation and ranking of risks. Results of risks measured in a ratio scale also conform with Measurement Theory.

None of above holds for "risk" classification matrices, which usually ignore or violate well established theoretical principles. Consequently, there is no place for such arbitrary matrices in serious safety and reliability studies.

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