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Development of neuro-fuzzy models for predicting shear behavior of rock joints

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Article

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Abstract

The purpose of this article is to present predictive models of dilation and shear stress of rock discontinuities by applying the neuro-fuzzy technique, which uses a) the high capacity of artificial neural networks (ANN) to understand and to model complex multivariate phenomena, and b) the concepts of fuzzy sets theory to consider the variability of the input parameters in the proposed models' responses. To develop the proposed models, experimental results were obtained from large-scale direct shear tests performed on different types of rock discontinuities and boundary conditions. The input variables of the proposed neuro-fuzzy models are the normal boundary stiffness, the ratio of fill thickness to asperity height, the initial normal stress, the joint roughness coefficient, the uniaxial compressive strength of the intact rock, the basic friction angle of the intact rock, the friction angle of the infill, and the shear displacement. The proposed models for dilation and shear stress provided results that fitted satisfactorily the experimental data, and the analyses of their performances indicated that they can represent the influence of the input variables on the shear behavior parameters of the rock discontinuities. The results from the neuro-fuzzy systems developed are also closer to the experimental data than those estimated by using traditional analytical methodologies existing in Rock Mechanics. This occurs because once considering the uncertainty of the input data, a more representative shear behavior prediction can be made by the neuro-fuzzy models.

1. Introduction

The discontinuities present in the rock masses are one of the main factors influencing their mechanical behavior. Distinct studies have aimed to estimate the shear behavior of the rock discontinuities, to provide parameters to analyze and design projects in Rock Mechanics realistically.

Several analytical models have been developed to represent the shear behavior of rock discontinuities. Some works worth mentioning are Barton (1973), Barton & Choubey (1977), Barton & Bandis (1990), Skinas et al. (1990), Papaliangas et al. (1993), Indraratna & Haque (2000), Indraratna et al. (2005, 2008, 2010, 2013), and Oliveira & Indraratna (2010). However, application of such analytical models is limited because they do not consider some key-factors governing the shear behavior of the rock discontinuities, such as the normal boundary stiffness imposed by the surrounding rock mass and the presence of infill material, or due to the difficulty in obtaining some of their parameters. Due to these limitations, other analysis methodologies have been used in Rock Mechanics, such as the intelligent systems that use the artificial neural networks (ANN), the fuzzy logic, and the neuro-fuzzy techniques to predict the shear behavior of the rock discontinuities (Dantas Neto et al., 2017; Leite et al., 2019a, b).

Besides the studies with the use of ANN have shown its excellent performance for predicting the shear behavior of rock discontinuities, some of the highlighted disadvantages, which can be also attributed to the analytical models, regards their deterministic character, i.e., the fact that they cannot consider the influence of the variability and uncertainties inherent to the input variables in their predictions. Thus, some systems which are developed based on the concepts of the fuzzy sets theory, or fuzzy logic, present themselves as an alternative for the development of predictive models that may consider the variability and uncertainties of the input variables in the models' responses without requiring a widespread field or laboratory investigation.

The fuzzy logic proposed by Zadeh (1965) is like a tool designed to address subjective problems, involving imprecise and vague data, in addition to being able to use prior knowledge on such studied phenomena. Using the

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potential of fuzzy logic, several studies have been done on Rock Mechanics to predict some rock mass and intact rock properties, such as Kayabasi et al. (2003), Sonmez et al. (2003), and Harrison & Hudson (2010). However, results presented by Matos et al. (2019a, b) indicate that the fuzzy logic has proven somewhat satisfactory for predicting the peak shear strength of the rock discontinuities but has not provided results that help to properly represent its variation with the shear displacement imposed on the unfilled discontinuities.

Therefore, to use the high learning potential inherent in the ANN and the capacity of the fuzzy sets in considering the variability or uncertainties of the input parameters in the predictive model responses, Jang (1993) proposes a neuro-fuzzy controller called ANFIS (Adaptive-Network-based Fuzzy Inference System), which is based on the construction of a set of fuzzy inference rules from appropriate membership functions, creating adjusted in-out patterns. In Rock Mechanics, some of the developed neuro-fuzzy systems were proposed by Gokceoglu et al. (2004), Singh & Singh (2006), Noorani et al. (2010), Jalalifar et al. (2011), and Yesiloglu-Gultekin et al. (2013) for modeling some properties of the rock masses and the intact rocks.

Matos (2018) and Matos et al. (2019a, b) proposed fuzzy and neuro-fuzzy models that provide predictions of the shear behavior of unfilled rock discontinuities. Although they supply satisfactory results, these models do not consider the effect of the infill material, which is one of the main factors influencing the shear behavior of rock discontinuities (Papaliangas et al., 1993; Haque, 1999; Indraratna et al., 2010, 2013; Oliveira & Indraratna, 2010; Shrivastava & Rao, 2018).

In this context, the objective of this article is to present predictive models of dilation and shear stress of the rock discontinuities based on neuro-fuzzy techniques, which use the high capacity of the artificial neural networks in representing complex and multivariate phenomena, and the concepts inherent in the fuzzy sets, allowing for consideration of the variability or uncertainties of the input data in the responses of the proposed systems.

2. Modeling in rock mechanics with intelligent systems

As an alternative to existing analytical models (Barton, 1973; Barton & Choubey, 1977; Barton & Bandis, 1990; Skinas et al., 1990; Indraratna et al., 2005, 2008, 2010, 2013, 2014, 2015; Oliveira & Indraratna, 2010; etc.), and to facilitate the prediction process of shear behavior of the rock discontinuities under different boundary conditions, intelligent systems using artificial neural networks, fuzzy logic, and neuro-fuzzy techniques have been increasingly applied in Rock Mechanics (Jalalifar et al., 2011; Ocak & Seker, 2012; Yesiloglu-Gultekin et al., 2013; Dantas Neto et al., 2017; Sadrossadat et al., 2018; Matos, 2018; Matos et al., 2019a, b; Leite et al., 2019a, b). The choice of these systems is usually based on several factors which consider the high capacity of understanding and modeling multivariate and non-linear complex problems.

2.1 Fuzzy logic

Zadeh (1965) introduced the concept of fuzzy sets, the role of which is to represent human knowledge on a determined phenomenon, or problem, by treating the information vaguely and imprecisely. According to the author, fuzzy sets are represented by membership functions, in which they associate each element of the set to the respective degree of membership, whose value is between 0 and 1. Unlike the deterministic approach, in which a single value is attributed to a certain input variable in a model, the fuzzy set theory attributes to it a set of possible values within a given membership level (Zadeh, 1965; Jang et al., 1997).

The basic structure of a fuzzy inference system consists of three conceptual components: a set of rules, representing the relations between the fuzzy sets; a database, which defines all membership functions used in the fuzzy rules; and a reasoning process, that executes the inference procedure over the fuzzy rules, producing a response or output. There are different types of fuzzy inference systems, the best-known being those proposed by Mamdani (1974), Tsukamoto (1979), and Takagi & Sugeno (1983).

Harrison & Hudson (2010) states that mathematics present in fuzzy logic can be a proper tool to solve Rock Mechanics problems, bearing in mind that it allows consideration of the uncertainties present in the rock masses and their structures. Using fuzzy logic features, some studies such as those presented by Sonmez et al. (2003), Kayabasi et al. (2003) have used the logic fuzzy to predict properties in the rock masses.

Based on a fuzzy inference system of the Mamdani (1974) type, Sonmez et al. (2003) estimated several parameters necessary for characterizing rock mass using the Geological Strength Index (GSI). The authors concluded that the fuzzy sets provide a more practical way of working with cases in which the data are limited and uncertain.

Kayabasi et al. (2003) estimated the rock mass deformation modulus based on simple regressions, multiple regressions, and a Mamdani (1974) fuzzy inference system. The authors' results show that the predictions made by the fuzzy system were more reliable compared to the experimental data.

Matos (2018) and Matos et al. (2019a, b) used the Mamdani (1974) and Takagi & Sugeno (1983) type fuzzy models for predicting the shear behavior of unfilled rock discontinuities. Based on the authors' results, it was found that first-order Takagi & Sugeno (1983) models were those that performed best in dilation and shear stress estimations, comparing the output data of these systems with the experimental data used. Besides, their results have shown good performance only to predict the peak shear stress.

According to the aforementioned works, it is possible to observe the difficulty of choosing the suitable membership functions for each input variable considered in any modeled phenomenon remains the main limitation of fuzzy logic systems. Besides, the higher the number of input variables the higher the computational effort necessary to perform the inference procedures during the modeling process.

2.2 Artificial Neural Networks (ANN)

Haykin (2008) defines an artificial neural network as a mechanism formed by processors distributed in parallel layers, consisting of processing units that are called artificial neurons, having the natural tendency to store knowledge and make it available for use. One of the main types of ANN used in engineering is the multilayer perceptron, which is a feed-forward neural network formed typically of three types of layers: the input layer, whose main function is to receive the external "stimulus"; the hidden layers, responsible for extracting more complex statistics from the modeled mechanism; and the output layer, which provides the results of the modeled phenomenon by the ANN (Haykin, 2008).

An artificial neural network is trained by alterations in their synaptic weights and biases, using a specific learning algorithm and the knowledge about the existing modeled phenomenon in a set of experimental data containing known input-output patterns. After the training phase, the performance of the neuronal model is checked at a phase called testing, using a set of input-output values that was not presented to the network during the alteration of synaptic weights and biases. In general, in the process of defining a neuronal model, various architectures are trained and tested until identifying a configuration that has the best performance in predicting the responses during the training phase, and which demonstrates a satisfactory capacity to generalize knowledge of the phenomenon modeled by the neuronal network during testing (Haykin, 2008; Dantas Neto et al., 2017).

The functionalities and high learning capacity of the ANN have led to the development of various studies in Rock Mechanics, for example, by Sonmez et al. (2016), Dehghan et al. (2010), Ocak & Seker (2012), Dantas Neto et al. (2016, 2017), Leite et al. (2019a, b) and others.

Dehghan et al. (2010) used regressions and neuronal models to predict the uniaxial compressive strength and the elastic modulus in samples of the rock mass. The authors concluded that the results of the models developed with ANN were closer to the experimental data used, emphasizing the capacity of those systems to represent the nonlinear aspects of the phenomena studied.

Dantas Neto et al. (2016, 2017) produced a model that uses ANN to predict dilation and shear stress found in unfilled discontinuities. Considering the main factor influencing the shear behavior of rock discontinuities, the model gave satisfactory results when compared to the experimental data used in their development, as well as allowing the user faster and more practical calculations of the estimations.

Dantas Neto et al. (2017) and Leite et al. (2019a, b) also proposed predicting models for the shear behavior of unfilled and infilled rock discontinuities under constant normal stiffness (CNS) and constant normal loading (CNL) conditions with multilayer perceptrons. It can be observed that these neuronal models have provided results closer to the experimental data than the estimations obtained from applying different analytical models used by the authors, showing the capacity of the ANN to predict the shear behavior of rock discontinuities.

2.3 Neuro-fuzzy systems

Jang et al. (1997) mention that the modeling process of a neuro-fuzzy problem is based on two segments: the artificial neural network, which recognizes patterns to adapt to the change in their medium, and the fuzzy inference system, which allows the incorporation of human knowledge and perform a role of inference and decision-making about a specific problem. Jang (1993) proposed a class of ANN that are functionally equivalent to a fuzzy inference system known as the Adaptive Network-based Fuzzy Inference System (ANFIS). Figure 1 illustrates an ANFIS model made up of two fuzzy inference rules, based on the concepts of the fuzzy inference system of the Takagi & Sugeno (1983) type for representing a specific phenomenon.

Jang (1993) differentiates each layer shown in Figure 1 according to its functions. The nodes in Layer 1 are not adaptable and the values of its nodes are defined according to Equation 1, where x and y are the input in the nodes, and A_i, or B_i, are the fuzzy sets associated with the nodes. In this example, "i" depicts the value of 1 and 2 in virtue of the number of fuzzy inference rules and sets used. Each output ($O_{1,i}$) of Layer 1 is the value of the degree of membership obtained from x and y, calculated by any preestablished membership function.

$$O_{1,i} = \mu_{A_i(x)} \; ou \; O_{1,i} = \mu_{B_i(y)}, \; i = 1,2 \tag{1}$$

Layer 2 is made up of fixed nodes, which function is to calculate the product of the input signals, according to

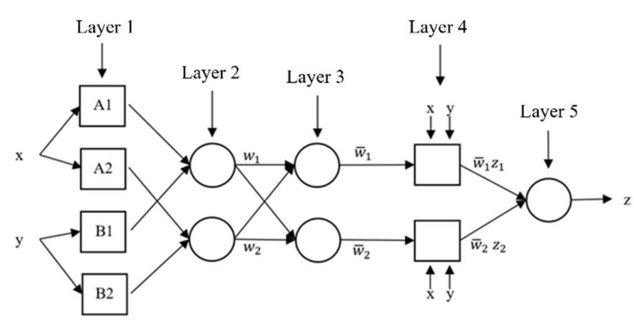

Figure 1. Diagram of an ANFIS neuro-fuzzy model (Jang, 1993).

Equation 2. Each output of this layer represents the weight of its fuzzy inference rule.

$$O_{2,i} = w_i = \mu_{A_i(x)} \cdot \mu_{B_i(y)},\ i = 1,2 \quad (2)$$

The nodes in Layer 3 are also fixed and they calculate the ratio between the weight of each fuzzy rule and the sum of the weights of all fuzzy rules according to Equation 3.

$$O_{3,i} = \overline{w}_i = \frac{w_i}{w_1 + w_2},\ i = 1,2 \quad (3)$$

Layer 4 contains adaptable nodes that have outputs computed according to Equation 4, in which $\overline{w}_i$ is the output from Layer 3 and p_i, q_i, and r_i are referred to as consequent parameters.

$$O_{4,i} = \overline{w}_i z_i = \overline{w}_i \left(p_i x + q_i y + r_i \right) \quad (4)$$

Finally, Layer 5 is characterized by a single fixed node, whose function is to calculate the output (z) of the neuro-fuzzy system by summing all its input signals together, according to Equation 5.

$$O_{5,i} = z = \sum_i \overline{w}_i z_i \quad (5)$$

Jang (1993) points out that when the premise parameters are fixed, the output (z) of the neuro-fuzzy system can be expressed through a linear combination of the consequent parameters, as presented in Equation 6.

$$z = (\overline{w}_1 x) p_1 + (\overline{w}_1 y) q_1 + (\overline{w}_1) r_1 + (\overline{w}_2 x) p_2 + (\overline{w}_2 y) q_2 + (\overline{w}_2) r_2 \quad (6)$$

The linear combination represented in Equation 6 allows the hybrid learning process proposed by Jang (1993). This process consists of two phases: the forward phase, when the outputs proceed to Layer 4, the consequent parameters being calculated by the least-squares method; and the backward phase when the error sign is defined by the difference between the output calculated by the ANFIS system and the experimental data spreads through the system and the premise parameters are calculated by the gradient descent method (Jang, 1993; Jang et al., 1997). Thus, the ANFIS neuro-fuzzy system tends to move closer to the desired response pattern, comprising the problem under analysis with the modification of the parameters inherent to the membership functions, which result in the development of optimized fuzzy sets.

In the training process of the ANFIS neuro-fuzzy systems, an initial structure is required with fuzzy sets and established membership functions. Jang et al. (1997) point to various methodologies that can be used in developing these initial structures, such as the grid partitioning method and the subtractive clustering technique. The grid partitioning method is based on dividing the input variables domain in equally spaced sizes from membership functions of the same format. In the case of the subtractive clustering method proposed by Chiu (1994), the clustering centers are created according to the distribution of the input data in the variable domains based on the concept of data density, which creates the number of membership functions necessary to represent the problem under analysis.

Neuro-fuzzy systems have been used in several Rock Mechanics applications, such as those presented by Gokceoglu et al. (2004), Singh & Singh (2006), Noorani et al. (2010), Jalalifar et al. (2011), Yesiloglu-Gultekin et al. (2013), Sadrossadat et al. (2018), Matos (2018).

Gokceoglu et al. (2004) used a neuro-fuzzy model to estimate the rock mass deformation modulus. From the study, it was noticeable that the results from the developed neuro-fuzzy system were closer to the experimental data than the predictions made by the empirical models used by the authors.

Singh & Singh (2006), in turn, developed neuro-fuzzy models and ANN to predict the Poisson's ratio and Young's modulus of intact rocks. The authors used as input variables in their models the property of intact rock, as the uniaxial compressive strength and tensile strength. Comparing the results from the developed models, the neuro-fuzzy system provided estimations closer to the experimental data.

Matos (2018) used ANFIS systems to predict the shear behavior of unfilled rock discontinuities submitted to CNS and CNL conditions. The models gave satisfactory results when compared to the experimental data used in their development. Despite the results, the neuro-fuzzy systems developed by these authors do not consider the presence of the fill material in the shear behavior of rock discontinuities restricting their use only to certain field situations.

3. Development of neuro-fuzzy models

3.1 Experimental data

Experimental data were obtained from 116 large-scale direct shear tests undertaken by Benmokrane & Ballivy (1989), Skinas et al. (1990), Papaliangas et al. (1993), Haque (1999), Indraratna & Haque (2000), Oliveira et al. (2009), Indraratna et al. (2010), Mehrishal et al. (2016) and Shrivastava & Rao (2018). This survey results in a set of data with 2098 input-output patterns to be used in developing and evaluating the neuro-fuzzy models for predicting dilation and shear stress of rock discontinuities. Different conditions for the rock discontinuities can be observed for the data available in the used dataset in terms of uniaxial compressive strength (soft to hard rocks), roughness profile (slightly to very rough), external boundary conditions (CNL and CNS), condition of infill (unfilled and infilled rock discontinuities), which show the wide variety of situations considered in the experimental data used in developing the proposed models. In direct shear tests carried out on CNL condition, the normal stress is constant during shearing, while in CNS conditions

3.2 Defining the input and output variables

The input variables of the neuro-fuzzy models were defined to take into consideration the main factors governing the shear behavior of rock discontinuities represented by the dilation (δ_v), in mm, and shear stress (τ_s), in MPa, during the shear process. They are:

- Normal boundary stiffness (k_n), in kPa/mm;
- Ratio of thickness of the infill material (t) to asperity height of the discontinuity (a) - t/a ratio;
- Initial normal stress (σ_{no}), in MPa;
- Joint roughness coefficient (*JRC*);
- Uniaxial compressive strength of intact rock (σ_c), in MPa;
- Basic friction angle of intact rock (ϕ_b), in degrees;
- Friction angle of infill material (ϕ_{infill}), in degrees;
- Shear displacement (δ_h), in mm.

The maximum and minimum values of the collected experimental data for the defined input and output variables are presented in Table 1. These values must be considered as the limits to which the models can be applied since they were used to establish the membership functions of the variables involved and to define the fuzzy inference rules.

3.3 Training of neuro-fuzzy models

The ANFIS models developed are based on the principles of the fuzzy inference systems of Takagi & Sugeno (1983) and the hybrid learning proposed by Jang (1993). The ANFIS systems for predicting the parameters that define the shear behavior of rock discontinuities were modeled first by neuro-fuzzy model training, in which 80% of the experimental data was randomly chosen from the available experimental. The remaining data (20%) were used later in the test phase of the developed models.

Concerning the initial structure, input variables, and membership functions adopted, different configurations were tested for the following neuro-fuzzy models:

- Model 1: created by the grid partitioning method, presenting two (2) Gaussian membership functions for the eight input variables;
- Model 2: created by the grid partitioning method, presenting three (3) Gaussian membership functions for the input variable *t/a*, and two (2) Gaussian membership functions for the remaining variables;
- Model 3: created by the grid partitioning method, presenting three (3) Gaussian membership functions for the input variables *t/a* and δ_h, and two (2) Gaussian membership functions for the remaining variables;
- Model 4: created by the grid partitioning method, presenting three (3) Gaussian membership functions for input variables *t/a*, δ_h, k_n and σ_c, and two (2) Gaussian membership functions for the remaining variables;
- Model 5: created by the subtractive clustering method, in which the number of Gaussian membership functions is obtained for each model itself;
- Model 6: variable ϕ_b is not considered as an input variable in the model, the initial structure is created using the grid partitioning method, presenting two (2) Gaussian membership functions for all remaining input variables;
- Model 7: variable ϕ_b is not considered as an input variable in the model, the initial structure is considered using the grid partitioning method, presenting three (3) Gaussian membership functions for the input variables *t/a* and δ_h, and two (2) Gaussian membership functions for the remaining input variables;
- Model 8: variables ϕ_b and ϕ_{infill} are not considered as input variables in the model, the initial structure is created using the grid partitioning method, presenting two (2) Gaussian membership functions for the other input variables;
- Model 9: variables ϕ_b and ϕ_{infill} are not considered as input variables in the model, the initial structure is created using the grid partitioning method, presenting three (3) Gaussian membership functions for the input variables *t/a* and δ_h, and two (2) Gaussian membership functions for the remaining variables.

Table 2 summarizes the main configurations established for the different neuro-fuzzy models evaluated in this paper

Table 1. Maximum and minimum values of the models' variables.

Input variables								Output variables	
k_n	t/a	σ_{n0}	JRC	σ_c	ϕ_b	ϕ_{fill}	δ_h	δ_v	τ
kPa/mm		MPa		MPa	degrees	degrees	mm	mm	MPa
0	0	0.05	2	3.5	30	0	0.02	-2.43	0.02
7515	2	46.5	20	150	37.5	35.5	26	4.97	6.68

Table 2. Configuration of studied neuro-fuzzy models.

Model	Initial structure method	Membership funcion	Number of membership functions for the considered input variables							
			K_n	t/a	σ_{no}	JRC	σ_c	ϕ_b	ϕ_{fill}	δ_h
1	Grid partitioning	Gaussian	2	2	2	2	2	2	2	2
2	Grid partitioning	Gaussian	2	3	2	2	2	2	2	2
3	Grid partitioning	Gaussian	2	3	2	2	2	2	2	3
4	Grid partitioning	Gaussian	3	3	2	2	3	2	2	3
5	Subtractive clustering	Gaussian	variable	variable	variable	variable	variable	variable	variable	variable
6	Grid partitioning	Gaussian	2	2	2	2	2	-	2	2
7	Grid partitioning	Gaussian	2	3	2	2	2	-	2	3
8	Grid partitioning	Gaussian	2	2	2	2	2	-	-	2
9	Grid partitioning	Gaussian	2	3	2	2	2	-	-	3

in an attempt to find out the model which presents the best performance in the shear behavior of the rock discontinuities. The grid partitioning method was used in all ANFIS models tested except for Model 5, in which the subtractive clustering partitioning was used to evaluate the performance of the ANFIS model when its training process is influenced by the distribution of the input data, as described in section 2.3. In this case, the number of membership functions is created according to the distribution of the input data values used in the modeling and not arbitrarily chosen by the expert. Then, it was tested whether the definition of the number of membership functions according to the distribution of available input data could improve the performance of the ANFIS models rather than the use of grid partitioning method.

With definition of such structures, one of the aims is to assess the influence of the input variables in the performance of the proposed neuro-fuzzy models. The choice of Gaussian membership functions in establishing the fuzzy sets representing the input variables was based on the satisfactory results from the various studies that adopted this function (Singh & Singh, 2006; Jalalifar et al., 2011).

The software used to develop ANFIS neuro-fuzzy models was MATLAB (Jalalifar et al., 2011; Yesiloglu-Gultekin et al., 2013). At the training phase, the different ANFIS models were developed by comparing the results obtained for each output variable (dilation or shear stress) with the experimental data, so that the parameters were created to form the membership functions and fuzzy inference rules to obtain the best performing neuro-fuzzy system possible.

3.4 Criteria for model selection and model validation

The selection criterion of the ANFIS systems was based on the comparison between the predictions made by the tested models and the experimental data used during the test phase by using the coefficient of determination (R^2). The neuro-fuzzy models that had R^2 values in the test phase higher than 0.95 were considered apt to be assessed at a later stage called the validation phase.

The validation phase consisted of predicting the shear stress and dilation of hypothetical rock discontinuities similar to the methodology used by Dantas Neto et al. (2017) to validate ANN models for predicting the shear behavior of unfilled rock discontinuities. This procedure allows checking whether the neuro-fuzzy models can represent satisfactorily the influence of the input variables on shear behavior of such discontinuities (Indraratna et al., 2014, 2015; Oliveira et al., 2009; Barton, 2013, 2016; Naghadehi, 2015; Shrivastava et al., 2011).

4. Results and discussions

4.1 ANFIS model training and testing

Tables 2-4 provide the values of the coefficients of determination (R^2) obtained during the training and testing phases of the neuro-fuzzy systems developed for predicting the dilation (*D*) and shear stress (*S*), respectively, for the different tested models. According to them, it is noticeable that the models showing the best performances are *D1*, *D2*, *D3*, and *D4* for predicting dilation, and *S1*, *S2*, *S3*, and *S4* for shear stress of rock discontinuities. The high R^2 values obtained in the training and testing phases express the excellent performance of the ANFIS models for predicting shear stress and dilation under CNL and CNS condition in rock discontinuities for a variety of conditions in terms of infill and roughness. Such results can be attributed to the consideration of uncertainties in the values of the input variables in the response of neuro-fuzzy models making them able to satisfactorily represent the phenomenon studied

Furthermore, it is found that the systems which did not consider all input variables, such as models *D6*, *D7*, *D8*, *D9*, *S6*, *S7*, *S8*, and *S9*, had inferior performances when compared to the models which use all input variables. This shows how important is to consider all the parameters governing the shear behavior of the rock discontinuities in the development of the proposed neuro-fuzzy models.

Table 3. Results in the training and test phases of the dilation prediction systems.

Model	Input variables	Initial structure	Membership functions	R^2 test	R^2 training
D1	8	Grid partitioning	2	0.99	0.99
D2	8	Grid partitioning	2, except t/a (3)	0.99	0.99
D3	8	Grid partitioning	2, except t/a and δ_h (3)	0.98	0.99
D4	8	Grid partitioning	2, except t/a, δ_h, k_n and σ_c (3)	0.98	0.99
D5	8	Subtractive clustering	14	0.92	0.90
D6	7	Grid partitioning	2	0.00	0.01
D7	7	Grid partitioning	2, except t/a and δ_h (3)	0.14	0.25
D8	6	Grid partitioning	2	0.00	0.01
D9	6	Grid partitioning	2, except t/a and δ_h (3)	0.01	0.02

Table 4. Results obtained in the training and test phases of the shear stress prediction systems.

Model	Input variables	Initial structure	Membership functions	R^2 test	R^2 training
S1	8	Grid partitioning	2	0.96	0.97
S2	8	Grid partitioning	2, except t/a (3)	0.96	0.97
S3	8	Grid partitioning	2, except t/a and δ_h (3)	0.97	0.98
S4	8	Grid partitioning	2, except t/a, δ_h, k_n and σ_c (3)	0.95	0.98
S5	8	Subtractive clustering	14	0.92	0.93
S6	7	Grid partitioning	2	0.10	0.28
S7	7	Grid partitioning	2, except t/a and δ_h (3)	0.37	0.50
S8	6	Grid partitioning	2	0.00	0.00
S9	6	Grid partitioning	2, except t/a and δ_h (3)	0.04	0.06

About the method used in the initial structure of the ANFIS systems, it is worth mentioning that models *D5* and *S5*, developed from the subtractive clustering technique, presented lower R^2 values than those obtained in the training and testing phases by models *D1*, *D2*, *D3*, *D4*, *S1*, *S2*, S3 and *S4*, which used the grid partitioning technique. This shows that by dividing the domain of input variables by the membership functions in equal sizes, it proved more efficient in predicting the shear behavior of rock discontinuities.

According to the results given in Tables 2-4, it is found that the increase in the number of membership functions did not correspond necessarily to an improvement in the performance of the ANFIS models. This can be confirmed by comparing the R^2 values obtained in the test phase in the *D2* and *D4*, and *S2* and *S4* systems, when there was a drop in the coefficients of determination even with the increase in the number of membership functions for more input variables.

4.2 Validation of neuro-fuzzy models

Neuro-fuzzy models used in the validation phase were *D1*, *D2*, *D3*, and *D4* to predict dilation, and *S1*, *S2*, *S3*, and *S4*, to predict the shear stress.

The hypothetical rock discontinuities used to validate the neuro-fuzzy models have the same characteristics considered by Dantas Neto et al. (2017) and Leite et al. (2019a, b). Therefore, for the hypothetical unfilled rock discontinuities the following parameters were considered: *JRC* = 5; σ_c = 12 MPa e ϕ_b = 37.5°. For hypothetical infilled discontinuities, it was considered that the shear strength of the infill material is characterized by a friction angle (ϕ_{infill}) of 35.5°.

The models with the best results in predicting the shear behavior of the hypothetical discontinuities were the ANFIS *D1* and *S2* systems, as shown in Figures 2-5 and in Figures 6-9 for dilation and shear stress results, respectively.

According to the results presented between Figure 2 and Figure 5, it is observed that the ANFIS *D1* model can satisfactorily represent the drop in dilation in the hypothetical rock discontinuity with the increase in normal boundary stiffness (k_n), initial normal stress (σ_{no}), and the *t/a* ratio. In addition, higher dilation values have been obtained with the increased roughness in the discontinuity, represented by the *JRC* value. Such results can be considered satisfactory to the extent that they express the trends seen in different studies (Indraratna & Haque, 2000; Indraratna et al., 2005, 2008, 2010, 2013, 2014, 2015; Oliveira et al., 2009; Oliveira & Indraratna, 2010; Barton, 2013, 2016; Naghadehi, 2015; Shrivastava et al., 2011; Shrivastava & Rao, 2018).

Figures 6-9 show the results obtained with the *S2* neuro-fuzzy model considering the input variables of the hypothetical rock discontinuities. These results show that the *S2* model is capable to express the increase of shear stress values with the normal boundary stiffness (Figure 6), initial normal stress (Figure 7), and roughness (Figure 9). It is also possible to observe its ability to represent the drop in shear stress with the increase in *t/a* ratio (Figure 8), and also due to the asperity

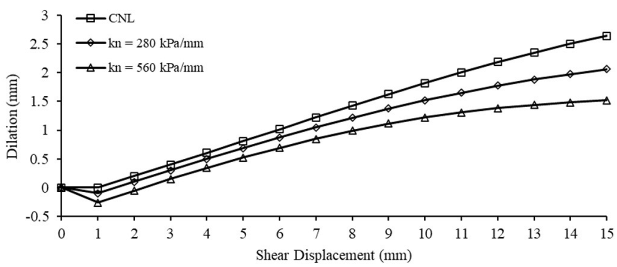

Figure 2. Effect of normal boundary stiffness on the dilation (σ_{no} = 0.5 MPa).

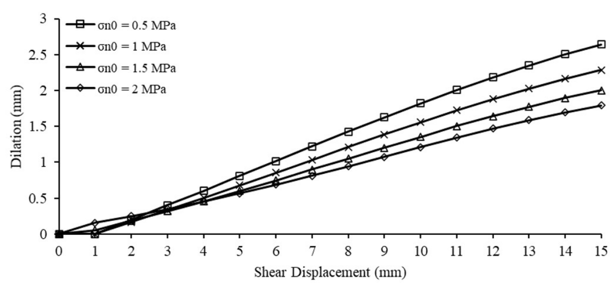

Figure 3. Effect of initial normal stress on the dilation (k_n = 0 kPa/mm).

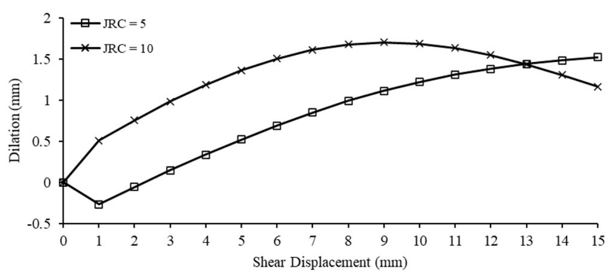

Figure 4. Effect of join|t roughness on the dilation (k_n = 560 kPa/mm and σ_{no} = 0.5 MPa).

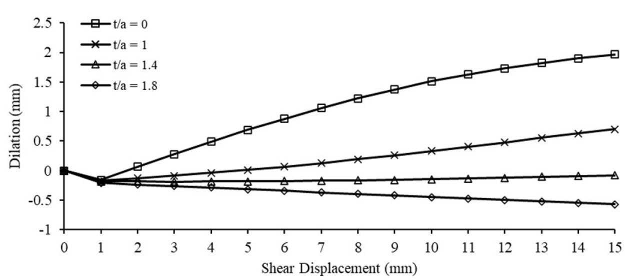

Figure 5. Effect of infill on the dilation (= 425 kPa/mm and σ_{n0} = 0.3 MPa).

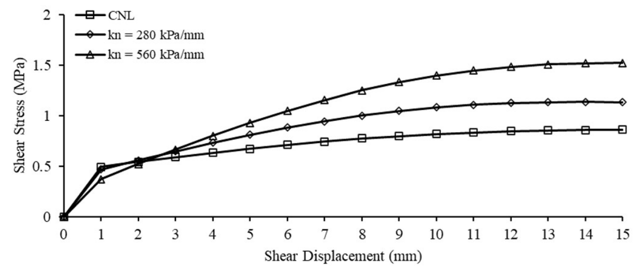

Figure 6. Effect of normal boundary stiffness on the shear stress (σ_{no} = 0.5 MPa).

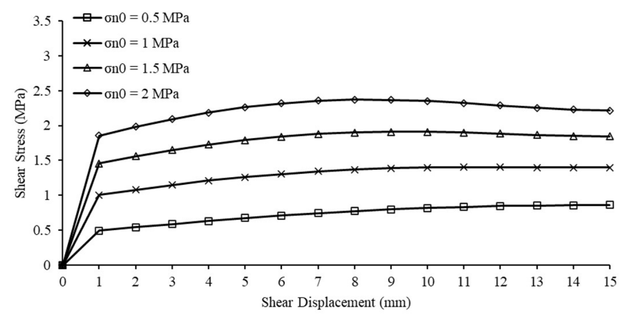

Figure 7. Effect of initial normal stress on the shear stress (k_n = 0 kPa/mm).

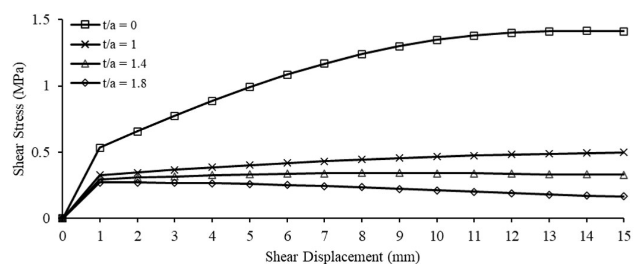

Figure 8. Effect of infill on the shear stress (k_n = 425 kPa/mm and σ_{n0} = 0.6 MPa).

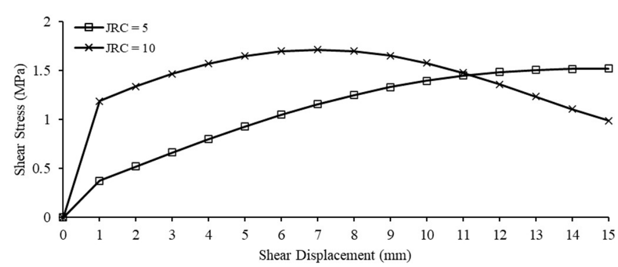

Figure 9. Effect of joint roughness on the shear stress (k_n = 560 kPa/mm and σ_{no} = 0.5 MPa).

damage occurring in rock discontinuities with high *JRC* values under CNS condition (Figure 9), similarly to what is expected when analyzing the influence of the input variables on the shear stress (Skinas et al., 1990; Papaliangas et al., 1993; Indraratna et al., 2015; Shrivastava & Rao, 2018).

5.3 Neuro-fuzzy models for predicting shear behavior of rock discontinuities

As shown and discussed previously, the neuro-fuzzy models referred to as *D1* and *S2* presented the best performances regarding the tests on experimental and hypothetical discontinuities, and were, therefore, chosen to predict the dilation and shear stress, respectively. It is worth mentioning the large number of the 2098 input-output patterns used in the concept and analysis of such models which consider different boundary conditions referring to the rock discontinuities, being developed from the grid partitioning technique using 80% of the experimental data for the training phase, and 20% for the testing phase.

Model *D1* consists of two (2) Gaussian membership functions for the eight (8) input variables that represent the main governing factors of the shear behavior of rock discontinuities, thereby creating 256 fuzzy inference rules. In the case of system *S2*, it presents 384 fuzzy inference rules, referring to the three (3) Gaussian membership functions for the *t/a* variable and two (2) Gaussian membership functions for the remaining input variables. In this context, dilation and shear stress are calculated by the respective fuzzy inference rules used in the models, when attributing input data that are within the intervals comprised by the established membership functions.

Figures 10-11 show the comparison of results obtained by applying the proposed *D1* and *S2* ANFIS models, those from the neuronal model proposed by Dantas Neto et al. (2017) to predict the dilation and shear stress, respectively, and the experimental data presented by Papaliangas et al. (1993) for an unfilled soft rock discontinuity under CNL condition, and σ_{no} = 0.05 MPa, *JRC* = 12, σ_c = 3.5 MPa and ϕ_b = 30°.

Figure 10 shows that both the ANFIS model (*D1*) and the neuronal model proposed by Dantas Neto et al. (2017) satisfactorily represented the variation of dilation with shear displacement. However, the results presented in Figure 11 show that the shear stress estimations provided by the ANFIS *S2* system were closer to the experimental data than those calculated by the ANN proposed by Dantas Neto et al. (2017). Its main limitation is the fact that it failed to represent the variation of shear stress with shear displacement in very soft rock discontinuities and submitted to low initial normal stress values, as already commented by the Dantas Neto et al. (2017).

Figures 12-13 show the comparison between the predictions from the proposed ANFIS systems, the neuronal model developed by Dantas Neto et al. (2017), and the experimental data presented by Benmokrane & Ballivy (1989), referring to unfilled hard rock discontinuity with the following characteristics and boundary conditions: σ_{no} = 1 MPa; *JRC* = 14; σ_c = 90 MPa; ϕ_b = 35°; and k_n = 315 kPa/mm (CNS condition). According to these results, both *D1* and *S2* ANFIS models fitted satisfactorily the experimental data regarding the variation in dilation (Figure 12) and shear stress (Figure 13) with the shear displacement, expressing the good performance of those neuro-fuzzy models in also estimating the shear behavior of hard rock discontinuities.

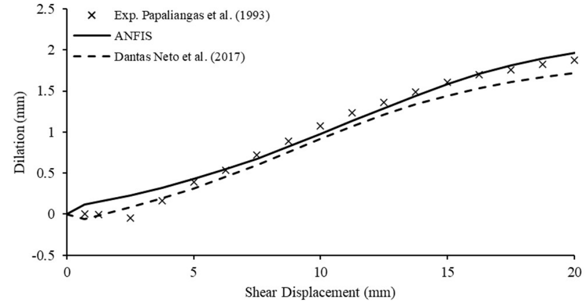

Figure 10. Comparison of experimental data for dilation of soft rocks and results from the *D1* ANFIS model and the ANN model proposed by Dantas Neto et al. (2017).

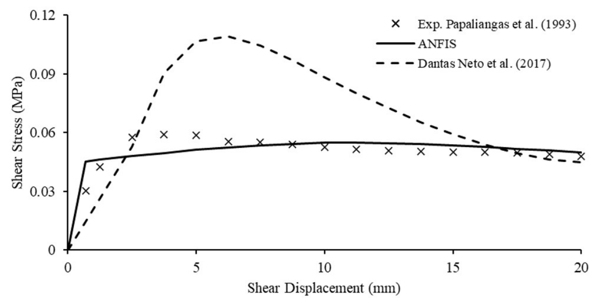

Figure 11. Comparison of experimental data for shear stress of soft rocks and results from the *S2* ANFIS model and the ANN model proposed by Dantas Neto et al. (2017).

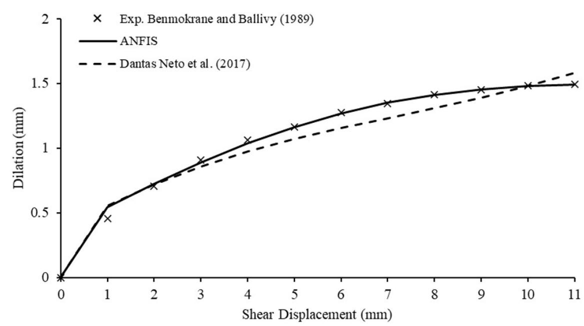

Figure 12. Comparison of experimental data for dilation of hard rocks and results from the *D1* ANFIS model and the ANN model proposed by Dantas Neto et al. (2017).

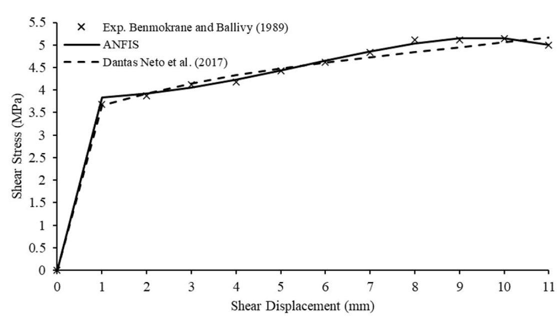

Figure 13. Comparison of experimental data for shear stress of hard rocks and results from the *S2* ANFIS model and the ANN model proposed by Dantas Neto et al. (2017).

5. Conclusions

Assessing the various neuro-fuzzy models developed in this study for the predictions of dilation and shear stress in filled and unfilled rock discontinuities, the *D1* and *S2* systems presented the best performances considering the tests and analyses made in experimental and hypothetical rock discontinuities. Such systems were designed based on the grid partitioning technique, which presented better results than the subtractive clustering technique, with 80% of the experimental data used for the training phase, the remaining 20% being used for the testing phase.

The input variables used in the developed neuro-fuzzy systems were the normal boundary stiffness (k_n), in kPa/mm; the ratio between the infill thickness and height of asperity (*t/a*); initial normal stress (σ_{no}), in MPa; joint roughness coefficient (*JRC*); uniaxial compressive strength of intact rock (σ_c), in MPa; basic friction angle of intact rock (ϕ_b), in degrees; friction angle of fill material (ϕ_{infill}), in degrees, and shear displacement (δ_h), in mm. Hence, the aim was to use the main governing factors of the shear behavior of the filled and unfilled rock discontinuities,

The model for predicting dilation (*D1*) consists of two (2) Gaussian membership functions for all input variables corresponding to a total of 256 fuzzy inference rules. Its coefficients of determination (R^2) were 0.99 in both training and testing phases indicating a satisfactory correlation between the experimental data and results obtained by the proposed neuro-fuzzy system. The neuro-fuzzy system developed for predicting the shear stress of rock discontinuities (*S2*) presents coefficients of determination of 0.97 and 0.96 in the training and testing phases, respectively, which also show the proximity between the estimations and the experimental data.

Using *D1* and *S2* ANFIS models to estimate dilation and the shear stress considering several characteristics of hypothetical infilled and unfilled rock discontinuities, it was possible to notice that the ANFIS systems satisfactorily represented the influence of the input variables on their shear behavior. This highlights the ability of neuro-fuzzy models to model multivariate, non-linear, and complex problems when compared to frequently used analytical models, and to consider the variability, or uncertainties, of the input values in the models' response.

Despite the functions of the neuro-fuzzy models in estimating the shear behavior of the infilled and unfilled rock discontinuities, the use of these systems is conditioned and limited to the intervals attributed to their input variables during the modeling process. Moreover, these models did not yet consider other key factors that influence the shear behavior of the rock discontinuities, such as the drainage condition, saturation degree, cohesion of the fill material, and weathering in the rock discontinuity walls.

Lastly, the developed neuro-fuzzy systems are not meant to substitute tests that still need to be performed on samples from rock masses. Here, the ANFIS models appear as a potential tool for the preliminary estimation of the shear behavior of rock discontinuities, by attributing values to the input variables used, providing rapid responses to help toward the design's assessment.

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Declaration of interest

The authors have no conflicts of interest to declare. All co-authors have observed and affirmed the contents of the paper and there is no financial interest to report.

Authors' contributions

Silvrano Adonias Dantas Neto: conceptualization, supervision, data analysis, writing – review. Matheus Cavalcante Albino: data curation, methodology, modeling, writing – original draft. Ana Raquel Sena Leite: investigation, methodology. Ammanda Aragão Abreu: data analysis, format revision, writing – editing.

List of symbols

a	asperity height of the discontinuity
i	depicts the value of 1 and 2 in virtue of the number of fuzzy inference rules and sets used
k_n	normal boundary stiffness
p_i	consequent parameter
q_i	consequent parameter
r_i	consequent parameter
T	thickness of the infill material
t/a	relation between t and a
$\overline{w}_i$	output from Layer 3
x	input in the nodes
y	input in the nodes
z	output of the neuro-fuzzy system
A_i	fuzzy set associated with the nodes
B_i	fuzzy set associated with the nodes
D	designation for ANFIS model for dilation
$D1$	designation for ANFIS model 1 for dilation
$D2$	designation for ANFIS model 2 for dilation
$D3$	designation for ANFIS model 3 for dilation
$D4$	designation for ANFIS model 4 for dilation
$D5$	designation for ANFIS model 5 for dilation

D6 designation for ANFIS model 6 for dilation
D7 designation for ANFIS model 7 for dilation
D8 designation for ANFIS model 8 for dilation
D9 designation for ANFIS model 9 for dilation
JRC joint roughness coefficient
$O_{1,i}$ value of the degree of membership obtained from *x* and *y*
R^2 coefficient of determination
S designation for ANFIS model for shear stress
S1 designation for ANFIS model 1 for shear stress
S2 designation for ANFIS model 2 for shear stress
S3 designation for ANFIS model 3 for shear stress
S4 designation for ANFIS model 4 for shear stress
S5 designation for ANFIS model 5 for shear stress
S6 designation for ANFIS model 6 for shear stress
S7 designation for ANFIS model 7 for shear stress
S8 designation for ANFIS model 8 for shear stress
S9 designation for ANFIS model 9 for shear stress
δ_h shear displacement
δ_v dilation
ϕ_b basic friction angle of intact rock
ϕ_{infill} friction angle of infill material
σ_{no} initial normal stress
σ_c uniaxial compressive strength of intact rock
τ_s shear stress

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