

Risk management for geotechnical structures: consolidating theory into practice

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Lecture

Keywords

Curves of risk acceptance and tolerance
Failure probability
Geotechnical structures
Impact and consequences
Probabilistic methods
Risk management

Abstract

This paper intends to consolidate the theory of risk management into practical applications in geotechnical engineering, presenting concepts, clarifying procedures and discussing openly its difficulties and trends. It brings the evolution of the risk concept and its application to engineering, worldwide and in Brazil, showing the trend of risk management as a decision-making tool in engineering with fair acceptance by the society. The probabilistic approach is discussed and compared to the deterministic one, focusing on the obtaining of reliability indexes and failure probabilities for engineering structures. For this, quantitative methods, such as event and fault tree analyses and probabilistic methods, are reviewed, discussing their applications and comparing their advantages and disadvantages. Risk metrics and the evaluation of its two components, failure probability and consequences due to failure, are presented, focusing on the need to quantify and monetise consequences, and, consequently, the engineering risks. From this derives the concept of overall cost, which is the structure cost or value added to its risk value, providing an efficient tool to compare engineering alternatives and solutions. Finally, the risk management scheme is discussed, focusing on the need to establish an intelligent risk management system, which incorporates an automatic and intelligent communication tool, to disseminate among professionals, company hierarchy and outside stakeholders, the structure risks, according to their levels in the Risk Diagram and guided by the company Risk Policy. This is illustrated by examples of applications in two geotechnical structures (a dam and an urban tunnel), showing its enormous potential as a decision-making tool in engineering, using risk-based or risk-informed approach.

1. Introduction

This paper presents the contents of the Pacheco Silva Conference, awarded by the Brazilian Society for Soil Mechanics and Geotechnical Engineering (ABMS), and delivered during the XIX COBRAMSEG (Brazilian Congress on Soil Mechanics and Geotechnical Engineering) in Salvador, August 2018. The theme of the conference was agreed between the ABMS and the author, considering the growing demand for geotechnical risk analysis and management in Brazil and worldwide. This paper aims to present the basic concepts related to the probabilistic approach and risk management, including probabilistic methods to evaluate the probability of failure modes of geotechnical structures, estimation of consequences, in case of failure occurrence, risk calculation and evaluation considering the acceptance and tolerance curves, taking into account a ro-

bust theoretical background applied to practical examples, using the simplest language and manner as possible. From that it comes the proposed target of consolidating theory into practice of risk management applied to geotechnical structures.

Ground property variability has been recognised for a long time (Lumb, 1966) and concepts of risk and reliability applied to geotechnical engineering (Ang & Tang, 1975, 1984; Harr, 1987) have been available for the last four decades. However, the consideration of this knowledge to analyse and design geotechnical structures is still not fully widely applied, struggling with a deterministic culture established and dominant for a long time. In addition to the deterministic culture, several factors may have been contributing to the difficulties of applying risk management currently in geotechnical engineering, such as: i) poor background in statistics and probability of the professionals

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involved; ii) difficulties to establish the probability distribution of geotechnical properties and loadings due to lack or few number of data; iii) feeling that risk calculation is too complicated, complex, time consuming or not accurate enough; and iv) no familiarity with risk acceptance concepts contrarily to safety factors (or similar concepts), whose recommended values are well defined in standards and guidelines.

The fact that traditional engineering has always considered the concept of exactitude has led professionals and society to believe in accurate and precise calculations, with no chances for errors and potential failures. This concept has implied calculations following the deterministic approach, where in any engineering calculation, defined by an empirical, analytical or numerical formulation, material properties and loadings are deterministically defined by specific values, giving a unique result for the engineering calculation. However, it is well known by engineering professionals that some properties and loadings are variable or present uncertainties and should not be defined as a unique value. For this reason, concepts of safety margins or factors have been proposed, which means that the engineering result calculated by the deterministic approach has to obey a safety margin in relation to its critical value that defines a potential failure for that particular structure. In other words, the recognised variability and uncertainties of some engineering properties and loadings have been dealt by safety margins or similar concepts.

In the 1950s, the nuclear energy engineering had to deal with uncertainties for the first time in a clear, simple and objective way to share information with society. The Brookhaven Report (USAEC, 1957) analysed the consequences of an eventual failure of a nuclear powerplant reactor and estimated potential losses and impacts. Despite some fatalities and economic losses were estimated, no methodology for evaluating the failure probability was presented. This report can be considered the first to clearly tackle risks of engineering structures using qualitative estimation. Later, the Rasmussen Report (USNRC, 1975) presented a review of the Brookhaven Report, incorporating quantitative methodologies for estimating risk, in terms of both failure probability and potential consequences. These reports played an important role in promoting the probabilistic approach and risk analysis to deal with uncertainties and variabilities, clearly opposite to the deterministic approach commonly adopted in traditional engineering. In the 1990s, the concepts of risk analysis and management became more common and widely applied to several types of structures, turning into a decision-making tool in engineering (risk-based or risk-informed approach), which means that the calculated risk is taken as one of the key aspects for selecting the best engineering alternative, which has been called the New Engineering.

In Brazil, the pioneers of risk management applied to geotechnical engineering introduced these concepts in the

1980s and 1990s, in particular Fernando Franciss, Hachich (Hachich, 1981; Hachich & Vanmarcke, 1983), Pacheco (1990) and Aoki (Aoki & Cintra, 1996). In 1995, a graduate course dedicated to probabilistic approach and risk management applied to geotechnical engineering was created at the University of Brasilia (Assis *et al.*, 2018), which has motivated applied research on this topic and several M.Sc. and Ph.D. theses have been completed (Espósito, 1995, 2000; Lauro, 2001; Maia, 2003, 2007; Perini, 2009; Hidalgo, 2013; Alarcón-Guerrero, 2014; Charbel, 2015; Mendes, 2017; Franco, 2019; Mendes, 2019; Yokozawa, 2019). Other institutions in Brazil have also contributed actively to risk management in geotechnical structures, such as the Federal University of Ouro Preto (UFOP), Pontifical University of Rio de Janeiro (PUC-Rio) and University of São Paulo (USP), to cite a few. Lately, this topic has been gaining importance in the Brazilian geotechnical community and widely disseminated by the ABMS, contemplating risk aspects, analysis and management in three Milton Vargas Lectures (Coutinho, 2010; Aoki, 2011; Hachich, 2018), two Pacheco Silva Conferences (this paper; Aoki, 2016) and two Victor de Mello Lectures (Mitchell, 2014; Morgenstern, 2018).

Once assuming that engineering calculations incorporate uncertainties and sharing this concept with society, this has brought a dilemma between engineering and society. In general, engineering professionals attempt to focus only in the failure probability of their structures, which it usually quite low. On the other hand, the society only considers the consequences of an eventual failure of these engineering structures, which in some cases can be quite considerable or even catastrophic. Both views are realistic, but antagonistic. Then, the concept of risk, which incorporates both the failure probability and its potential consequences, is the only one able to consider the demands of both sides. In other words, the risk concept is the common denominator between engineering and society, therefore able to be the promising key parameter for decision of acceptable engineering solutions for the society.

It is worth recalling the evolution of engineering approaches during the last decades. In the past, the engineering approach focused basically on the technical benefits and costs of structures, which means that the dimensional view of engineers was only the engineering structure itself. Later, the environmental impacts caused by the implantation and operation of engineering structures have jointed the aspects of benefits and costs. One can say that the engineering view widened to two dimensions, focusing on the structures and their environmental impacts. More recently, the evolution of the current engineering approach has included engineering risks, which consider its failure probability and all dimensions of consequences due to its eventual failure. This means that the best engineering alternative nowadays has to take into account aspects of technical benefits and costs of the structure, its environmental impacts for implantation and during operation, and its potential con-

sequences to the society in case an eventual failure occurs, which is the risks of the engineering structure.

Since risk analysis and management is an essential aspect of the New Engineering, it is desirable to present the important components of risk theory and its practical application to geotechnical structures. As risk is based on uncertainties and variabilities, the first step is to discuss how to incorporate them in engineering calculations, which is done using the probabilistic approach in opposition to the deterministic one, commonly used in traditional engineering up to now.

2. Probabilistic approach

Before discussing the probabilistic approach, it is worth recalling the concept of failure criterion usually adopted for engineering structures. Among all engineering calculations, some are of interest for analysing the behaviour or checking the safety of structures. They are called performance indicators and examples in geotechnical engineering can be the flow rate of a dam, settlements of a foundation, safety factor of a slope, construction schedule, costs and so on. Each of these performance indicators is calculated by an engineering formulation, which can be empirical, analytical or numerical, and is generally expressed as:

$$y = f(x_1, x_2, \dots, x_n) \quad (1)$$

where y is the performance indicator, x_i are the input parameters (material properties, loadings etc.) and f is the function that defines the engineering formulation for this performance indicator.

For each performance indicator, a failure criterion can be defined. The concept of failure here has a very broad meaning, indicating deficient or total loss of the engineering structure performance (structural, functional, schedule overtime, over costs etc.). A critical value (y_{crit}) for the performance indicator is defined, which means that the structure would not perform satisfactorily if the performance indicator calculated value exceeds its critical value. Failure criteria can be expressed as:

$$\begin{aligned} y = f(x_1, x_2, \dots, x_n) &> y_{crit} \\ \text{or} \\ y = f(x_1, x_2, \dots, x_n) &< y_{crit} \end{aligned} \quad (2)$$

This concept is quite common in traditional engineering, which adopts the deterministic approach, calculating the performance indicator using constant values for all input parameters. The assumed values for input parameters are a choice of the engineer and commonly are taken as the mean, most likely or any other value according to his or her experience and common sense. As all input parameters are taken as constant values, the calculated value of the performance indicator is unique. In some occasions, to better understand how input parameters may affect the calculated value of the performance indicator, parametric or sensitive

analyses can be done to complement the deterministic calculation. Safety margins are defined between the performance-indicator calculated value and its critical value to cover eventual uncertainties and variabilities of input parameters and engineering processes (assumptions, modelling adequacy and so on). The concept of safety margins is very consolidated and well-accepted in traditional engineering and typical values are commonly suggested or prescribed by guidelines and standards.

On the other hand, the probabilistic approach can be taken as an alternative to the deterministic one, where uncertainties and variabilities of input parameters are considered in the evaluation of the performance-indicator engineering formulation (Eq. 1), using probabilistic methods. As some input parameters are taken as variables, and not constant values, the calculated value of the performance indicator is also a variable and can be described as a probabilistic distribution function. Besides the probabilistic function statistics (mean, standard deviation, etc.), this permits an additional and very important information, which is the failure probability (p_f), defined as the probability of the performance-indicator probabilistic function exceeding its critical value prescribed by the failure criterion. The statistics of the performance-indicator probabilistic function allow the evaluation of the reliability index β (Christian *et al.*, 1992, 1994) and the failure probability, expressed as:

$$\beta = \frac{(y_m - y_{crit})}{\sigma_y} \quad (3)$$

$$p_f = p(y > y_{crit}) \quad \text{or} \quad p_f = p(y < y_{crit}) \quad (4)$$

where β is the reliability index; y_m is the mean value of the performance indicator; y_{crit} is the critical value of the performance indicator as prescribed by the failure criterion; σ_y is the standard deviation of the performance indicator; and p_f is the failure probability.

It can be noted that the reliability index (β) has a similar concept of the safety margin, defined by the difference between the mean and critical values, normalised by the standard deviation. In other words, it is the number of standard deviations from the mean to the critical values of that performance indicator. Its main advantage is that it is independent of the performance-indicator probabilistic function, which is helpful for suggested or prescribed values in guidelines. On the contrary, the failure probability (p_f) can be only calculated for a certain probabilistic function. The main aspects and comparison between the deterministic and probabilistic approaches are shown on Table 1.

Figure 1 illustrates the schematic process of the probabilistic approach. Some input parameters assumed as variable (x_i) can be described by a probabilistic function that best fits its variability distribution. Input-parameter probabilistic functions are considered in the calculations of the performance-indicator engineering formulation using probabilistic methods. The result is the performance-in-

Table 1. Comparison between engineering approaches.

Engineering approach	Deterministic	Probabilistic
Input parameters of the engineering formulation (x_i independent variables)	Values of input parameter are assumed constant	Some input parameters are assumed as variables
Dependent variable y (performance indicator)	Result is a unique or a range of values for parametric or sensitivity analyses	Result is a probabilistic function or a mean value and its standard deviation
Failure criterion	Comparison between the calculated and critical values of the performance indicator y and check with the prescribed safety margin	The reliability index and failure probability of the performance indicator y are calculated and used in risk analyses

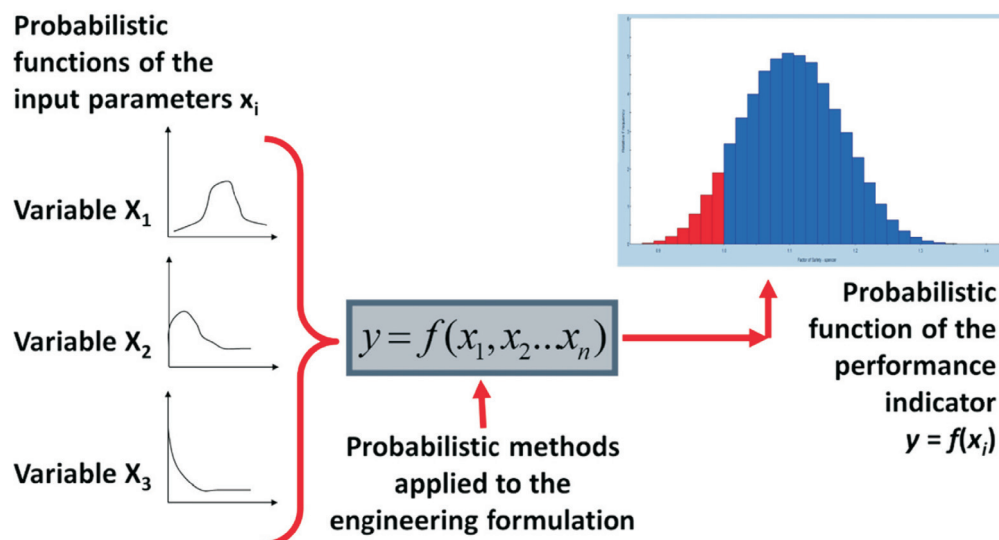
indicator probabilistic function and its statistics, which permits to calculate its mean value, standard deviation, reliability index, failure probability and so on.

As mentioned, one of the main advantages of the probabilistic approach is the evaluation of the failure probability or reliability index of the performance indicator, in addition to its mean value. Traditional engineering based only on deterministic approach usually takes decisions founded on the mean value or similar. When also taking into account the failure probability or reliability index, besides the mean value of the performance indicator, the input parameter variabilities (standard deviations and type of probability functions) are also considered. Figure 2 depicts this concept considering two slopes, with different mean and standard deviation values of the Factor of Safety (FS). Slope B has a greater FS mean value ($FS = 2.0$) than Slope A ($FS = 1.5$), but due to the greater scatter of its input parameters (standard deviation of 0.85 for Slope B against 0.25 for Slope A), Slope B also presents a higher value of the failure probability. Considering only the FS mean values, which is commonly the practice in traditional engineering, one may erroneously decide that Slope B is safer than Slope A. This reinforces that any attempt to establish

correlations for different structures between the mean value of performance indicators and their probabilities of failure may lead to misjudgements, because it misses an important information, which is the input data scatter, as depicted in Fig. 3.

Despite the advantages of the probabilistic approach applied to engineering, there are several challenges to be overcome in order to ensure reasonable and reliable results from it. The estimation or calculation of the failure probability can be done by different methods, ranging from the simplest to more complex ones, such as:

- Qualitative analyses are the simplest methods, where the failure probability is qualified by adjectives (for instance, ranging from practically impossible to very likely).
- Numerical values, where the adjectives qualifying the failure probability are replaced by a range of numbers (for instance, number 1 means practically impossible, increasing to number 5, which means very likely).
- Event-tree and fault-tree analyses describe the logical path of events leading to failure and allow to attribute probabilities to each event, finally quantifying the failure probability according to the relation among all participating events.

**Figure 1.** Schematic process of the probabilistic approach applied to engineering.

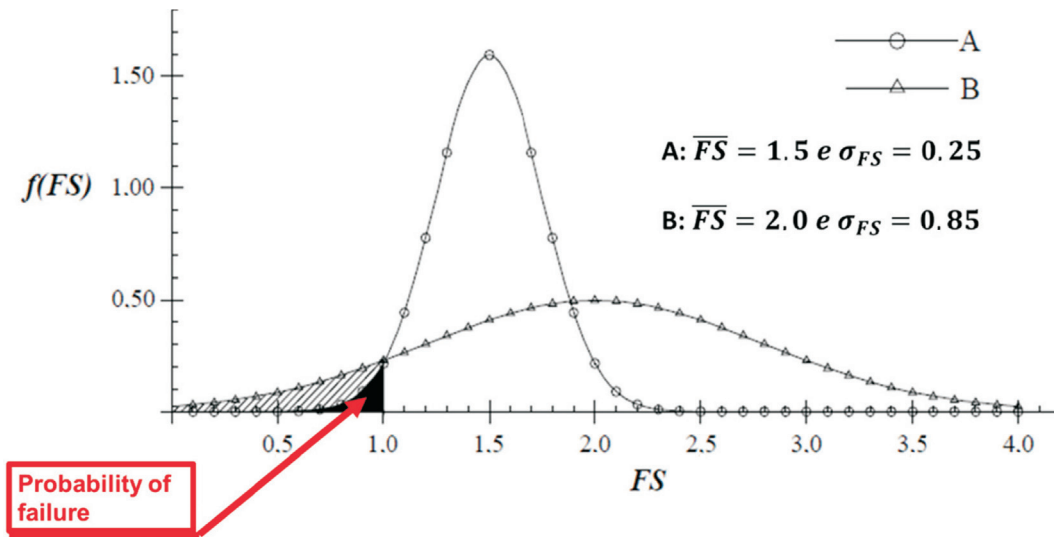


Figure 2. Comparison of the factor of safety of two slopes.

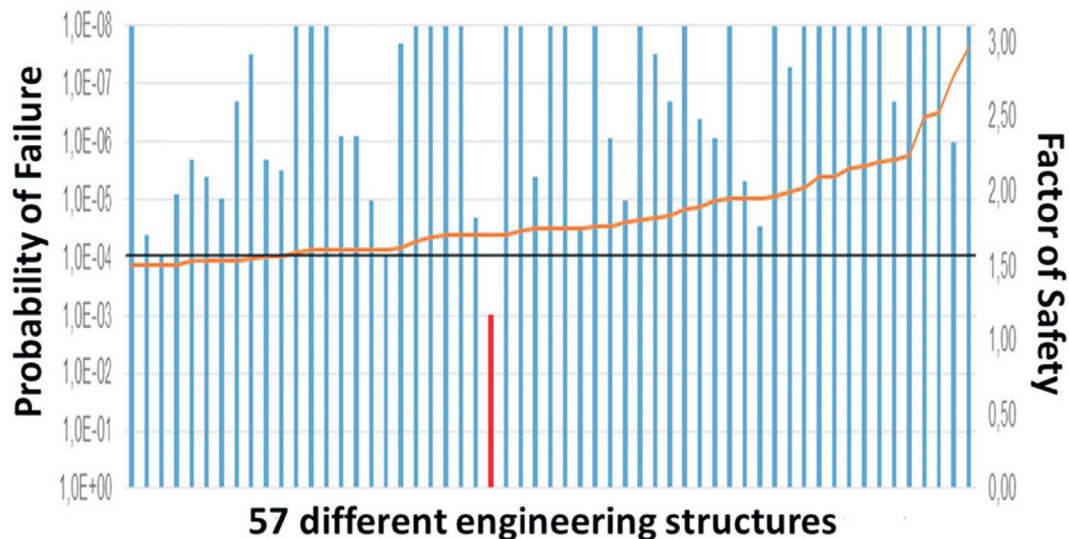


Figure 3. Comparison between factors of safety and failure probabilities of 57 different engineering structures, showing no correlation between these two variables.

- Probabilistic methods are the most complex ones, using the scheme depicted in Fig. 1.

The first challenge of applying probabilistic approach in engineering is the selection of the type of method to estimate or calculate the failure probability of structures. This depends on the availability and quality of input data, importance and complexity of the structure (dimensions and potential impacts in case of an eventual failure), level of engineering studies and knowledge and maturity of professionals and companies involved. At a first trial, the main point is to use the probabilistic approach, no matter how simple the chosen method is. However, there are many gains moving to quantitative methods (Assis *et al.*, 2019; Oboni & Oboni, 2020), which are the focus of this paper, more precisely the use of probabilistic methods to evaluate

the failure probability. As shown in Fig. 1, the quantification of the reliability index and failure probability of a certain performance indicator requires:

- Selection of a performance indicator, its engineering formulation and critical value (defined by the adopted failure criterion), what is also a common practice in traditional engineering.
- Definition of the probabilistic functions or statistics (mean and standard deviation) of input parameters (material properties), loadings, etc., which are taken as variables for the calculation of the performance indicator.
- Choice of the most adequate probabilistic method to obtain the probabilistic function or statistics of the performance indicator, and, consequently, its reliability index and failure probability.

- Interpretation of the probabilistic analysis results by the evaluation of the reliability index and failure probability, and, consequently, the risk management of the engineering structure.

Before discussing probabilistic methods, a word on event-tree and fault-tree analyses. These tools are quite important when the failure probability has to be quantified, but there is no well-defined engineering formulation to evaluate that performance indicator, so that probabilistic methods cannot be applied. In addition, event and fault trees are excellent engineering exercises, helping establish the logical sequence of events towards the final failure event (event tree) or backwards from the failure event, identifying the previous events necessary to cause the failure (fault tree). Once the event or the fault tree is set, these tools are useful and effective to understand the whole engineering process that may lead to the final failure event. Then, probabilities can be attributed to each event, using expert experience and perception or any other calculation tool. At the end, the probability of the failure event may be calculated using logical operators, following probability rules for events that may occur alternatively, simultaneously, in series and so on. Figure 4 shows an example of event tree for calculating the failure probability due to piping in a dam. Two aspects are of particular interest in this example: i) firstly, piping is a phenomenon that hardly is defined by a precise engineering formulation, so that the event tree analysis is required; ii) secondly, some probabilities of the branches of this event tree have been calculated using fault tree analysis. So, this example combines both event and fault tree techniques and, finally, the failure probability due to piping is quantitatively calculated. More detailed discus-

sions on event and fault tree analyses applied to piping in dams are presented by Fell *et al.* (2015) and Caldeira (2018).

Contrarily, when the performance indicator is well-defined by an engineering formulation (empirical or analytical expressions, or numerical calculations), the use of probabilistic methods is recommended and presents reliable results. Probabilistic methods can be defined as those able to determine the probabilistic distribution function or its statistics of a dependent variable (performance indicator), which is defined by an engineering formulation, based on the probabilistic distribution functions or their statistics of input variables (material properties, loadings, etc.). There are many probabilistic methods available and the most commonly used in geotechnical engineering are the Monte Carlo Method – MCM (Harr, 1987; Baecher & Christian, 2003; Fenton & Griffiths, 2008), First Order Second Moment – FOSM (Harr, 1987; Baecher & Christian, 2003), Point Estimate Method – PEM (Rosenblueth, 1975, 1981; Harr, 1987; Baecher & Christian, 2003), Hybrid Point Estimate Method – HPEM (Gitirana, 2005; Franco, 2019; Franco *et al.*, 2019; Yokozawa, 2019), First Order Reliability Method – FORM (Baecher & Christian, 2003; Fenton & Griffiths, 2008) and so on. All of them require the variability data (probabilistic distribution functions or statistics) of the input parameters, which is the second challenge of the probabilistic approach applied to engineering to be discussed in this paper.

The problem is how to obtain the probabilistic distribution functions of input parameters and loadings assumed as variable, considering that usually engineering input parameters and loadings are quite limited in values due to test-

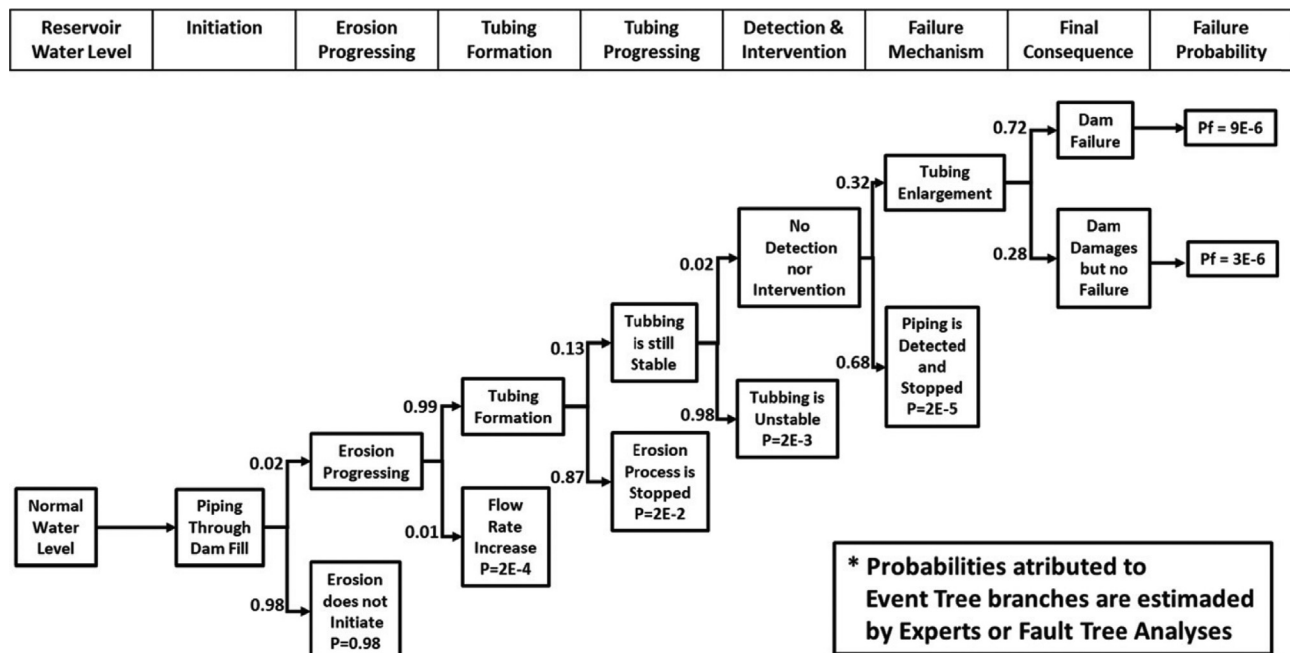


Figure 4. Example of an event tree for estimating the failure probability due to piping of a dam.

ing complexity, time and costs. There are some options to cope with this challenge. Firstly, consider that there are plenty of data for a certain input parameter, which is a sample statistically representative of that variable. In this case, data are treated by descriptive statistics, following these stages: i) Calculate the mean, standard deviation and any other moments of the sample data; ii) Organise the data in a histogram; iii) Fit any probabilistic distribution function; iv) Use any type of fitting test (minimum square of errors, Chi-square, Kolmogorov-Smirnov, etc.) to check which function better represents the sample data; v) Choose the probabilistic distribution function and its statistics for that particular input variable. More commonly, there is lack of enough data to obtain the probabilistic distribution functions of some input variables. In this case, the options to represent the variability of the input parameters are the following:

- The few existing data of a certain variable are used to estimate the mean value and the standard deviation is obtained by the coefficient of variation (CoV), as suggested in the literature and listed in Table A1 (several authors have reported typical or a range of CoV values for different geotechnical properties, as CoV values present much less scatter than the mean and standard deviation themselves); considering the obtained mean and standard deviation values, a probabilistic distribution function can be adopted for that variable (some authors have suggested the type of probabilistic functions that better fits some geotechnical properties, as shown in Table A2).
- In case of no suggested CoV value for a certain variable and, consequently, its standard deviation cannot be estimated, some simplified probabilistic functions can be adopted such as Constant or Triangular functions, which requires the minimum and maximum values of the variable (these limit values might be defined by expert and experienced engineers).

Some comments now follow on the above options to overcome the lack of enough data to describe the variability of input parameters taken as variables. The CoV values and their ranges suggested in Table A1 are the best option to overcome the lack of data. However, it is highly recommended that the chosen value be the result of a critical analysis from experienced engineers, considering their expectation of the variability of that parameters. For instance, the CoV range for the cohesion varies from 20 to 80 %, and 40 % is commonly taken for most cases; however, if the target is the cohesion CoV for a compacted soil, the lower values of the range can be considered, such as 20-25 %, but on the contrary, if the target is the cohesion CoV for a natural saprolitic soil, maybe the higher values of the range are more adequate, such as 60 % or more due to its enormous variability. Other important point to consider is that the simplified probabilistic functions (constant or triangular) do not require a defined standard deviation value, however, some probabilistic methods require this value even so. A

common approximation to estimate the standard deviation for these functions is to assume its value equal to 1/6 of the difference between the maximum and minimum limit values of that variable (this concept comes from the fact that variables described by the normal distribution present almost all its variability within the limits of three standard deviations around the mean value, totalling six standard deviations for the whole range).

Other important point to consider for representing the input variables by probabilistic distribution functions is the truncation of their value ranges. This is needed because some probabilistic functions are unbounded on both sides or on one side, requiring that limits are imposed on unbounded sides to avoid variable values outsider their realistic possible range. Two types of truncations are possible, the statistical and the engineering ones. The statistical truncation is usually done considering a number of standard deviations around the mean value (mean value minus m standard deviations and mean value plus n standard deviations, where commonly m is equal to n). As already mentioned, three standard deviations around the mean value for variables well-described by the Gaussian distribution represent almost the totality of the possible range (99.7 %) of that variable, so that this is a very commonly adopted truncation. Statistical truncations using smaller numbers of standard deviations around the mean values have to be taken with care, because, if the range of values for input variables are made narrower, the standard deviation of the dependent variable (performance indicator) evaluated by probabilistic methods may be done artificially smaller, leading to lower failure probabilities than the realistic ones (Hammah *et al.*, 2010). On the other hand, sometimes the statistically truncations define a range of values for a particular input variable that is not realistic, which means that the possible values of that variable are not physically acceptable. In this case, it is highly recommended to apply engineering truncation, defining the minimum and/or maximum acceptable limit values of the variable. It is helpful to have the expertise of experienced geotechnical engineers to define these limit values for the geotechnical properties to be represented by probabilistic distribution functions. In summary, the good practice is to firstly define the statistical truncation (three standard deviations around the mean value is recommended) and then, check if its range of values does not violate the possible engineering range of values for that input variable; if so, the range of values defined by the statistical truncation has to be corrected by the engineering truncation.

Once all input variables have their probabilistic distribution functions or their statistics (mean value and standard deviation) defined, probabilistic methods can be applied to obtain the probabilistic distribution function or its statistics of the dependent variable (performance indicator). Several probabilistic methods are available with their advantages and disadvantages and comments are made on the three

most popular methods applied to geotechnical engineering, which are the Monte Carlo Method (MCM), the First Order Second Moment Method (FOSM) and the Point Estimate Method (PEM).

The MCM is a probabilistic method that may be considered exact in obtaining the probabilistic distribution function of the dependent variable, since it solves randomly the engineering formulation for the dependent variable (performance indicator) N times, generating a sample of results, and when N is a number large enough, the statistics of the resulting sample do not change any more, achieving its stability, indicating that the results are final and considered exact. The aspects and steps for running the MCM (Fig. 5) can be summarised as follows:

- The MCM aims to obtain an approximate numerical simulation of the probabilistic distribution function of the dependent variable y (performance indicator), which is defined by an engineering formulation (empirical, analytical or numerical).
- The MCM requires the probabilistic distribution functions of all input variables.
- Independent and random values for each input variable x_i are obtained and an evaluation of the engineering formulation of the dependent variable y is done; for each input variable selection, its probabilistic distribution function is taken in its accumulative probability form, and a random number between 0 and 1 is obtained, applied to the accumulative probability curve, obtaining the value of the input variable x_i , which is used in that particular MCM simulation.
- Repeating this procedure N times, a sample of N discrete values of the dependent variable y is obtained; this process may require considerable computational effort, depending on the complexity of the engineering formulation.

- Taken this sample with N values of the dependent variable (performance indicator), a histogram can be plotted and the statistics (mean, standard deviation and other moments) calculated, as well the best-fit probabilistic distribution function found and the failure probability evaluated.
- Increment the number of MCM simulations until the resulting statistics is stabilised (they do not change compared to immediate previously ones as shown in Fig. 6); when that happens, the MCM results can be considered final and exact.

As soon as a sample of N results of the dependent variable (performance indicator) is obtained, the sample statistics can be calculated using descriptive statistics of discrete variables, as expressed below for the mean value, standard deviation and failure probability:

$$E(y) = \bar{y} = \frac{\sum (y_i)}{N} \quad (5)$$

$$\sigma_y = \sqrt{\frac{\sum (y_i - E(y))^2}{N}} \quad (6)$$

$$p_f = \frac{N_f}{N} \quad (7)$$

where: $E(y)$ is the mean value of the dependent variable y ; y_i are the discrete values obtained by MCM simulations; N is the total number of MCM simulations; σ_y is the standard deviation of the dependent variable; p_f is the failure probability of the dependent variable; N_f is the number of MCM simulations that indicates failure ($y_i < y_{crit}$ or $y_i > y_{crit}$).

A key point is to know when the number N of MCM simulations is large enough and the results can be considered already stabilised. Statistically speaking there is a for-

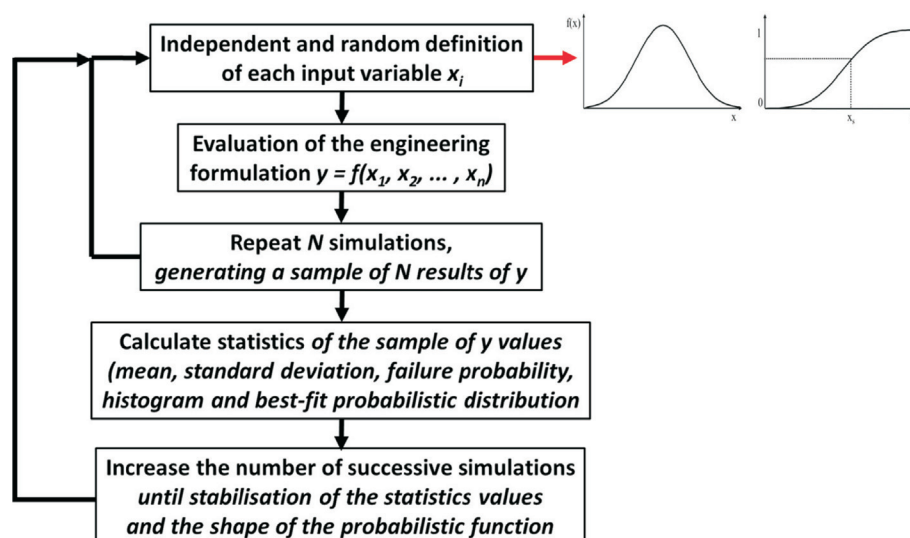


Figure 5. Main steps of the MCM simulations.

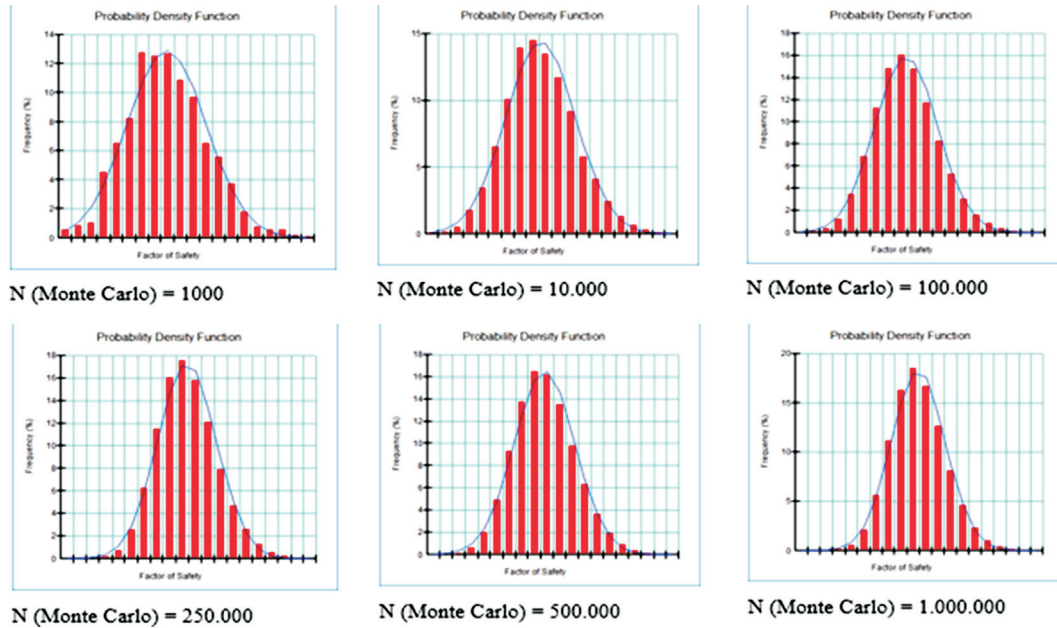


Figure 6. MCM results with different N simulations, showing that increasing N the results tend to stabilize, which can be considered final and exact.

mulation to estimate the number of MCM simulations, assuming that an error α may occur, which is expressed by:

$$N = \left(\frac{Z_{\alpha/2}^2}{4\alpha^2} \right)^n \quad (8)$$

where N is the desirable number of MCM simulations assuming that an error α in the results may occur; $Z_{\alpha/2}$ is the number of standard deviations around the mean value to reach an error $\alpha/2$ at each distribution tail; n is the number of input variables.

This statistical formulation usually yields an enormous number of MCM simulations N . For example, if an error α of 5 % is intended, Z is 2 standard deviations and assuming four input variables ($n = 4$), the number N of MCM simulations estimated is over 25 billion. Therefore, commonly the criterion to verify the stabilisation of the MCM statistics is observational, as shown in Fig. 7. In this case, one can notice that 200,000 simulations are enough to stabilise all statistics resulting from the MCM simulations.

Some important aspects related to the MCM results, in particular to the failure probability calculated by the frequentist formulation presented by Eq. 7, are worth some comments. It is recommended that the failure probability calculated from Eq. 7 should only be accepted if the number N of MCM simulations is at least one order of magnitude (10 times) greater than the inverse of the failure probability calculated. For example, if p_f is calculated as 10^{-4} , it requires at least 10^5 MCM simulations. Other common mistake in reporting the failure probability calculated by the MCM is to take it as zero, when after a certain number of simulations N , no failure case is found. This does not guarantee

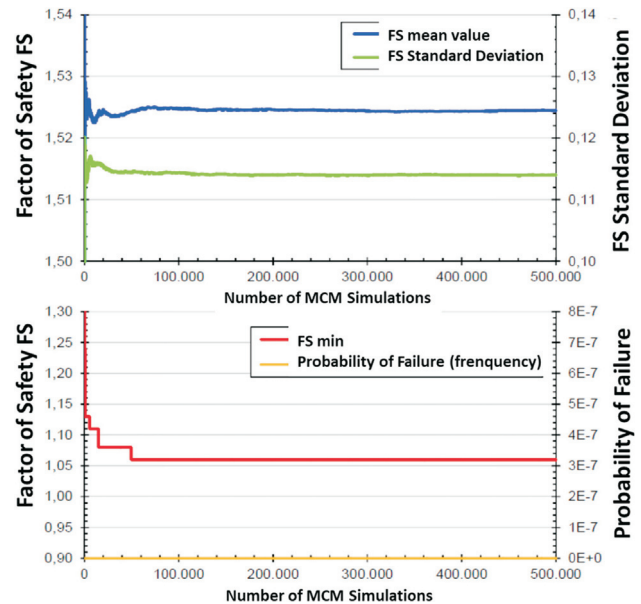


Figure 7. Observational criterion to verify the MCM simulation results stabilisation.

that p_f is zero but, simply, that within the number of simulations carried out, no failure event could be discovered by that particular MCM simulation process. The best form to report this result is that the failure probability is simply smaller than $1/N$ ($p_f < 1/N$). This recalls another problem related to the acceptance of the failure probability calculated by the frequentist formulation (Eq. 7), when the number of MCM simulations N is not large enough. This is quite common due to the enormous computational efforts required by the MCM to evaluate many engineering formulations. In

this case a sample of N results is obtained, but the results are not stabilised and are not enough to evaluate the failure probability. For this reason, a good alternative to evaluate the failure probability is to assume that the best-fit probabilistic distribution function derived from the sample histogram is the best estimator for the real one, and calculate the failure probability using this distribution. The best-fit technique can be done using the statistics calculated by Eqs. 5 and 6 from the sample of MCM results and applying them to different probabilistic distribution functions, choosing the one that best-fits the histogram of MCM results (best-fit by moments). Alternatively, the best-fit of the MCM results can be done by minimising the square of the errors between the histogram and different probabilistic functions, selecting the best-fit probabilistic function as the one with the minimum square of errors (maximum likelihood estimator), and, then, calculating its statistics, as described by Van Gelder (2000). This best-fit technique is the most used in commercial software. Other question commonly raised is that if the best-fit technique should focus on the overall histogram or on the tail of the histogram where the failure probability is calculated, as illustrated in Fig. 8. There are no simple answers to these questions, but a best-fit technique of the overall histogram is preferred over the best-fit of the histogram tail, simply because it fits the whole variability phenomenon and not a part of it.

In summary, the MCM is demanding, requiring the complete definition of the probabilistic functions of input variables and may need computational efforts to reach acceptable results, in function of the complexity of the engineering formulation to be solved. In doing so, the outcome results are also complete (statistics and definition of the probabilistic function of the dependent variable) and can be considered final and exact. However, in many cases, the necessary computational efforts may be excessive or even not acceptable for a realistic schedule of engineering tasks.

The alternative to overcome this problem is to use approximate probabilistic methods in place of the MCM, being the most popular in geotechnical engineering the FOSM and PEM.

The FOSM is a method that considers the first-order approximation of the Taylor Series expansion applied to the equation of the second statistical moment (variance). It requires only the mean and standard deviation values of input variables, but also returns only the mean and standard deviation values of the dependent variable (performance indicator). Considering some assumptions, such as the probabilistic distribution functions of the input variables are symmetric and these variables are independent among themselves, the FOSM equations for the performance-indicator mean and standard deviation values can be expressed as:

$$E(y) = \bar{y} = f(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n) \quad (9)$$

$$V(y) = \sum_{i=1}^n \left(\frac{\partial y}{\partial x_i} \right)^2 V(x_i) \quad (10)$$

where $E(y)$ is the mean value of the dependent variable y (performance indicator); f is the engineering formulation to calculate the dependent variable y as a function of the independent input variable x_i ; \bar{x}_i are the mean values of the independent input variables; $V(y)$ is the variance value of the dependent variable y ; $V(x_i)$ are the variance values of the independent input variables x_i ; and $(\partial y / \partial x_i)$ are the partial derivatives of the dependent variable y in relation to each independent input variable x_i .

The mean value of the dependent variable y is given by Eq. 9, inputting the mean values of the independent variables into the engineering formulation defined to calculate the performance indicator. In other words, the method assumes that the best estimator of the mean value of y is given

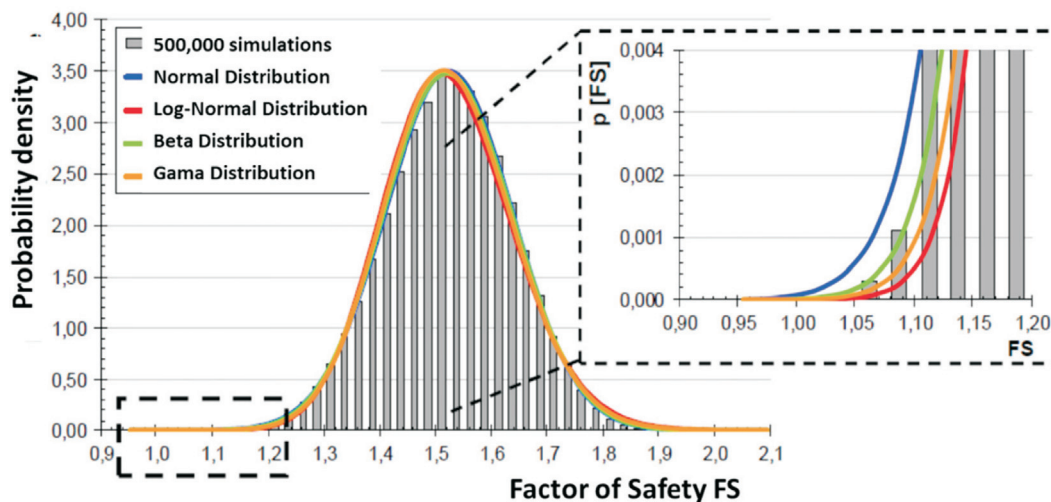


Figure 8. Possible alternatives of the best-fit technique of the probabilistic distribution function, focusing on the overall histogram or the histogram tail.

by the function f evaluated using the mean values of the input variables \bar{x}_i . In fact, this is a similar procedure when the deterministic approach is adopted, calculating the performance indicator using the mean values of input parameters and loadings. The FOSM major difficulty is to obtain the partial derivatives ($\partial y / \partial x_i$) of the engineering formulation used to calculate y as a function of each independent input variable (Eq. 10). In many cases, these partial derivatives may be not easily determined or are even not possibly defined. The problem was solved replacing the partial derivatives by a numerical approximation (Christian *et al.*, 1992, 1994; Christian, 1999, 2004; Baecher & Christian, 2003). The partial derivatives intend to evaluate how the engineering function of y is affected by each input parameter; in other words, they evaluate the mathematical weight of each input parameter in the engineering formulation used to calculate the performance-indicator variance. To do so, the numerical approximation proposes to increase by a small increment the value of each input parameter, independently, keeping the other input parameters at their mean values, and calculate a new value of the y_i . The numerical approximation of a certain partial derivative is given by:

$$\frac{\partial y}{\partial x_i} = \frac{y_i - \bar{y}}{\Delta x_i} = \frac{\Delta y_i}{\Delta x_i} \quad (11)$$

where y_i is the new value of the y -function calculated with the incremental value of a certain input parameter ($x_i = \bar{x}_i + \Delta x_i$); \bar{y} is the mean value of the dependent variable, given by Eq. 9; and Δx_i is a small increment added to the mean value of each input parameter.

In the literature, this small increment given for each input variable is usually reported as 10 % of its mean value. In fact, the exact value of this increment is not relevant because it is only used to calculate the new value of the y -function, and, then, to estimate the value of the partial derivative around the mean value of dependent variable \bar{y} , taking the increment of the y -value and dividing it by the increment of the input variable x_i , yielding the dependence of the dependent variable y per unit of that particular input parameter. Farias & Assis (1998) analysed the effect of the increment size and concluded that the value of 10 % is appropriate, but it could be any other value, except extremely small values, which could induce numerical errors in evaluating Eq. 11, or very large values, which could erroneously evaluate the derivative when its shape departs to far from a linear dependency between the y -function and that particular input variable. In this case, the evaluation of the partial derivatives by numerical approximation using two points around the mean value of the dependent variable \bar{y} is highly recommended, and given by:

$$\frac{\partial y}{\partial x_i} = \frac{y_i^+ - y_i^-}{\Delta x_i} = \frac{\Delta y_i}{\Delta x_i} \quad (12)$$

where y_i^+ is the new value of the y -function evaluated with an incremental value of a certain input parameter ($x_i = \bar{x}_i + \Delta x_i / 2$); y_i^- is the new value of the y -function evaluated with a decremental value of a certain input parameter ($x_i = \bar{x}_i - \Delta x_i / 2$).

Figure 9 illustrates how the two-point evaluation of the numerical approximation of the partial derivatives fits much better for any shape of the derivative function of the dependent variable y in relation to a particular input variable x_i . Duncan (2000) suggested the two-point numerical approximation of the partial derivatives, using the increment and decrement of each input variable equal to one standard deviation. The two-point numerical approximation technique requires a larger computational effort, since the single-point numerical calculation implies $N = n + 1$ calculations of the y -function, where n is the number of independent input variables, and the two-point numerical approximation needs $N = 2n + 1$.

At the final evaluation of Eq. 10, the variance of the dependent variable $V(y)$ (performance indicator) is calculated by the summation of the product between the partial derivatives and variances of input parameters, which means that the y -variance is given by the sum of the mathematical weight (partial derivative) multiplied by the statistical weight of each input variable (its individual variance). This type of calculation allows to estimate the individual weight of each input-variable variance to the total y -variance, simply dividing them, usually reported in percentage (%), as depicted in Fig. 10. This result helps understand the effect of each input-variable variance in the total variance, allowing to focus on those input variables that play a more important role in the process. The final outcome of the FOSM is only the mean and standard deviation values of the dependent variable, but it also only requires these statistics of input variables. The computational effort is very low, requiring only $N = n + 1$ calculations for the one-point numerical approximation of the partial derivatives or $N = 2n + 1$ for the two-point alternative numerical approximation. The main disadvantage is that the method does not return any information on the type of the probabilistic function of the dependent variable, which has to be assumed, and this is required to calculate the failure probability for that performance indicator.

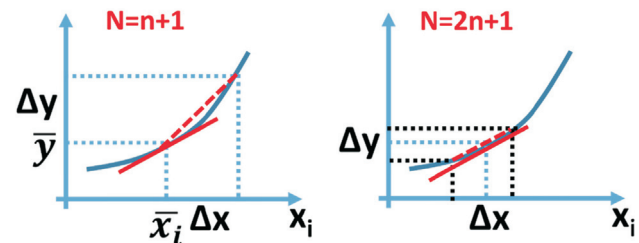


Figure 9. Numerical approximation schemes (one-point and two-points), illustrating the better fitting of the two-point technique for derivative functions departing from linear dependency.

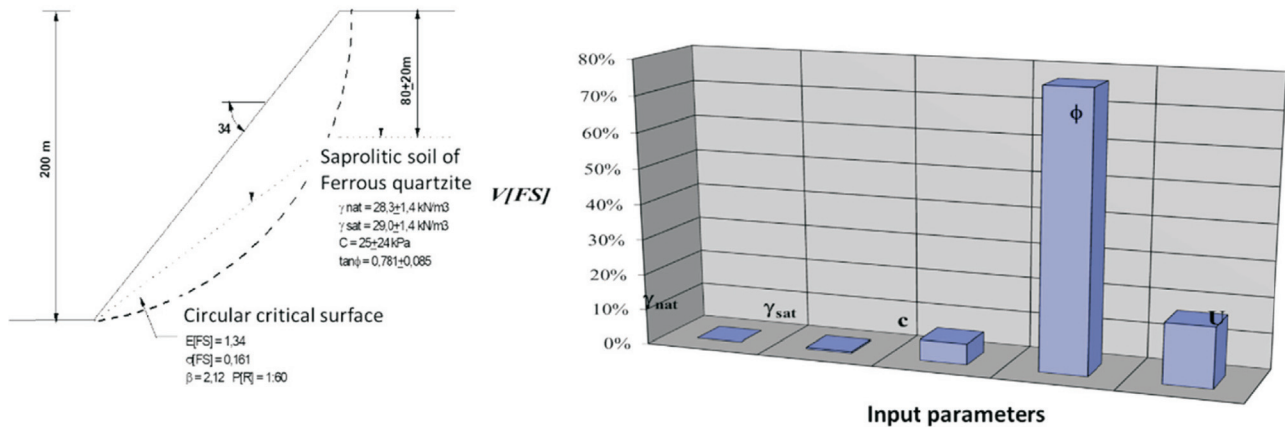


Figure 10. Influence of the variances of each input variable (unit weight, cohesion, friction angle and pore pressure) on the final variance of the dependent variable (factor of safety) (modified from Sandroni & Sayão, 1993).

The other alternative probabilistic method is the Point Estimate Method (PEM), derived by Rosenblueth (1975; 1981). The PEM is based on the Gaussian quadrature (Christian & Baecher, 1999; Baecher & Christian, 2003) to numerically calculate the moments of the probabilistic distribution function of the dependent variable y , based on all possible combinations of two estimate points of the input variables. For each input variable x_i , two estimate points are defined, being its mean value plus and minus one standard deviation value, such as $x_i^+ = \bar{x}_i + \sigma x_i$ and $x_i^- = \bar{x}_i - \sigma x_i$. So that, the PEM also only requires the mean and standard deviation values of all input variables. Once the two estimate points are defined for all input variables, the engineering function for the dependent variable y is solved N times, considering all possible combinations among the estimate points of all input variables, what gives $N = 2^n$ calculations (where n is the number of input variables). This means that the PEM generates a sample of N results of the dependent variable y . Then, the statistical moments of the dependent variable are calculated using descriptive statistics for discrete values, as given by Eq. 5 for the mean value and Eq. 6 for the standard deviation value, simply replacing N by 2^n . Moments $M3$ (symmetry) e $M4$ (kurtosis) can also be calculated using similar equations. These expressions are for independent input variables and corrections based on the correlation coefficients may be applied when these variables are dependent among themselves. The PEM also only returns the mean and standard deviation values, and other statistical moments, as it only requires similar data of the input variables. As the FOSM, the PEM does not yield any information on the type of the probabilistic function of the dependent variable (performance indicator), which has to be assumed to calculate the failure probability.

Table 2 presents a comparison of these probabilistic methods. The MCM and PEM generate a sample of results of the performance indicator y (dependent variable), allowing each individual calculation to explore the most critical response for that set of input parameters, including possible

changes in the failure mechanism, which means that multiple mechanisms may be considered. The final variance of the dependent variable is calculated from all their individual results in relation to its mean value. In this aspect, the FOSM is more limited, since it calculated a unique failure mechanism for the mean values of input parameters and the final variance is evaluated only in relation to this mean-value failure mechanism of the dependent variable. There are several publications in the literature (for instance, Griffiths & Fenton, 2007), comparing these methods and their applicability in geotechnical engineering. In general, the FOSM results indicate a lower failure probability than the other two methods, exactly because it explores less the scatter of input data and potential failure mechanisms. The HPEM presents the advantages of the FOSM in terms of the low number of required simulations and the influence of each input parameter in the final result, and explores all variability of parameters and mechanisms as done in the PEM, yielding similar reliable results as the PEM (Gitirana, 2005; Franco, 2019; Franco *et al.*, 2019; Yokozawa, 2019).

In summary, probabilistic approach and quantitative evaluation allow obtaining the failure probability value of a certain performance indicator. This can be done by Event and Fault Tree Analyses, when the performance indicator is not well-defined by an engineering formulation (empirical or analytical formula, or numerical solutions), or, on the contrary, by probabilistic methods, when the engineering formulation is well set for that performance indicator. As it can be noticed, the whole probabilistic approach and methods are not perfectly defined and some assumptions might be needed. However, estimations of the variabilities of input parameters yield better and more complete engineering results than the assumptions that input parameters and loadings are taken as constant values and the final result is unique, as commonly done in the deterministic approach. Paraphrasing Warren Buffett and adapting his thought, it is preferable to have an approximate probabilistic result than a precise deterministic one that is certainly wrong. This re-

Table 2. Comparison of the most common used probabilistic methods in geotechnical engineering (Assis *et al.*, 2018, 2019).

Probabilistic method	Advantages	Disadvantages
Monte Carlo (MCM)	Final results may be exact Obtains all statistics and the probabilistic function of the dependent variable	Requires complete probabilistic functions of all input variables May imply considerable computational effort
First Order Second Moment (FOSM)	Very fast computations Requires only the mean and standard deviation values of input variables Obtains the influence of each input variable on the final variance of the dependent variable	Requires the assumption of a probabilistic function for the dependent variable to evaluate its failure probability Final variance is limited to the influence of the variances of input variables around the mean value (does not change the failure mechanism for each set of input parameters)
Point Estimate (PEM)	Computational efforts are reasonable Requires only the mean and standard deviation values of input variables	Requires the assumption of a probabilistic function for the dependent variable to evaluate its failure probability

inforces that probabilistic results in terms of failure probabilities should not be taken as exact numbers, but as an indication of their magnitude, mainly focusing firstly on their order of magnitude (power of 10, for instance 10^{-4} or 10^{-5}) and, then, on their decimal digits, avoiding to report non-significant digits (5×10^{-5} is preferred to 5.15×10^{-5}). Following similar logic, very small calculated probabilities may not have enough accuracy, so that it is recommended to report 10^{-7} or 10^{-8} as the lowest possible failure probability for geotechnical structures (this means that if any lower failure probability than those values is calculated, it is reported at the suggested lowest limit), as suggested by Mitchell (2014).

Once the reliability index or failure probability for a certain performance indicator is determined, the question is how to consider their values within the scope of engineering decision making. A first and easy attempt is to correlate the results of the probabilistic approach to the conventional concepts of safety margins and factors, commonly used in conventional engineering (deterministic approach). This leads to erroneous findings, simply because the variabilities of input data are not considered in this correlation. The only and truly alternative is to integrate the results of the probabilistic approach, in particular the reliability index or failure probability, into the concept of risk, as defined and applied to engineering.

3. Risk metrics and analyses

The risk concept, as defined in engineering, comes from an uncertain event that, if it happens, may lead to a structure behaviour different from that forecasted and expected, generating consequences from this unexpected behaviour, which can be better or worse than those forecasted. Uncertain events that may lead to better consequences are

called opportunities and those causing worse and undesirable consequences are hazards. This risk concept implies two main variables that are taken together, the probability of occurrence of the uncertain event and the potential consequences caused, if the uncertain event occurs (damage, impacts and so on). It is very important to understand and consider this engineering risk concept clearly, to ease the communication among all stakeholders and the society about a certain structure. This reminder is even more important considering that colloquial language usually takes the word *risk* as synonymous of chance, likelihood, probability (for instant, what is the risk to have a storm today?). The engineering concept of risk has to take, both, the probability of occurrence and potential consequences. This concept is well accepted implicitly in our mind. For instance, when deciding to take a plane, the consequences of a crash is disastrous (loss of all lives on board), but it is usually accepted because the probability of a crash (failure) is very small (taken as 10^{-6} to 10^{-7}). The same should apply to all engineering structure. There is no engineering structure with risk equal to zero, there is always a failure probability to all structures. In the same manner as the decision making to take the plane, the failure probability has to be analysed in conjunction with the potential consequences of an engineering structure failure, in case it occurs, and to be accepted by engineers and society if the failure probability and consequences are within certain limits (acceptance and tolerance curves).

A first and easy approach to carry out a risk analysis is by qualitative methods, where the failure probability and consequences are described by adjectives, according to their susceptibility and severities. An example of this qualitative method is the risk index, which results from the multiplication of the probability and consequence factors

(Table 3). This is the method currently adopted by the dam safety legislation in Brazil. Several characteristics of the dam are considered using a point summation system to classify its susceptibility to failure in three categories (high, moderate and low). Similarly, the potential consequences sum points considering the presence of people and environment downstream, size of the reservoir and so on, also classifying it in three categories (high, moderate and low). Table 4 presents the risk categories (A, B and C) that consider the susceptibility to failure of the dam and its potential damage (consequences). It is important to note that, in the dam safety legislation in Brazil, the term risk unfortunately is erroneously applied only to the failure probability of the dam, and not to the joint product of failure probability and its consequences, which must be corrected for ensuring precise risk communication among all.

The evolution from qualitative to quantitative analyses of risk requires a risk metrics that could be applied to all type of engineering structures, including the different types of consequences. The expected outcome from the engineering risk concepts yields to:

$$R = p_f \cdot C \quad (13)$$

where R is the engineering risk; p_f is the failure probability of the engineering structure; and C are the potential consequences due to its failure, if it occurs.

Equation 13 can be expanded to include cases where the failure is given by a sequence of independent events, as

defined in Event Tree Analyses, leading to the final failure probability as a result of the product of individual event probabilities towards the structure failure. Also, the consequences due to a failure of complex engineering structures involve different types of impacts, which are usually categorised in spheres of consequences, such as those related to:

- The structure itself - all costs due to the physical loss of the structure (works for cleaning the failure and rebuilding the structure) and the outgoing profit during the period of time that the structure is not operating;
- Health and Safety of People – costs due to medical treatments of injuries and compensations for life losses of workers and outsiders;
- Public and private properties – indemnity costs for partial or total losses of vehicles, housing, commercial, business, educational, industrial and agribusiness facilities, and all types of infrastructure (roads, bridges, water supply, sewage treatment plants, etc.);
- The environment – indemnity and recovery costs of environmental protected areas and parks, woods and forests, rivers and lakes, and so on.
- Reputational Damages – this is usually related to the losses of the company value, for instance in the stock markets, and future legal difficulties and constraints, such as obtaining of permits to engage new projects, or to enlarge, update or keep operation of on-going facilities.

Table 3. Example of Risk Index to qualitatively carry out risk analyses.

Risk Index Probability and Consequence Matrix	Consequence Factor			
	Insignificant	Low	Moderate	High
Probability Factor	1	2	3	4
Insignificant	1	2	3	4
Low	2	4	6	8
Moderate	3	6	9	12
High	4	8	12	16
4				

Table 4. Example of qualitative risk analysis for evaluating dam safety in Brazil.

Risk Category	Potential associated damage (consequences)		
Failure probability	High	Moderate	Low
High	A	B	C
Moderate	A	C	D
Low	A	C	E

All these consequences may happen and have to be summed, considering that each type of consequence has its own vulnerability (ability of a particular consequence to occur due to the structure failure). One point that commonly arises is how to sum different types of consequences. The best and most efficient alternative is the monetisation of the consequences and sum their values. For this, it is necessary to establish methodologies on how to monetise each type of consequence, which is not a complicated issue, except when dealing with life losses of people. Despite all aspects related to social, culture, religion, economic income, age and so on, it is highly recommended to not define a value for people lives, but simply take the value paid for compensation of life loss, with no difference among people, no matter how different they might be. In other words, each life loss will be compensated by the same amount, despite age, gender, education, social or professional position, and so on. As people life losses bring an enormous impact in the society, it is recommended to assume the highest compensation value possible, in order to make people lives the one most important or one of the most important consequences in the risk calculation. In doing so, the engineering risk expression becomes:

$$R(\$) = p_f \cdot C(\$) = \prod p_{fi} \cdot \sum p_j \cdot C_j(\$) \quad (14)$$

where $R(\$)$ is the monetised value of the engineering risk; p_{fi} are the individual failure probabilities of serial events leading to the final failure of the engineering structure, which are multiplied; p_j are the vulnerabilities (value from zero to 1) for each type of consequence (when equal to 1 means that that consequence will happen, if the failure occurs); $C_j(\$)$ are the monetised value for each type of consequence.

As examples of consequence monetisation, Oboni & Oboni (2020) reported that the Fundão Dam failure, occurred in Mariana, Brazil, in 2015, may cost over US\$ 40 billion to the owners (BHP and Vale mining companies) to cover all spheres of consequences and the oil spill that happened in the Gulf of Mexico, in 2010, where 5 million barrels leaked into the ocean, has costed to BP company around US\$ 65 billion in controlling, cleaning and recovery measures, and penalties.

The major advantage of risk monetisation is that the risk value of a certain engineering structure can be added to its own construction cost or value, leading to the concept of overall cost, given by:

$$OC(\$) = CC(\$) + R(\$) \quad (15)$$

where $OC(\$)$ is the overall cost of a certain engineering structure or alternative; $CC(\$)$ is the construction cost or value of the structure or alternative; and $R(\$)$ is the monetised risk value for that particular structure or alternative.

The overall cost is a very powerful concept, because by integrating the risk for each engineering alternative, it allows a better analysis of all engineering alternatives. This

certainly avoids the common mistake in simply selecting the lowest price offer, which probably presents higher risks (quality, maintenance, durability, contractor bankruptcy and so on). On the other hand, a more expensive offer for the structure could indicate better engineering data, design and construction, leading to lower risks. And an easy solution to solve this dispute is to analyse the overall cost (Eq. 15) and take it as a decision-making tool, as done in the bidding for the contractors of a subway line in Copenhagen, Denmark, as shown in Fig. 11.

4. Risk management applied to geotechnical structures

As risks can be qualified or quantified, the following task is to implement a risk management system. Figure 12 presents a scheme of sequence tasks for risk management (ABNT, 2018) that goes from the identification of the risk event (uncertain event that may cause risk), to risk calculation and evaluation, to the measures for risk elimination or mitigation, if necessary. This is a cyclic scheme, indicating that risks have to be re-evaluated from time to time, and when any changes in circumstances occur.

The first task of the risk management scheme (Fig. 12) is the Identification of Risk Events. This can be done initially by all involved professionals (owners, designer, contractors, operational teams) or with the help of a board of expert and experienced engineers, using provocative methodologies, such as SWOT (Strengths, Weaknesses, Opportunities and Treats), Delphi or Brainstorming. Examples of risk events are slope instability, piping, overtopping and liquefaction for an earth or tailings dam. Each type of geotechnical structure presents characteristic risk events. When risk events have been identified and listed, it is recommended that they are organised in order from the highest to the lowest potential risks. Then, each risk event identified has to be qualified in terms of causes, likelihood of occurrence, potential impacts and possible solutions, and then it has to be recorded in the Risk Register. The most important annotation for each risk in the Risk Register is the nomination of its technical responsible and its owner. The Risk Responsible is usually a technical and competent pro-

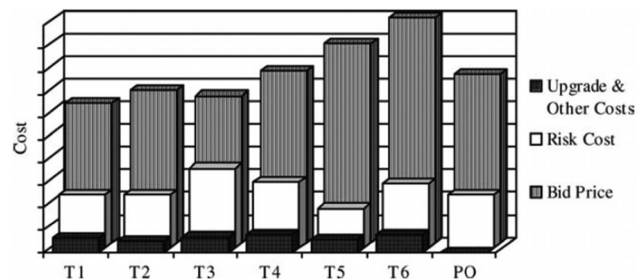


Figure 11. Concept of overall cost for deciding the bidding of a subway line (Eskesen *et al.*, 2004).

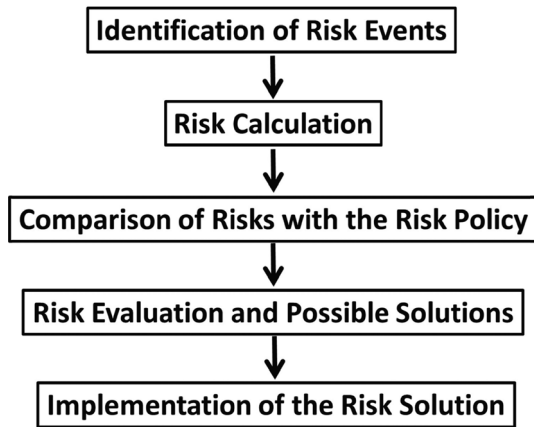


Figure 12. Risk management scheme.

professional in charge of taking care and following the risk for all its existence, contracting and implementing risk solutions, and monitoring them. The Risk Responsible has to do or follow the risk calculation and evaluation as described in Fig. 12 and report the risk status to the Risk Owner. The Risk Owner is a professional who has the power in the company hierarchy to authorise budget for implementing the necessary risk solutions as demanded by the Risk Responsible. The Risk Owner is the ultimate professional in charge of the risk management. Despite these nominations being essential for an efficient risk management in any company, unfortunately, in many cases, the whole process failed because some people may not feel comfortable with this transparency required for the risk management process. The most common mistake is the attempt of higher hierarchy officials to impose the risk ownership to technical professionals that do not have power to decide on budget issues.

The next step of the risk management process is the Risk Calculation, which has already been discussed in this paper, in terms of, both, the quantification of the failure probability by event and fault tree analyses or probabilistic methods, and the monetisation of all different types of consequences. Once the risk components are calculated, they are usually plotted in the Risk Diagram, also called Farmer's diagram, which is a bi-log graphic, with the value of the consequences in the x -axis and the failure probability in the y -axis (Fig. 13). As engineering risk is defined by Eq. 13 or 14, in a bi-log graphic, the product between failure probability and consequences becomes a sum of the logs of these variables; then, any diagonal line represents a certain risk value. For instance, taken the same diagonal with a certain risk value, this risk value can be achieved by a higher failure probability and lower consequence value, or vice versa. One can also note that risk mitigations, moving from a higher risk value (upper diagonal) to a lower one, can be done using active engineering solutions, which decrease the failure probability of the structure (vertical arrow), or by passive solutions, which decrease the consequence va-

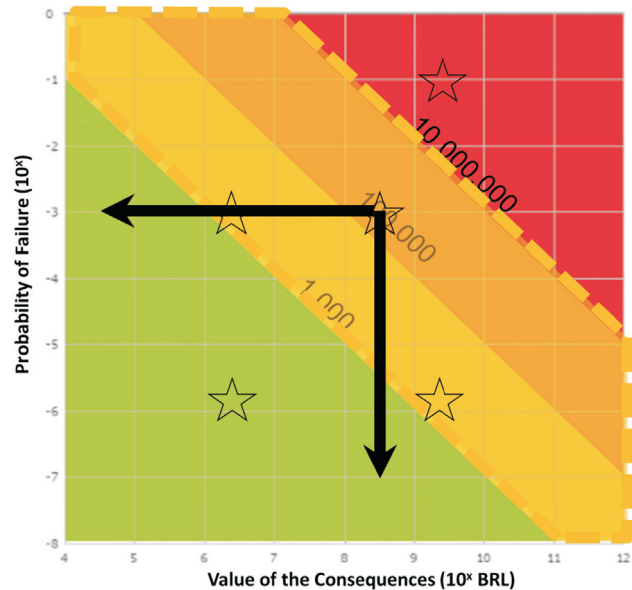


Figure 13. Risk (Farmer's) diagram used to plot risk values.

lue (horizontal arrow). For instance, considering a slope stability problem with high risk, the solution could be any stabilisation measure (active solution), such as drainage or anchors, that increases the safety factor and, consequently, reduces the failure probability of that slope, or the installation of barriers that does not affect the safety of the slope, but minimises the consequences in case the slope fails.

When plotting risk values in the Risk Diagram, one immediate question is raised, which is the level of acceptable risks. This is defined by the Risk Policy that establishes the acceptance zone (usually painted in green colour), the intolerable zone (red colour) and the attention zone (yellow colour), also referred as ALARP, which stands for *as low as reasonably practical*. Originally this denomination was established in the United Kingdom for risks whose engineering complexity, time and cost for reducing them were not worth or unreasonably high. All efforts should be done to reduce these risks to a level as low as possible, considering reasonable engineering solutions, time and costs. If they still remain at a level considered high, but no more engineering solutions are reasonably practical, these risks have to be closely monitored and potentially affected consequences, especially people, trained to follow safe protocols if warned previously to any major problem. In practice, the zone between the acceptance diagonal line (upper limit of the accepted or insignificant risks) and the tolerance diagonal line (lower limit of the intolerable risks) is preferably named as attention zone, despite the concepts of ALARP being still valid.

There is no consensus for the limits of these zones, but considerable advances have been achieved in the last decades and years, mainly led by the Anglo-Saxon countries (UK, Australia, USA, The Netherlands and so on). The first application of the Farmer's diagram showing limits of

tolerable risks for geotechnical structures was presented by Whitman (1984). Since then, acceptance and tolerance curves have been proposed, and more recently they became stricter, more severe, indicating lower risk acceptance by the society. Presently, most risk criteria limit the acceptance zone of one potential life loss (consequence) to a probability of 10^{-4} to 10^{-5} (FEMA, 2015; Morgenstern, 2018), as shown in Fig. 13, assuming a life loss compensation of R\$ 10 million (Brazilian Real - BRL). Some Risk Diagrams show explicitly an additional x -axis with the number of potential life losses, due to its importance and concern to society, so that one can see the total consequence value, but, separately, also the number of potential life losses. The tolerance curve is usually assumed one or two orders of magnitude above the acceptance diagonal line, depending on the Risk Policy of the company, guidelines or standards (of professional societies) or legislation. In Brazil, there are no guidelines, standards or legislation prescribing acceptance and tolerance limits, which recalls the necessary and important role to be taken by professional societies and regulatory agencies. More recently, two complementary concepts have been applied to the definition of acceptance and intolerance zones, which is a truncation of the maximum failure probability accepted for each type of engineering structures and a truncation of the maximum consequence value accepted by the company owning the engineering structures. The first truncation is a horizontal line in the Risk Diagram that limits the maximum failure probability. In practice, the diagonal line that defines the acceptable risk limit cannot continue its trend and failure probabilities higher than that value indicated by the truncation line for maximum probability are not acceptable. The second truncation is a vertical line in the Risk Diagram that limits the maximum consequence value accepted by the company, otherwise, in case that failure occurs, the company could not deal with that loss amount, indicating higher chances of bankruptcy. This truncation for consequence value is usually calculated based on the company annual profit or as a percentage of its total value. Oboni & Oboni (2020) present a very complete discussion on all types of acceptance and tolerance risk criteria. Figure 14 shows an example of risk zones with complementary truncation lines.

In addition to the acceptance and tolerance limits, the Risk Policy, which is defined by the highest hierarchy of the company, has to establish how the risk information is communicated among the company hierarchy, according to risk levels, and to all stakeholders and authorities. It has to be revised periodically, taking into consideration the past experience of risk management and new demands from the society.

Having the risk policy defined and the structural risks calculated, the Risk Responsible and Owner have all elements for risk evaluation and making decisions of the most suitable treatment solutions for the risks. However, in com-

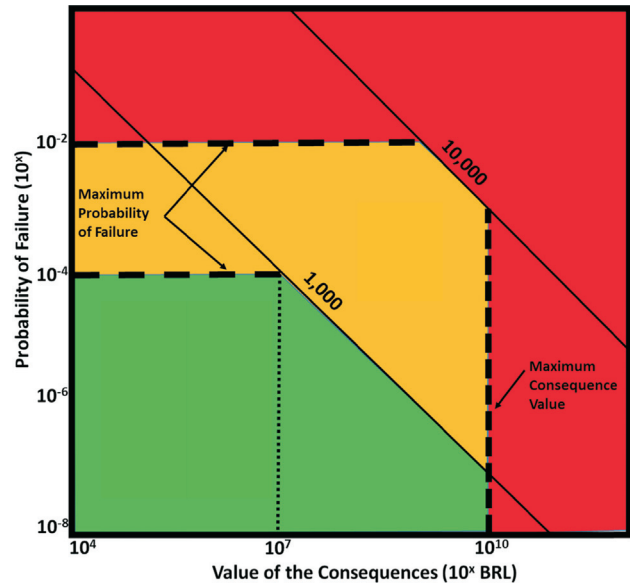


Figure 14. Risk Diagram showing an example of acceptance and intolerance zones using complementary truncations for maximum failure probability and consequence value.

plex structures or large enterprises with several structures, which demand a large number of people involved in the risk management, there are chances of lack or miscommunications among professionals, stakeholders and the society that may jeopardise the whole process. In this case, it is highly recommended an intelligent risk management process, as depicted in Fig. 15.

An intelligent risk management system is guided by the Risk Policy defined by the highest hierarchy of the company, or by guidelines or standards prescribed by professional associations or legislation. The Risk Management Office is in charge of executing the Risk Policy inside the company, providing personnel training and methodologies for all processes related to risk management. It should supervise all risk management processes done by company

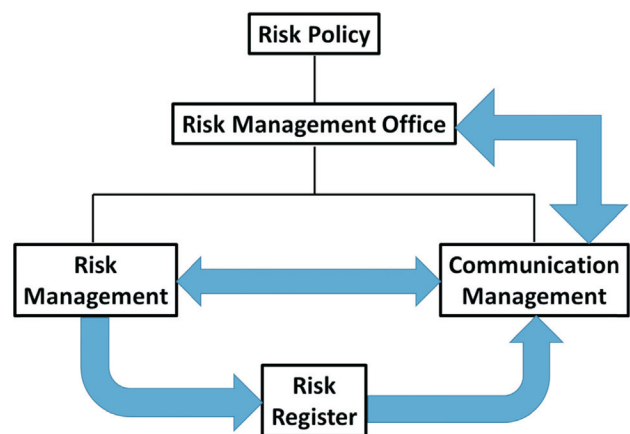


Figure 15 Example of the structure of an intelligent risk management system (Assis *et al.*, 2019).

staff or contracted from outside companies (in this case, it has to specify the terms of reference for these works done by consultant companies). It is important that the Risk Management Office be directly linked to the highest hierarchy of the company, in order to be independent of any intermediate control or inappropriate censorship. The Risk Management involves all steps of risk identification, calculation and evaluation, as already described in this paper and illustrated in Fig. 12. Each risk identified and calculated has to be annotated in the Risk Register, including the nomination of the Risk Responsible and Risk Owner. Any change in the risk is written in the Risk Register, which works as an actual register of the risks for all their existence. When the risk is in the Risk Register, the Communication Management reads it and disseminates its information to all professionals involved and company hierarchy officials, according to its level and zone in the Risk Diagram (Figs. 13 and 14). This communication is done automatically by an IT software, which is programmed according to the definitions of the Risk Policy. It is important to note the need for an automatic communication to avoid any personnel interference, provoking lack or miscommunications. In some cases, risk communication should go outside the company, reaching stakeholders and the society (for instance, civil defence and state authorities). More details on an intelligent risk management system can be found in Assis *et al.* (2019).

5. Examples of risk management applied to geotechnical structures

The first example presented in this paper is a tailings dam, where the Event Risk Identification stage recognises four potential failure modes: i) slope instability; ii) piping; iii) overtopping; and iv) liquefaction. All of them, if they happen, could lead to severe damage to the structure and, consequently, to downstream population, facilities and environment. For each possible failure mode, a performance indicator and a failure criterion have to be selected. For instance, the factor of safety (FS) is taken as performance indicator for the slope stability and liquefaction and its critical value indicating failure (failure criterion) is set to FS smaller than 1. For overtopping, the water level on the dam reservoir could be chosen as performance indicator and its critical value could be set as the topographic level of the dam crest (in this case, it is assumed that if overtopping

occurs, the downstream slope is eroded, leading to the whole structure failure). All these three failure modes have engineering formulations to evaluate their performance indicators, so that probabilistic methods are applied to calculate their failure probabilities. For piping, the performance indicator is not so evident and there is no clear engineering formulation to evaluate the whole process, considering the hydraulic gradient, characteristics of the soil to be eroded and to progressively evolve to piping formation, until leading to dam failure. Therefore, event and fault tree analyses are used to estimate the failure probability due to piping, as exemplified by Figure 4 and described in detail by Fell *et al.* (2015) and Caldeira (2018). The failure probabilities for these four possible failure modes are presented in Table 5.

It is worth a word on the FS statistics and its failure probability obtainment. The engineering formulation chosen to evaluate the FS was the Spencer Method and the probabilistic method was the Monte Carlo (MCM was preferred to be potentially exact and with a computational effort for this type of analyses considered acceptable; it takes about 2 days in a standard configuration computer for running the full analysis). For each set of input parameters, taken for consolidated and drained conditions, a critical failure mechanism was searched and its FS calculated. Many commercial software set as default a fixed failure mechanism obtained by the mean values of input parameters and, then, do all dispersion analyses using this failure mechanism, only varying the values of input parameters. This is not appropriate since, in geotechnical engineering, many failure mechanisms are dependent on geotechnical parameters, and may be changing their shape and position inside the ground mass. Therefore, careful setting of the software is mandatory in order to make sure that the MCM is fully exploring all possibilities of parameter variabilities and failure mechanism options. Nowadays, there is a trend, with promising advances, for searching for alternative methods to the MCM, which are faster, require much less computational effort, and provide similar reliability of the results.

Other important aspect to mention is related to the evaluation of the FS and its failure probability due to liquefaction. The condition for liquefaction to occur assumes that actions happened, called liquefaction triggers, which can be static or dynamic, that changed soil conditions from drained to undrained behaviour. Then, the stability analy-

Table 5. Example of risk analysis applied to a tailings dam.

Failure mode	Failure probability (p_f)	Consequences (BRL(\$) $\times 10^6$)	Risk (BRL(\$) $\times 10^6$)
Slope instability	10^{-5}	3,000-4,000	0.03-0.04
Piping	5×10^{-4}	3,000-4,000	1.5-2
Liquefaction	10^{-3}	4,000	4
Overtopping	10^{-4}	3,000-4,000	0.3-0.4

ses are executed using undrained strength parameters for submerged materials that are potentially susceptible to liquefaction. As described, the liquefaction instability may only occur if a series of independent events happens successively: first the trigger event has to happen, followed by the undrained failure of the structure at its peak-undrained strength values, and, finally, the structure failure overcoming its liquefied undrained strength. So, the failure probability due to liquefaction is given by a product of three failure probabilities (the occurrence probability of the trigger event, the failure probability using peak and undrained-strength parameters and the failure probability using the liquefied strength; $p_{f_liquefaction} = p_{trigger} \times p_{f_peak_undrained} \times p_{f_liquefied}$). The evaluation of failure probabilities using peak undrained and liquefied strengths is similar to the procedure used for any slope stability analysis. The main unknown in this calculation is the definition of the trigger event and estimation of its occurrence probability. For dynamic events, the most common trigger is related to earthquakes and, in this case, it is possible to study or measure their magnitudes and frequencies, determining a certain magnitude for a specific time frequency, which is taken as its occurrence probability. For static trigger events, this evaluation is much more complicated or unknown. Commonly, its occurrence probability is estimated based on the frequency of accidents already registered for the same type of geotechnical structure.

For evaluating the consequences due to a dam failure, dam break analyses, which are hydraulic studies, have to be carried out, implying in the following considerations:

- The amount of reservoir mass that will outflow due to the dam breach has to be assumed or estimated; in case of water reservoir, 100 % of the total mass is usually taken, but in case of tailings reservoir, more complex assumptions or studies are necessary, and common values range from 33 to 100 %.
- The hydraulic breach formation in the dam, due to slope instability, piping, liquefaction or overtopping, requires assumptions of its geometry (shape, width and depth) and evolution time; this is important to evaluate how much and how fast the reservoir mass flows.
- As the reservoir mass flows downstream, its propagation is extremely influenced by fluid parameters, which could be water or slurry (mix of water and solids), topography, which requires a precise digital model of the terrain, and roughness characteristics of the terrain surface, which is related to the type of vegetation or soil use, such as green field, pasture for cattle raising, paved surfaces, water bodies, building structures and so on.
- The results of dam break analysis provide information on the likely flooded area, including, for each geographical position, the flood depth and velocity, and the flood arrival time; the product of flood depth and velocity gives an estimation of the energy of the flow mass, called hydrodynamic risk or flood hazard factor, which is related

to potential damage (Fig. 16), and the flood arrival time is extremely helpful for preparing emergency plans, including establishing evacuation routes and training people.

- These results are overlapped with all information related to population, housing, educational, commercial, industrial and agribusiness facilities, environmental protected areas and parks, and so on, using databases available in governmental agencies (for instance, the database of the Brazilian Institute for Geography and Statistics – IBGE, or similar ones).
- To verify potential damage, the hydrodynamic risk (flood hazard) factor for each geographical position is checked against the occupation and use of that area; for each type of occupation and use, there are threshold limits or vulnerability curves of hydrodynamic risk (flood hazard) factors that indicate partial or total loss, or failure, applied to people, vehicles, different types of buildings, and so on (Fig. 16). Flood hazard criteria and vulnerability curves are discussed in detail by AIDR (2012) and Oboni & Oboni (2020).
- The final result is the inventory of all potential losses and damages, which are monetised using the social and economic values registered in the governmental databases; an important point to discuss is the possibility to have or not any warning prior to structure failure, which may affect enormously the number of life losses; this depends on the type of failure mode and on the efficiency of the emergency plans, including training, drills and full transparency of the risk information.

The consequence values of this dam failure example are shown in Table 5. All failure modes, except liquefaction, present two values, the first one, considering a warning at least 4 h before the dam failure, and the second one, assuming warning at the failure moment. For the liquefaction failure mode, as it happens suddenly, the only option is the warning at the failure moment. As one can conclude from Table 5, the highest risk in this example is due to liquefaction and this highest risk value should be the one plotted in the Risk Diagram for that particular structure.

The second example is derived from the feasibility studies of an urban tunnel for a metro system presented by Alarcón-Guerrero (2016). Other researchers have studied risk analysis and management applied to tunnelling as Einstein (1996), Sturk *et al.* (1996), Shahriar *et al.* (2008), Meng *et al.* (2010), Sousa (2010), Mollon *et al.* (2013), Jarek (2016) and Napa-Garcia *et al.* (2017). The performance indicator chosen by Alarcón-Guerrero (2016) was the distortion angle, defined by the difference of settlements estimated for two locations divided by their distance. The failure criteria prescribed, in general, a limit value of 1:300 for partial structural damages and a critical value of 1:100 to total structural damages. However, these limit values could change depending on the type and age of the structures, since the metro line runs through different nei-

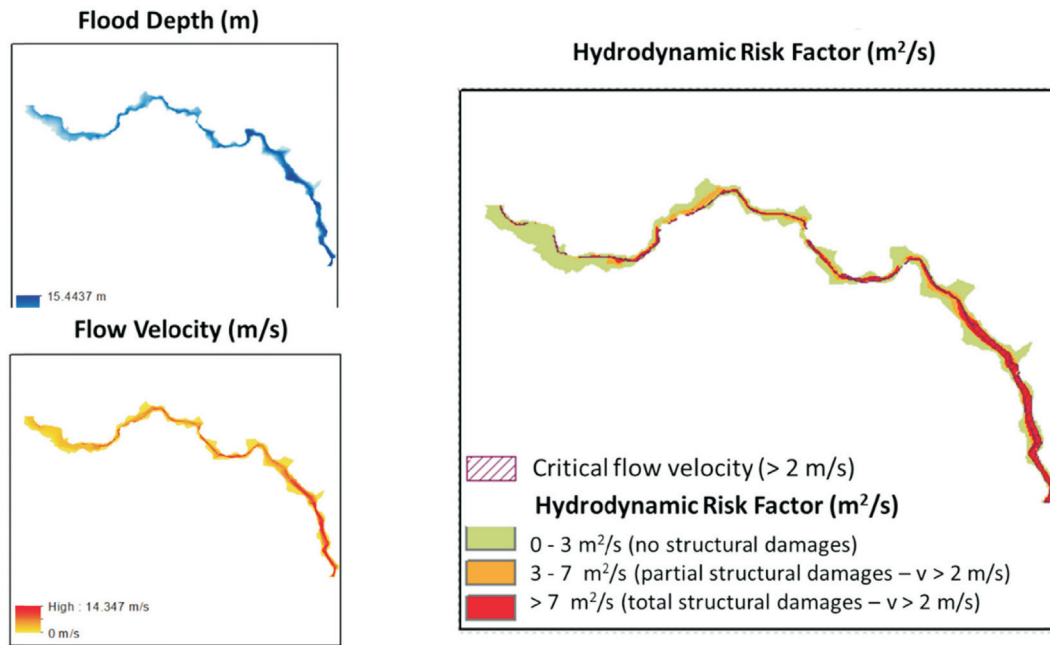


Figure 16. Results from dam break analyses in terms of flood height and flow velocity, resulting in hydrodynamic risk factors and damage criteria.

ghbourhoods, ranging from historical and old buildings to very modern skyscrapers. The engineering formulation was the tunnelling-induced settlement calculation by 3D numerical simulations, using the Finite Element Method. Input variables include deformability and strength parameters for all geologic lithotypes, water table, tunnel geometry and position inside the ground, and tunnelling and support system parameters related to conventional and mechanised methods. As the 3D numerical simulations impose enormous computational efforts, the use of the MCM was not feasible, leading to the adoption of approximate probabilistic methods. Also, the number of input parameters was initially tremendous, which could cause computational problems even to some approximate methods. So, as a first step, the FOSM was applied to identify the most relevant input variables to the performance indicator (settlement and consequently the distortion angle) variance. Then, with the number of input variables limited according to the FOSM findings, the PEM was carried out to calculate the statistics of the performance indicator. Finally, the failure probability was estimated assuming a Gaussian distribution for the dependent variable.

The estimation of the consequence values considered the tunnelling-induced damages to all structures and according to their failure criteria (historical and cultural buildings, conventional housing, low-height buildings and modern skyscrapers), consequences were estimated and monetised. Vulnerability probabilities (ability to suffer damage) were applied to people according to their age and position inside buildings, and to the type of structure and foundation. The final result is a risk-zone map indicating

risk acceptance and intolerance for the chosen tunnelling method, as shown in Fig. 17. Indeed, risk management is a powerful tool for decision making.

6. Closing remarks

This paper intends to bring the theory of risk management to practical applications in geotechnical engineering, consolidating concepts, clarifying procedures and discussing openly its difficulties and trends. Most comments, recommendations and conclusions have been already written along the text, so that only the most relevant ones are listed here.

Probabilistic approach and risk analyses and management bring additional and helpful information, challenging the conventional decision making in engineering, breaking paradigms, but requiring training, culture and setting new acceptance and tolerance criteria.

Risk management in complex structures and in enterprises with different types of structures requires risk quantification and monetisation. In doing so, the concepts of monetised risk and overall costs become a powerful decision tool for evaluating different alternatives of engineering solutions or structures.

Preliminary qualitative risk analyses can play an important role in qualifying risk events and organising them in a priority list, recommending those to be submitted to a more detailed risk evaluation, using quantification and monetisation methodologies.

Monetised risks are efficient tools for decision making, because a common language is used and understood by

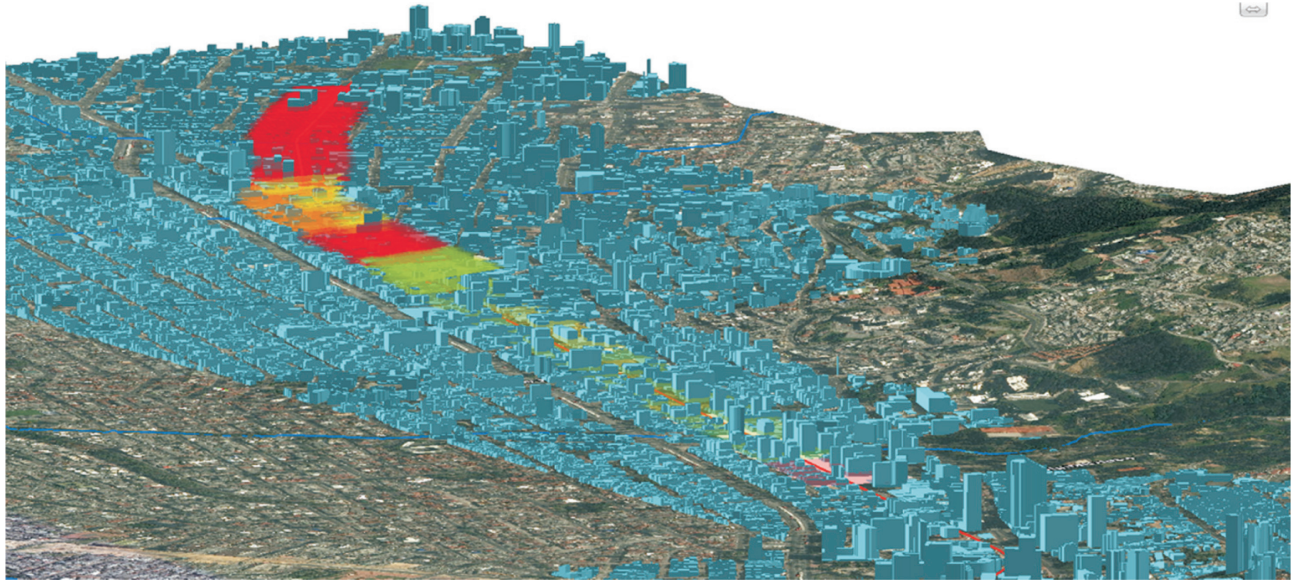


Figure 17. Risk acceptance and intolerance zones for an urban tunnel of a metro system (Alarcón-Guerrero, 2014).

all stakeholders, and may be used as metrics for contingencies, insurances and security deposits.

Monetised risks allow to aggregate all corporate risks (Assis *et al.*, 2019), despite their types and structure differences, leading to a unique risk value for the whole company.

Risks, that are evaluated for a period of time (for instance, annually) and may be recurrent along time, have to be estimated for the life span of that engineering structure, which yields the chance to have a failure during the entire structure lifetime.

Risk management is not a protection shield against all accidents, it does not avoid all failures, but it is an efficient tool that helps control and diminish them, and minimise consequences, if they occur.

Risk management is a tool towards a better engineering and, when done correctly and communicated transparently to all professional and stakeholders, it is essential to discuss new projects, their benefits and risks, leading engineering to regain its paramount role for the needs of modern societies.

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Appendix

Table A1. Suggested values of Coefficients of Variation (CoV) to some geotechnical properties.

Geotechnical properties	Range of CoV (%) and most likely value	References
Unit weight γ	3-7; < 10 5	Harr (1987); Kulhawy (1992); Uzielli <i>et al.</i> (2007)
Moisture content w	8-30 20	Uzielli <i>et al.</i> (2007)
Atterberg limits w_L, w_p	6-30 20	Phoon & Kulhawy (1999a and 1999b); Uzielli <i>et al.</i> (2007)
Void ratio e and Porosity n	7-30 20	Uzielli <i>et al.</i> (2007)
Cohesion c	20-80 40	Baecher & Christian (2003)
Undrained strength S_u	13-40 Triaxial UU \rightarrow 10-30 Triaxial CU \rightarrow 20-55 Triaxial CIU \rightarrow 20-40 25	Harr (1987); Kulhawy (1992); Lacasse & Nadim (1997); Phoon & Kulhawy (1999); Duncan (2000); Uzielli <i>et al.</i> (2007)
Undrained strength ratio S_u/σ_v'	5-15 10	Harr (1987); Kulhawy (1992); Duncan (2000)
Friction angle ϕ	2-13; 5-15 10	Harr (1987); Kulhawy (1992); Baecher & Christian (2003); Uzielli <i>et al.</i> (2007)
Deformability modulus E_0	10-30 20	Baecher & Christian (2003); Mollon <i>et al.</i> (2012)
Coefficient of consolidation c_v	33-68 50	Duncan (2000); Uzielli <i>et al.</i> (2007)
Index of compression C_c	10-37 25	Harr (1987); Kulhawy (1992); Duncan (2000); Uzielli <i>et al.</i> (2007)
Overconsolidation ratio OCR	10-35 20	Harr (1987); Lacasse & Nadim (1997); Duncan (2000); Baecher & Christian (2003); Uzielli <i>et al.</i> (2007)
Coefficient of earth pressure at rest K_0	40-75 50	Phoon & Kulhawy (1999)
Coefficient of permeability K	68-90; 130-240; 200-300 200	Harr (1987); Benson <i>et al.</i> (1999); Duncan (2000); Baecher & Christian (2003); Uzielli <i>et al.</i> (2007)
SPT blowing count N_{SPT}	15-45; 25-50 30	Harr (1987); Kulhawy (1992); Uzielli <i>et al.</i> (2007)
CPT mechanical q_c	15-37	Harr (1987); Kulhawy (1992); Uzielli <i>et al.</i> (2007)

Geotechnical properties	Range of CoV (%) and most likely value	References
	Clay 20-40 30	
	Sand 20-60 40	
CPT q_t	Clay < 20 10	Harr (1987); Kulhawy (1992); Uzielli <i>et al.</i> (2007)
CPT electrical q_c	5-15 10	Harr (1987); Kulhawy (1992); Uzielli <i>et al.</i> (2007)
DMT (resistência de ponta) q_{DMT}	5-15 10	Kulhawy (1992)
Vane Test VST S_v	10-20 Clay 10-40 25	Kulhawy (1992); Uzielli <i>et al.</i> (2007)
Pressuremeter PMT P_L	Clay 10-35 25	Uzielli <i>et al.</i> (2007)
	Sand 20-50 35	
Pressuremeter PMT E_{PMT}	Sand 15-65 40	Uzielli <i>et al.</i> (2007)

Obs. The Most Likely Values of CoV reported in bold are those more frequently assumed in geotechnical engineering design and reports.

Table A2. Types of probabilistic functions commonly suggested to some geotechnical properties (modified from Uzielli *et al.*, 2017).

Geotechnical property	Soil type	Probabilistic distribution function
Water content	All	Normal/Log-normal
Liquidity limit	All	Normal/Log-normal
Plasticity limit	Sand/Silt	Normal/Log-normal
Void ratio	All	Normal
Porosity	All	Normal
Consolidation coefficient c_v	All	Normal/Log-normal
CPT strength	Sand	Log-normal
Undrained strength	Clay	Normal/Log-normal
Ratio between undrained strength and effective principal stress	Clay	Normal/Log-normal
Cohesion	Clay	Normal/Log-normal
Unit weight	All	Normal
Friction angle	Sand	Normal