Determination of Depth Factors for the Bearing Capacity of Shallow Foundations in Sand

Armando Nunes Antão, Mário Vicente da Silva, Nuno M. da Costa Guerra

Abstract. The bearing capacity of shallow foundations is a traditional problem in geotechnical engineering. Many authors have contributed to the solution of this problem using an equation valid under ideal conditions, such as strip foundation under vertical and centred loading and assuming the superposition of the separate effects of self-weight and surcharge. Successive corrections are made to this equation using factors which take into account conditions different from the ideal ones. Among these corrections are the depth factors, which consider the resistance of the soil above the foundation level. In this work, the shallow foundation is considered in sand, and its strength is modelled by an associated Mohr-Coulomb criterion. Approximations to the depth factors are determined using a finite element formulation based on a strict implementation of the upper bound limit analysis theorem, which allows to obtain an optimal failure mechanism and to determine the limit loads. A comparison with previously published solutions is presented, and values for the depth factors are proposed. Following proposals by other authors, depth factors which take into account the superposition of effects of the bearing capacity equation are presented.

Keywords: bearing capacity, depth factors, upper bound limit analysis, sand.

1. Introduction

The bearing capacity of a strip footing in sand deposit acted upon by a vertical centred load can be expressed by (Terzaghi, 1943):

$$q_u = 0.5\gamma B N_\gamma + q N_q \tag{1}$$

where γ is the soil unit weight below the footing base level, B is the footing width, q is the surcharge at the footing base level and N_γ and N_q are bearing capacity factors which depend on the soil friction angle ϕ'. If the soil above 47-52ting base has the same unit weight and the footing is embedded to a depth D, the surcharge q is equal to γD.

Equation (1) is an approximation:

• it assumes that the bearing capacity in the described conditions is the sum of the bearing capacity in two idealised situations: the first one ($0.5\gamma\, B\, N_\gamma$) assumes that the surcharge q is null; the second one considers the soil unit weight below the footing base level equal to zero; the superposition of both effects is not theoretically correct, but this is a traditional solution;

• it does not consider the resistance of the soil above the footing base level, which means that this soil is considered in the calculations by its weight only ($q = \gamma D$).

An exact value of the bearing capacity factor N_q is known (Brinch Hansen, 1970), assuming an associated flow rule:

$$N_q = \tan^2\left(\frac{\pi}{4} + \frac{\phi'}{2}\right) e^{\pi \tan\phi'} \tag{2}$$

but there is not a known exact solution for N_γ. Recently, some excellent approximations have been found (Hjiaj *et al.*, 2005; Martin, 2005). The values obtained by Hjiaj *et al.* (2005) can be approximately determined by the following equation, proposed by them:

$$N_\gamma = e^{\frac{\pi + 3\pi^2 \tan\phi'}{6}} (\tan\phi')^{\frac{2}{5}\pi} \tag{3}$$

The second of the assumptions presented above can be addressed by using depth factors d_γ and d_q:

$$q_u = 0.5\gamma B N_\gamma d_\gamma + q N_q d_q \tag{4}$$

These depth factors account for the resistance of the soil above the footing base and several proposals have been made. Amongst these proposals, are (Meyerhof, 1963):

$$d_\gamma = d_q = 1 + 0.1 \tan\left(\frac{\pi}{4} + \frac{\phi'}{2}\right)\frac{D}{B} \tag{5}$$

and from Brinch Hansen (1970) and Vesic (1973):

$$d_\gamma = 1 \tag{6}$$

$$d_q^{\frac{D}{B}\leq 1} = 1 + 2\tan\phi'(1-\sin\phi')^2 \frac{D}{B} \tag{7}$$

$$d_q^{\frac{D}{B}>1} = 1 + 2\tan\phi'(1-\sin\phi')^2 \tan^{-1}\frac{D}{B} \tag{8}$$

In the present work the depth factor d_γ will be considered equal to 1. This was assumed by Brinch Hansen and by Vesic and seems to be the appropriate theoretical value for this factor.

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In fact, if depth factors take into account the resistance of the soil above the footing base, the first part of Eq. (4), being obtained using $q = 0$, should need no depth correction.

The paper will, then, deal with the depth factor d_q and its determination.

2. Using Limit Analysis for Determining Bearing Capacity of Footings

The bearing capacity calculations which will lead to the determination of depth factors d_q that are used in this paper are performed using numerical limit analysis.

A finite mixed element formulation which implements the upper bound theorem of limit analysis was used. External forces are considered in two types: fixed forces and variable ones, which are affected by a collapse load multiplier. Scaling the mechanism by setting the work rate of the variable external forces equal to one, the optimisation algorithm performs the minimisation of the difference between the plastically dissipated work rate and the work rate of the fixed external forces. The calculations of the present paper were made using a parallel implementation of the above mentioned tool (Vicente da Silva & Antão, 2008), which allows the use of very fine meshes and, therefore, good approximations of the collapse loads.

As is traditionally considered in the determination of depth factors, only 2D analysis were performed. The influence of the length of the footing on the bearing capacity expression is usually considered by the use of shape factors s_γ and s_q, which are not covered in this work.

Initial calculations are performed to evaluate the accuracy of the method and of the level of refinement of the finite element mesh. These calculations considered the situation presented in Fig. (1a), using a null unit weight for the soil. The bearing capacity $q_{u,a}^{UB}$ was numerically determined for $q = 1$ [FL^{-2}], $B = 2[L]$ and for ϕ' equal to 25, 30, 35, 40 and 45°. This made it possible to determine the values of the bearing capacity factor N_q using the second part of Eq. (1):

$$N_q = \frac{q_{u,a}^{UB}}{q} \qquad (9)$$

In these calculations, as in all other presented in this work, the footing was considered rigid and the contact of the base of the footing with the soil below was assumed as rough (Fig. 2). In these initial calculations there is no soil above the footing base.

The mechanism represented in Fig. 3(a) for $D/B = 0$ is the one obtained from those calculations for the case $\phi' = 35°$. The obtained results of N_q for all analysed values of ϕ' are presented in Fig. 4 and show a very good agreement with the theoretical values given by Eq. (2).

Calculations for the determination of depth factors d_q assumed the geometry presented in Fig. 1(b). Soil below the footing base was considered weightless ($\gamma_1 = 0$), and soil above this base had a unit weight $\gamma_2 = 20$ [FL^{-3}], therefore corresponding to a surcharge $q = \gamma_2 D$. The ratios D/B were considered in the range [0.1;2]. Contact between the footing and the soil was assumed as in Fig. 2: rough at the base and smooth laterally.

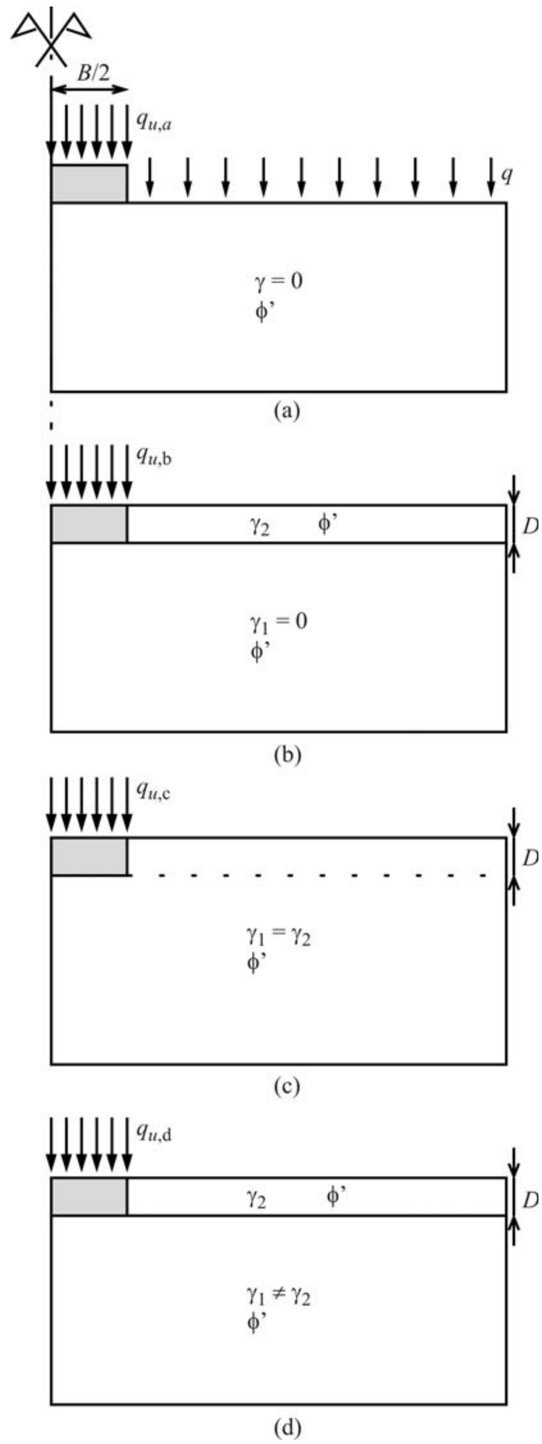


Figure 1 - Geometry considered in the calculations.

In all calculations about 10^6 3-noded triangular linear finite elements were used. The size of the analysed soil was adapted in function of the *D/B* ratio and friction angle in order to adapt the size of the mesh to the size of the plastic zones when failure is obtained.

Later in the paper the situations presented in Figs. 1(c) and (d) will also be considered for comparison with other results.

3. Results

For the cases shown in Fig. 1(b) the bearing capacity $q_{u,b}^{UB}$ was determined in the calculations and the second part of Eq. (4) was used to determine the depth factor d_q:

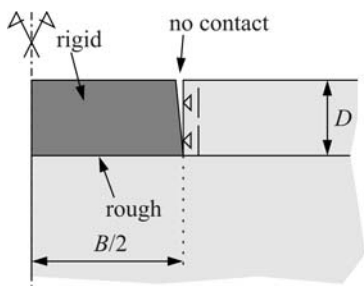


Figure 2 - Details of the footing-soil contact modelling.

$$d_q = \frac{q_{u,b}^{UB}}{qN_q} \tag{10}$$

The theoretical values of the bearing capacity factor N_q given by Eq. (2) were used in this equation. Results of

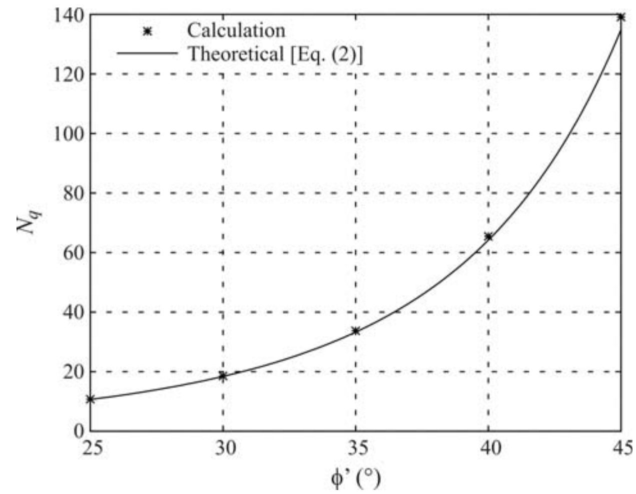


Figure 4 - Comparison between the values of the bearing capacity factor N_q obtained from limit analysis calculations and the theorical ones.

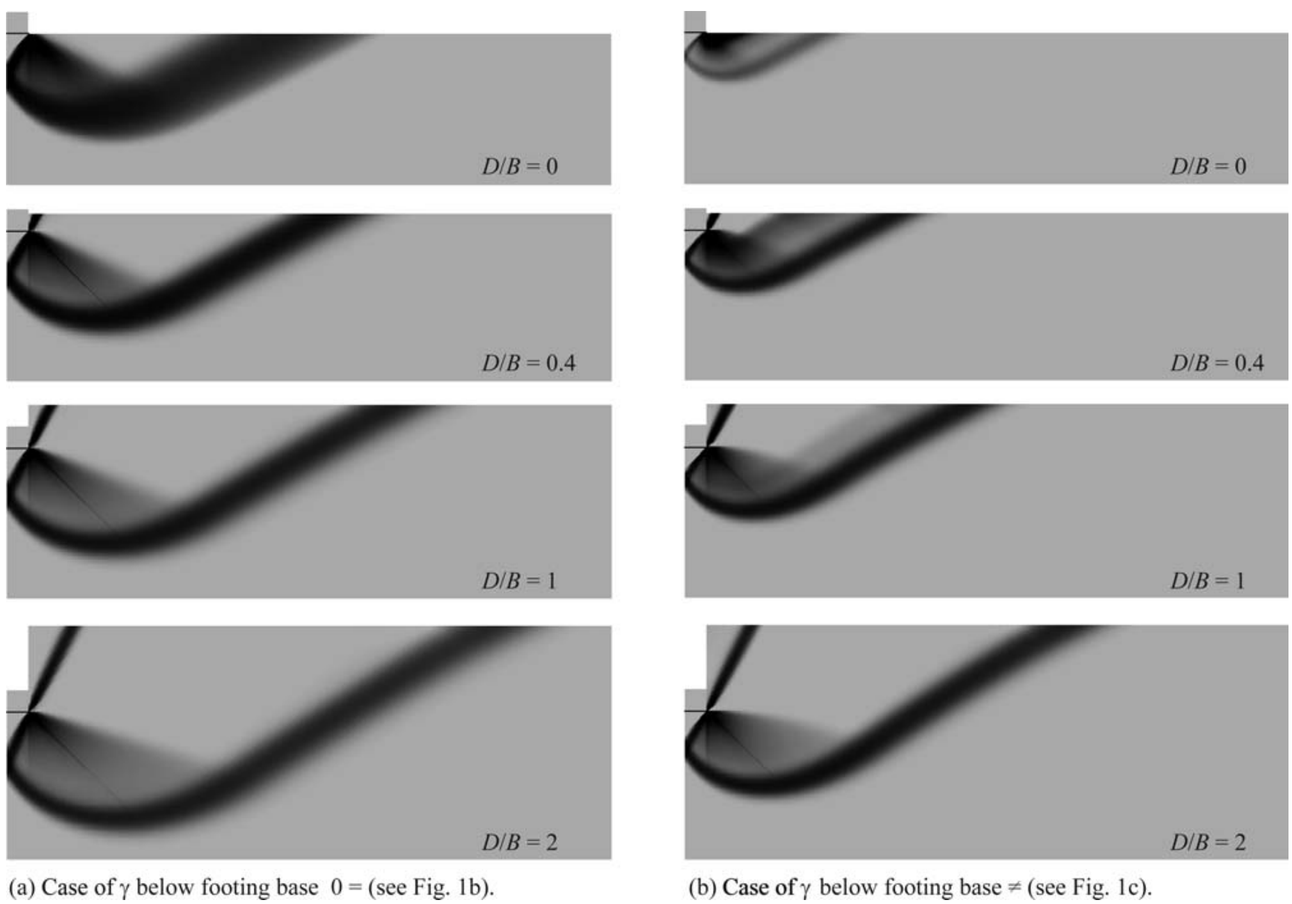


Figure 3 - Failure mechanisms for different *D/B* ratios and for ϕ' = 35° [note: for figure (a) *D/B* = 0 a surcharge *q* was applied on the soil surface].

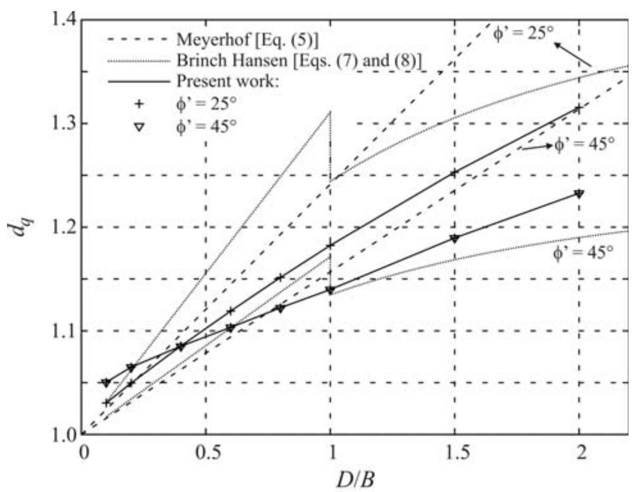
the depth factor d_q are presented in Fig. 5. Figure 5(a) compares the results obtained for two values of the soil friction angle (25° and 45°) with classical solutions and Fig. 5(b) presents values for all friction angles analysed in the present study.

Analysis of Fig. 5 allows the following remarks:

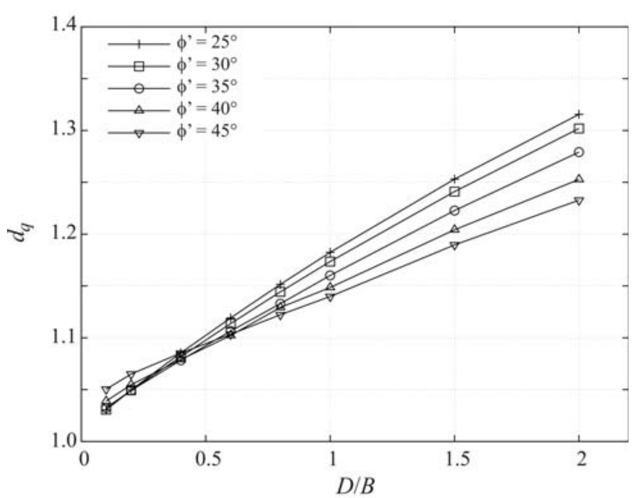
• Meyerhof's values are consistently greater than those obtained by the numerical calculations performed for this work, and therefore they seem to be unsafe, particularly for lower values of the friction angle;

• Brinch Hansen's values are closer to those obtained by the numerical calculations; however, for D/B less than 1 they also give unsafe results; this is also particularly true for the lower values of the friction angle;

• For a given value of D/B, numerical results are less variable with the friction angle of the soil than the ones obtained by either method (Meyerhof or Brinch Hansen);



(a) Comparison with traditional proposals



(b) All values determined in the present study

Figure 5 - Values of the depth factor d_q obtained from calculations.

• The depth factor d_q is greater for greater values of the ratio D/B; it is close to the unity for D/B close to zero and can reach 1.3 for ϕ' = 25° and $D/B = 2$;

• The depth factor d_q is greater for lesser values of the friction angle.

The influence of the D/B ratio on the depth factor d_q can also be observed by analysing the failure mechanisms for a given value of friction angle. This is represented in Fig. 3(a), for the case of ϕ' = 35°. It should be noticed that the graphics in this figure are for representation purposes and were obtained using a simplified finite element mesh (of about 10^5 elements) and the width of the mesh was kept constant.

The analysis of this figure makes it possible to see that there is a clear influence of the D/B ratio on the failure mechanism. This influence is not only and most obviously seen on the soil above the footing plane but also on the soil below: a greater D/B ratio results on a wider failure mechanism (even below the footing plane) but also on a deeper one. It should, however, be noticed that this influence is moderate: in fact, for the case represented (ϕ' = 35°), the mechanism for $D/B = 2$ is only about 15% wider (below the footing base) and 20% deeper than the one for $D/B = 0$. This can probably explain the more or less linear dependence of the depth factor from the D/B ratio.

4. Comments on the Validity of the Superposition of Effects

Equations (1) and (4) are approximations which consider the superposition of the effects given by the first and the second portions of the sum. It is well known that this approximation underestimates the collapse load of the problem represented in Fig. 1(c) (Terzaghi & Peck, 1967). This situation was also considered in a new set of calculations, so that a collapse load $q_{u,c}^{UB}$ could be obtained.

The failure mechanisms for this situation is shown (for ϕ' = 35°) in Fig. 3(b). It can be seen that the failure mechanisms are clearly not the same as those obtained for the determination of d_q. Their width and depth are lower than the ones previously obtained. The influence of D/B is much more clear in the mechanism: for $D/B = 2$ is about 75% wider (below the footing base) and deeper than the one for $D/B = 0$.

Figure 6 shows the results obtained by these calculations divided by the sum of $q_{u,b}^{UB}$ with the first portion of Eq. (4):

$$\chi_c = \frac{q_{u,c}^{UB}}{q_{u,b}^{UB} + 0.5\gamma_1 B N_\gamma} \tag{11}$$

where N_γ was determined using Eq. (3). It can be seen that values of χ_c range between 1.16 and 1.31 for the cases analysed. It should be noticed that χ_c - 1 can be interpreted as a measure of the error of Eq. (4) if the values of d_q determined in this paper and shown in Fig. 5 are used. This means, therefore, that bearing capacity obtained from the numerical cal-

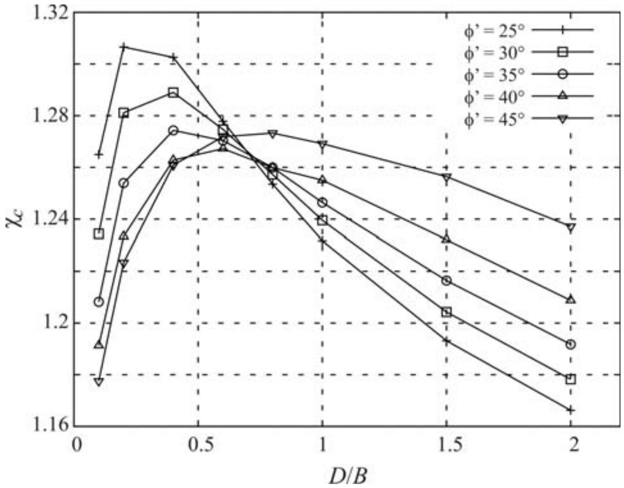


Figure 6 - Ratio χ_c obtained from Eq. (11).

culations are about 15 to 30% greater than the one estimated by Eq. (4). It is interesting to notice that *D*/*B* has a greater influence on this error estimation for the lower values of the friction angle and that it decreases with increasing *D*/*B* from a value of 0.2 to 0.8, depending on the friction angle (Fig.6). An approach where a depth factor (in the present work d_q^* will be used) takes into account the superposition effects (Lyamin *et al.*, 2007) can also be considered. This was achieved by using the following equation:

$$d^*_{q,c} = \frac{q^{UB}_{u,c} - 0.5\gamma_1 BN_\gamma}{qN_q} \quad (12)$$

Figure 7 presents the comparison between the results obtained from this equation using the (upper bound) calculations from the present study and the results obtained from published upper bound solutions (Lyamin *et al.*, 2007). It should be noticed that those authors performed both upper bound and lower bound calculations.

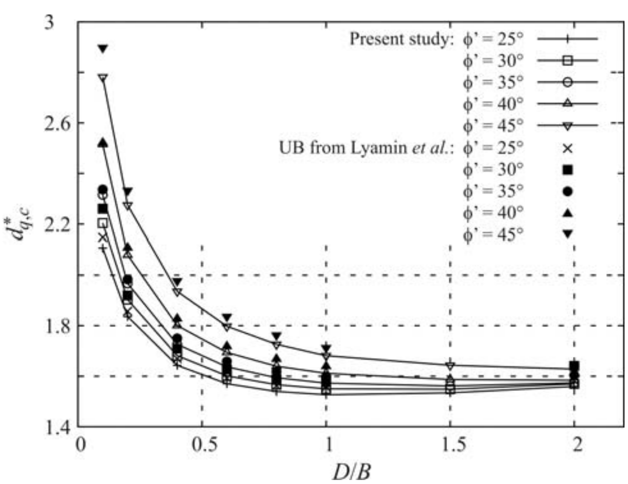


Figure 7 - Comparison between depth factor $d^*_{q,c}$ (Eq. 12) obtained from the calculations of the present work and the upper-bound ones from Lyamin *et al.* (2007).

It can be seen from the analysis of this figure that results are very similar, with a slight improvement in the results from the present work.

Figures 6 and 7 were obtained for the case presented in Fig. 1(c), where $\gamma_1 = \gamma_2$. Results for $\gamma_1 \neq \gamma_2$ will be different and will depend on the ratio between the two unit weights. Soils, however, do not usually have significant differences in the unit weight and, therefore, an idealized model where γ_1 would be much different from γ_2 is not realistic, except for the case where the water level is coincident with the footing base. For this situation calculations can be made using γ_1 equal to the effective unit weight of the submerged soil. The following results assume that γ_1 = 10 [FL^{-3}] and γ_2 = 20 [FL^{-3}] and case (d) of Fig. 1 was considered for the determination of the bearing capacity. New values of χ can, therefore, be computed using the following equation:

$$\chi_d = \frac{q^{UB}_{u,d}}{q^{UB}_{u,b} + 0.5\gamma_1 BN_\gamma} \quad (13)$$

and results are compared in Fig. 8 with the previously obtained ones. It can be seen that values of χ_d are lesser than those of χ_c. This could be expected, as results obtained with a smaller value of γ_1 will naturally be closer to the one obtained using γ_1 = 0, which means that the values of χ are closer to the unity. In fact, bearing capacity obtained from the numerical calculations for this case are about 10 to 30% greater than the estimate given by Eq. (4).

Obtaining new values of the bearing capacity for the case γ_1 = 10[FL^{-3}] also means that new values of d_q^* can be obtained:

$$d^*_{q,d} = \frac{q^{UB}_{u,d} - 0.5\gamma_1 BN_\gamma}{qN_q} \quad (14)$$

Results of this depth factor are presented in Fig. 9, where they can be compared with those previously obtained.

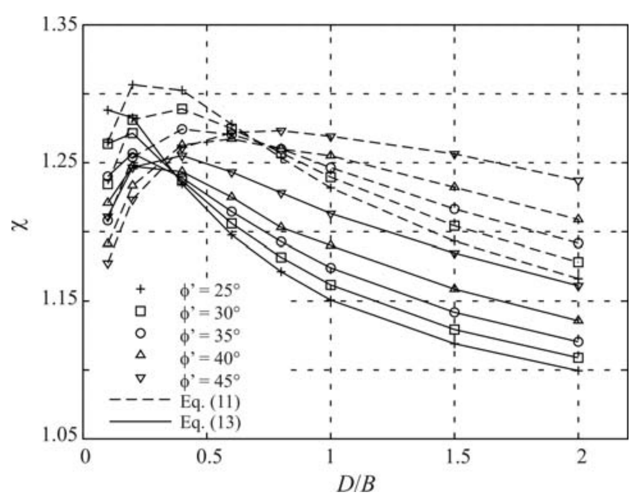


Figure 8 - Comparison between ratios χ obtained from Eq. (11) for γ_1 = 20[FL^{-3}], and Eq. (13), for γ_1 = 10[FL^{-3}].

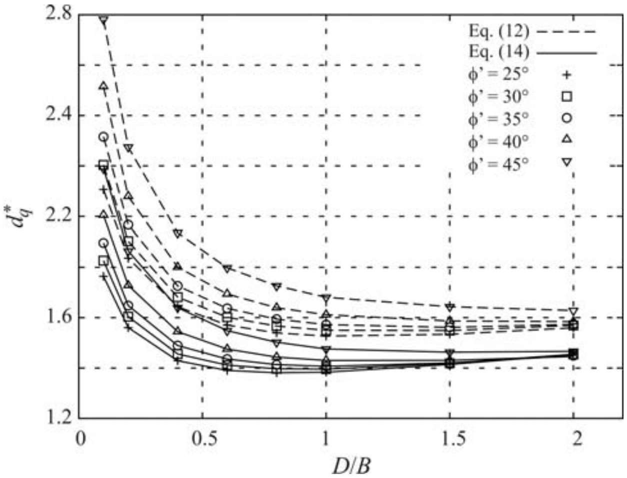


Figure 9 - Comparison between depth factor d_q^* obtained in the present work using Eq. 12, for $\gamma_1 = 20[FL^{-3}]$, and Eq. (14), for $\gamma_1 = 10[FL^{-3}]$.

It can be seen that results now obtained for this factor are significantly lower than the ones previously determined, which shows the influence of the value of the soil unit weight.

5. Conclusions

Resistance of the soil above the footing base can be taken into account by using depth factors, d_γ and d_q which correct, for practical purpuses, the bearing capacity formula for this effect. Commonly used proposals for these factors have been made by Meyerhof (1963) and Brinch Hansen (1970) and Vesic (1973).

Following previous proposals, depth factor d_γ was assumed equal to 1. Using two-dimensional upper bound numerical limit analysis, a proposal for depth factor d_q was presented and compared with the classical ones.

Further analysis made it possible to assess the validity of the superposition of effects classically assumed in bearing capacity formulas. For the analysed situations, bearing capacity is about 10 to 30% greater than the one determined by those formulas.

The same calculation results also allowed to determine values for a different depth factor d_q^* – originally defined by other authors – which correct the underestimation of the classic bearing capacity formula. The results obtained for the case where soil below and above the footing base have the same unit weight are quite similar to the ones obtained using upper bound methods by other authors, slightly improving them.

It could also be established that, for the case of submerged soil below the footing base, lower values should be used and were determined.

Acknowledgments

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References

Brinch Hansen, J. (1970) A revised and extended formula for bearing capacity. Bulletin of The Danish Geotechnical Institute, v. 28 p. 5-11.

Hjiaj, M.; Lyamin, A.V. & Sloan, S.W. (2005) Numerical limit analysis solutions for the bearing capacity factor N_γ. International Journal of Solids and Structures, v. 42:5-6, p. 1681-1704.

Lyamin, A.V.; Salgado, R.; Sloan, S.W. & Prezzi, M. (2007) Two- and three-dimensional bearing capacity of footings in sand. Géotechnique, v. 47:8, p. 647-662.

Martin, C.M. (2005) Exact bearing capacity calculations using the method of characteristics. Proc. 11th Int. Conf. IACMAG, Turin, v. 4, pp. 441-450.

Meyerhof, G.G. (1963) Some recent research on bearing capacity of foundations. Canadian Geotechnical Journal, v. 1:1, p. 16-26.

Terzaghi, K. (1943) Theoretical Soil Mechanics, 2nd ed. John Wiley and Sons, New York, 510 pp.

Terzaghi, K. & Peck, R.B. (1967) Soil Mechanics in Engineering Practice, 2nd ed. John Wiley and Sons, New York, 729 pp.

Vesic, A.S. (1973) Analysis of ultimate loads of shallow foundations. ASCE Journal of Soil Mechanics and Foundation Division, v. 99:1, p. 45-73.

Vicente da Silva, M. & Antão, A.N. (2008) Upper bound limit analysis with a parallel mixed finite element formulation. International Journal of Solids and Structures, v. 45:22-23, p. 5788-5804.

Symbols

B: footing width

d_γ, d_q: depth factors

d_q^*: depth factor taking into account the superposition of effects, as defined by Lyamin *et al.* (2007)

$d_{q,c}^*$, $d_{q,d}^*$: depth factor taking into account the superposition of effects for the cases of Figs. 1(c) and 1(d).

D: depth of the footing base

N_γ, N_q: bearing capacity factors

q: surcharge at the footing base level

q_u: bearing capacity

$q_{u,a}^{UB}$, $q_{u,b}^{UB}$, etc.: upper bound bearing capacity calculation for the case of Figs. 1(a) and 1(b), etc.

s_γ, s_q: shape factors

χ_c, χ_d: ratio between $q_{u,c}^{UB}$, $q_{u,d}^{UB}$ and the bearing capacity determined by the classical bearing capacity equation

ϕ': soil friction angle

γ: soil unit weight

γ_1: soil unit weight below the footing base level

γ_2: soil unit weight above the footing base level