The Use of Fuzzy Sets in the Evaluation of the Effectiveness of Earth Dam Reinforcement

Fernando Saboya Jr., Jair Ferreira de Resende Filho

Abstract. This paper shows the use of fuzzy logic to evaluate the effectiveness of the reinforcement of Santa Branca dam. The embankment reinforcement was found imperative once the dam had shown signs of instability process, as undesirable seepage and unexpected movements, just after the first impoundment. Stability analyses were carried out considering fuzzy variation of the strength parameters in order to assessing the non-random uncertainties associated with their evaluation. These uncertainties are mostly connected to values varying within a specified interval, whose boundaries express the possibility of occurrences that, in turns, account for vagueness in information. Finally, the fuzzy Factor of Safety distribution resulting from these analyses is then compared to pre-defined fuzzy class intervals in order to classify the failure potentiality of the embankment, before and after the reinforcement has been constructed, reflecting, thus, its effectiveness.
Keywords: fuzzy sets, earth dam, stability analysis.

1. Introduction

Generally, earth dams have been designed for multiple propose applications; however power generation and fresh-water storage are amongst the main objective of these structures.

Unfortunately, in developing countries, due to lack of specific laws, a considerable number of these dams have not always been built with the desirable safety and care, putting in risk goods and lives of people who live downstream.

Several factors, associated to the earth embankment alone, can lead a dam to undermine. Unsuitable materials used as fill and filter, poor compaction, use of overestimated parameters for stability analysis, erroneous design of the spillway (overtopping) and animal activities, can be pointed out as the main reasons for failures and incidents of earth dams.

Stability analysis of an embankment dam is normally made using well-recognized methods where input parameters are derived from investigation using deterministic method. Statistical approach is less common nevertheless nowadays its use is gradually becoming popular in geotechnical field.

However, in order to take into account uncertainties that are not originated from random, but yes, from cognitive errors, it has been proposed herein the use of fuzzy sets concept.

This methodology was, thus, used to assess the effectiveness of the embankment reinforcement used to improve the stability of Santa Branca dam, built in 1959 in Sao Paulo, Brazil, which, since its construction, has been showing worrying signs of instability.

After the occurrence of unexpected important leakages followed by undesirable movements, experts concluded that Santa Branca dam had to be reinforced in order to reach a stable condition.

Regarding that the reinforcement comprised itself as an additional flatter embankment placed at downstream part of the dam, in which the main aim was to improve the stability of the dam, the new factor of safety could be evaluated using standard stability analysis procedures. However, for a more comprehensive analysis, it seems to be desirable to take into account the intrinsic uncertainties of the parameters used in such analysis. In addition, if these uncertainties are of random-type, this can be done using the well-known reliability approaches. However, in case in which uncertainties are of cognitive source, the approach is more effective when based on possibility analysis instead of probabilistic one.

Therefore, in order to allow for vagueness in information and consider this feature in a stability analysis, Fuzzy Set theory can be used as a tool to "classify" the results obtained before and after the installation of the reinforcement, clearly pointing out, thus, its effectiveness by means of linguistic terms instead using numbers only.

2. Fuzzy Logic and Sets

Around 350 b.C., a Greek philosopher Aristoteles has established a Logical Science which is a set of rigid rules to logically validate conclusions. Later, this was become known as Occidental Logic that is based on "false" or "true" statement.

However, in many situations, problems cannot be solved considering only binary approach, because of the presence of ambiguities and vagueness forcing, thus, the answer to be somewhere between "true" or "false" statement.

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Submitted on May 26, 2009; Final Acceptance on November 4, 2009; Discussion open until April 29, 2011.

In order to overcome this situation and to take into account the ambiguity raised in several common engineering situations, Zadeh (1965) developed a methodology based on fuzzy sets theory, which considers states between "true" and "false" interval. This is done through the degree of belief that is inferred by so-called membership functions that varies from 0 to 1.

The membership function $\alpha_a(x)$ for a fuzzy set A can be defined as:

$$\alpha_a(x): X \to [0,1] \tag{1}$$

where X is the Universe of Discourse.

Membership function reflects the degree of belief of x in A. If $\alpha(a)$ is equal to zero the object x definitely does not belong to X. On the other hand if $\alpha(a)$ is equal to unity, the object x fully belongs to X. Values between zero and unity imply that x partially belongs to X with a degree equals to $\alpha(a)$ The membership function can be represented by convex shapes, but the most common are of triangular and trapezoidal. The higher is the membership function value, the more possible is the occurrence of it. The width of the interval defines its possible variation, which, in turn, reflects the embedded uncertainty.

This method is very suitable to be used in geotechnical engineering where problems are commonly defined by means linguistic terms to treat vagueness. Therefore, the cognitive uncertainties that are always present in design and evaluation can be assessed. This technique allows the mathematic to mimic the human thinking during the process of decision-making. This is particularly appropriated in geotechnical engineering where linguistic terms are commonly used to define the statement of a soil mass as compressibility and density or to define conditions as stability and strength, among others (Giasi *et al.*, 2003).

However, despite its enormous advantage, Fuzzy Sets Theory is not commonly used in geotechnical stability analysis. This can be attributed to the fact that engineers are trained and used to interpreting numbers and Fuzzy Theory deals with, basically, linguistic terms (Dodagoudar & Venkatachalam, 2000)

This paper has as the main aim to show the potentiality of Fuzzy Sets Theory in geotechnical and dam engineering, provided that it is a different manner of interpreting results using possible interval of occurrences instead using classic statistical probability to infer the impact of the uncertainties on final results.

3. Methodology

Santa Branca Dam, built in State of São Paulo, southeastern of Brazil, had its construction completed in the 1959. It comprises a 55 m high dam with a crest extension of 320 m. The cross section of the dam is composed of compacted clayey soil with a vertical and horizontal traditional sand drain/filter system. The upper third portion of the upstream slope is protected with a thin layer of rockfill and the downstream slope has grass as a protective measure against erosion. Details can be reached in (Dell'avanzi, 1995).

After the first reservoir filling in 1959, unexpected problems were observed, including unwanted infiltration with water emerging from downstream slope surface with worrying signs of subsidence. Table 1 gives some details of all occurrences since the first filling.

Therefore, based on these occurrences it was found that the drainage system was not fully operational, showing that repair was necessary followed by a complete reinforcement of downstream slope.

After the expert surveillance conclusion that the vertical filter was not properly operating, for some unknown reason, remedial actions were lately taken only after 1989, when the situation appeared to have reached a critical stage. Thus, a major reinforcement was built in order to increase the dam safety to an acceptable level. This reinforcement consisted basically of an additional downstream embankment covering all over the existing downstream embankment, with flatter slope. A drainage element between the old and the new embankments was placed to keep ground water surface low and to prevent further leakage and piping, as well.

Concerning that Santa Branca Dam had not an instrumentation program to furnish in-situ data, the assessment of the efficiency of the reinforcement had to be checked using traditional methods as slope stability analysis and other less popular such as statistical analysis.

Table 1 - Occurrences observed during the 20 years of the dam's life.

February, 1962	Two shallow failures take place in downstream slope due to saturation
January, 1965	Another two shallow failures appear in downstream slope, but in a different point from those past ones
December 1966	A new shallow failure in the central portion of the downstream slope, takes place
Mar 1967	Water emerging from the central part of the downstream slope
August, 1967	Water emerging from the central part of the downstream slope just above the first berm
February, 1969	Soaking wet spot becomes visible at left abutment
July, 1969	Water emerging again from the central portion of the downstream slope, at mid height of the embankment
August, 1971	Soaked zoned above level 595 m. Important subsidence is found at level 615 m in the central portion of the embankment
November,1983	Soaked zones are found in the abutments and on the central part of the dam. A "waved" deformed surface at the dam crest has been observed

Due to the lack of a comprehensive geotechnical investigation, the main variables involved in analysis had their values estimated by experts. This has stimulated the use of a method based on "possibility" instead of those methods based on "probability" theory. Therefore, the Fuzzy analysis was chosen as a tool suitable for this kind of approach.

In such analysis the variation of the input parameters shall be based on interval of possible values, where a pair of parameters is considered for several membership degrees, varying from 0 to 1.

Therefore, the chosen parameters to be used in the assessment of the reinforcement effectiveness are those believed to have stronger influence on the stability of the embankment, that are: effective cohesion, shear strength angle and reservoir level. Their respective statistical values, from where the Fuzzy distribution was defined, are presented on Table 2.

Thus, a triangular fuzzy distribution is proposed herein in order to take into account the possible values assumed for the variables shown in Table 1. The narrower is the membership triangle function the lesser is the expected variability of the parameters.

The fuzzy distributions for reservoir water level, shear strength angle and cohesion, used in stability analysis are presented in Figs. 1, 2 and 3, respectively.

4. Fuzzy Assessment of Factor of Safety (Fos)

For each membership degree, all parameter values are cross-combined and the resulting factor of safety is then evaluated. Thus, the full analysis involves 2n combination of parameters for each membership function α and n is the number of parameters considered. In the case shown herein five membership degrees were considerable (for simplicity), forming, thus, 33 different combinations that are necessary to complete the analysis, which were carried out using the software Geoslope[R], as shown in Table 3. This technique is known as "vertex method" and is fully explained by Juang *et al.* (1998).

After all calculations were performed, the obtained Factors of Safety were sorted in crescent order. After that, both major and minor Factors of Safety are picked up for each membership function and a resulting fuzzy distribution is then defined, as shown in Fig. 4. It can be noticed that, due to the fact that parameters vary in a linear way, the output data can be expected to vary in the similar manner resulting in a regular triangular distribution.

Table 2 - Parameters to be used in stability analysis (Dell'avanzi, 1995).

Parameter	Expected values	Standard deviation
c' (kPa)	83.61	17.05
φ' (degree))	24.43	1.11
Water level (m)	40	1.0

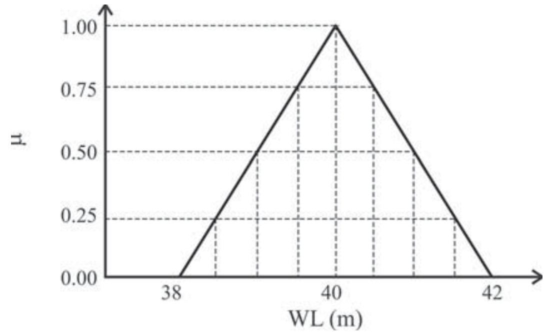

Figure 1 - Fuzzy distribution of the reservoir level (WL).

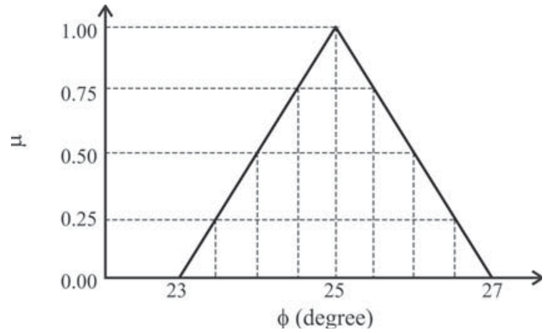

Figure 2 - Fuzzy distribution of the shear strength angle of the dam material.

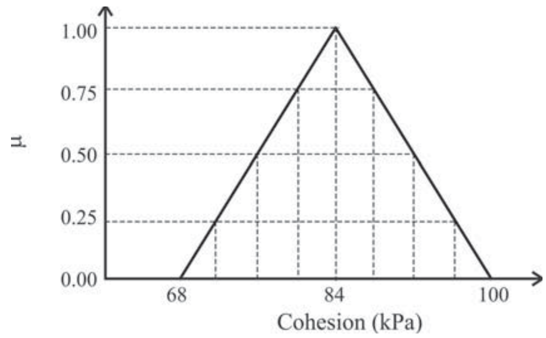

Figure 3 - Fuzzy distribution of cohesion of the dam material.

4.1. Stability state and resemblance degree

The next step to be followed is to compare the resulting fuzzy distribution with pre-defined fuzzy arrangement that describes the "stability state" in order to come up with a final comparison based on so-called "resemblance degree".

Stability state (Fig. 5) is a set of fuzzy distribution defined based on expert's opinion of engineers and geologist that identifies states of stability according to the factor of safety (Juang, 1998). The main objective of the stability state distribution is to create a pattern that can be understood as a standard model in order to classify the obtained fuzzy distribution of the Factor of Safety. This is the subjective portion of the analysis and therefore it strongly depends on judgment (and expertise) and, therefore, it can be

Table 3 - Cross-combined values for each membership degree.

Membership degree	Combined values (WL, C, φ)
μ(0.00)	(38, 68, 23), (38, 68, 27), (38, 100, 23), (38, 100, 27), (42, 68, 23), (42, 68, 27), (42, 100, 23), (42, 100, 27)
μ(0.25)	(38.5, 72, 23.5), (38.5, 72, 26.5), (38.5, 96, 23.5), (38.5, 96, 26.5), (41.5, 72, 23.5), (41.5, 72, 26.5), (41.5, 96, 23.5), (41.5, 96, 26.5)
μ(0.50)	(39, 76, 24), (39, 76, 26), (39, 92, 24), (39, 92, 26), (41, 76, 24), (41, 76, 26), (41, 92, 24), (41, 92, 26)
μ(0.75)	(39.5, 80, 24.5), (39.5, 80, 25.5), (39.5, 88, 24.5), (39.5, 88, 25.5), (40.5, 80, 24.5), (40.5, 80, 24.5), (40.5, 88, 24.5), (40.5, 88, 25.5)
μ(1.00)	(40, 84, 25)

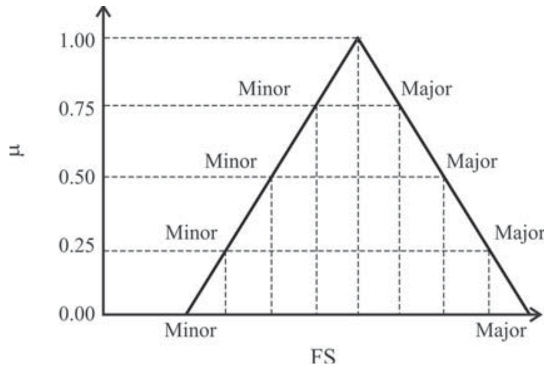

Figure 4 - Definition of Fuzzy distribution from resulting analysis.

changed for adjustment any time during the calibration. Herein the Stability State was divided in four categories as f (failure); pf (probable failure), ps (probable stable) and s stable. The number of categories and also their defined range are (analyst) dependent on the operator decision that constitutes itself the subjective part of the analysis (judgment). Figure 6, aiming to illustrate the method, shows a hypothetical comparison between obtained and pre-defined fuzzy distributions of Factor of Safety.

After superimposing the obtained fuzzy distribution on pre-defined pattern it is now necessary to use an algorithm in order to find which pre-defined distribution is most similar to that obtained one. This is known as resemblance degree.

The simplest way to carry this analysis is to subdivide the distributions in t equal parts and to compare the summation of the correspondent membership degree to each pre-defined FoS distribution. However, if the obtained distribution has other shape than symmetrical, the division must be

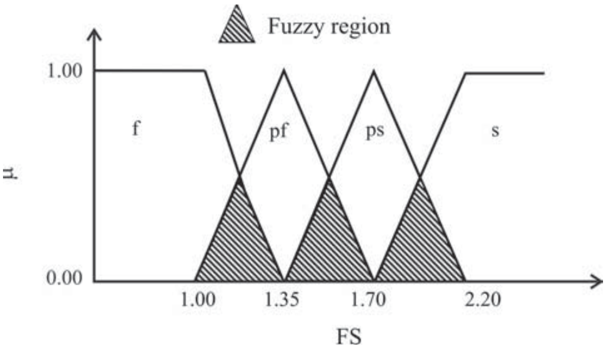

Figure 5 - Pre-defined rules for stability state.

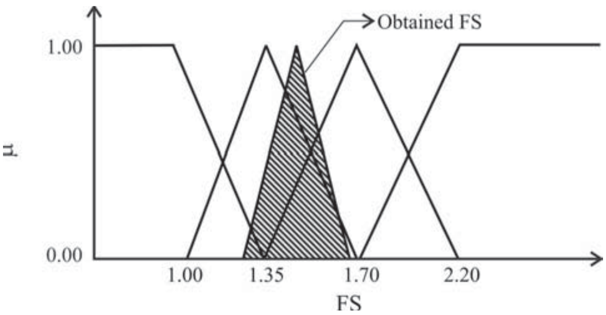

Figure 6 - Hypothetical superimposition of obtained FoS.

done for each side of the distribution in equal parts. Figure 7 shows details of this operation whose algorithm is given by Eq. (2).

$$S_j = \sum_{j=1}^{2t+1} \mu(fsp) \times \mu(fsr) \tag{2}$$

where j refers to the number of attributes and t is the number of subdivisions of each class of factor of safety. $\mu(fsp)$ represents the membership degree of the factor of safety corresponding to pre-defined curve and $\mu(fsr)$ is the membership degree of the resulting factor of safety, after the analysis had been carried out.

In order to define the percentage that describes how the obtained distribution of stability state is most similar to the predefined one, Eq. (3) is used.

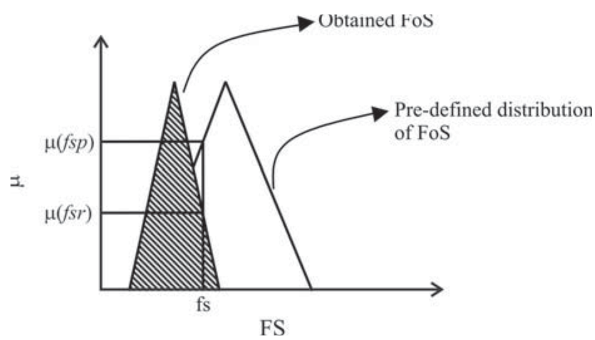

Figure 7 - Example of individual analysis.

$$w_j = \frac{S_j}{\sum_{1}^{4} S_j} \quad (3)$$

Therefore the highest values will point out the stability state amongst those represented in Fig. 5.

This methodology was, then, used for the Santa Branca Dam in order to assess the stability state before and after the reinforcement was placed.

5. Results and Discussions

Following the parameters cross-combination, stability analyses were carried out and the resulting factors of safety are presented in Table 4 for different membership degrees. It is important to mention that the values in bold are those corresponding to the maximum and minimum factor of safety for that specific membership degree, which were used to construct the final fuzzy distribution, shown in Fig. 8.

By comparing the fuzzy distribution for the factor of safety obtained from the analyses before and after the reinforcement, it is clear to verify that the reinforcement really improved the stability conditions of the dam. Before the placement of the reinforcement the resemblance degree calculated using Eqs (2) and (3) was around 61,2% for the possible failure distribution, meanwhile, after the reinforcement, the resemblance degree was around 69,4% of the stable distribution (Table 5).

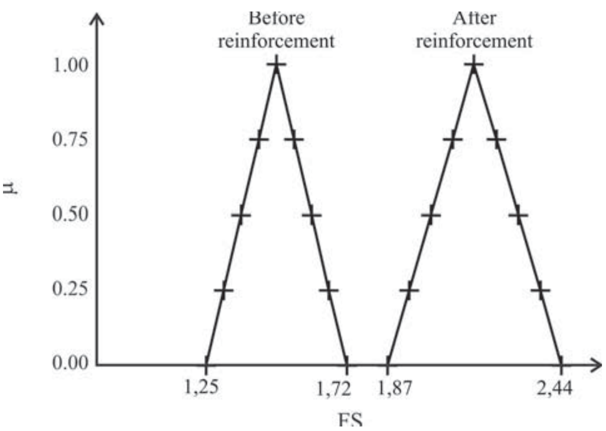

Figure 8 - Final fuzzy distribution of the Factor of Safety for Santa Branca Dam.

Table 4 - Factor of safety before and after the reinforcement.

Membership degree	(WL, C, φ)	FoS before reinforcement	FoS after reinforcement
μ(0.00)	(38, 68, 23)	1.284	1.858
	(38, 68, 27)	1.405	2.113
	(38, 100, 23)	1.594	2.198
	(38, 100, 27)	1.720	2.437
	(42, 68, 23)	1.249	1.856
	(42, 68, 27)	1.365	2.090
	(42, 100, 23)	1.562	2.180
	(42, 100, 27)	1.681	2.414
μ(0.25)	(38.5, 72, 23.5)	1.333	1.946
	(38.5, 72, 26.5)	1.423	2.125
	(38.5, 96, 23.5)	1.565	2.188
	(38.5, 96, 26.5)	1.659	2.367
	(41.5, 72, 23.5)	1.308	1.928
	(41.5, 72, 26.5)	1.396	2.104
	(41.5, 96, 23.5)	1.542	2.171
	(41.5, 96, 26.5)	1.632	2.347
μ(0.50)	(39, 76, 24)	1.384	2.013
	(39, 76, 26)	1.445	2.132
	(39, 92, 24)	1.541	2.174
	(39, 92, 26)	1.602	2.293
	(41, 76, 24)	1.366	2.001
	(41, 76, 26)	1.425	2.119
	(41, 92, 24)	1.522	2.163
	(41, 92, 26)	1.582	2.281
μ(0.75)	(39.5, 80, 24.5)	1.434	2.080
	(39.5, 80, 25.5)	1.449	2.139
	(39.5, 88, 24.5)	1.512	2.161
	(39.5, 88, 25.5)	1.543	2.220
	(40.5, 80, 24.5)	1.426	2.074
	(40.5, 80, 24.5)	1.455	2.133
	(40.5, 88, 24.5)	1.503	2.155
	(40.5, 88, 25.5)	1.534	2.214
μ(1.00)	(40, 84, 25)	1.484	2.144

Table 5 - Resemblance degree before and after reinforcement.

Resemblance degree – before reinforcement (in %)	Resemblance degree – after reinforcement (in %)	Stabilty state
0,44	0	Failure
61,12	0	**Possible Failure**
38,44	30.52	Possible Stable
0	**69.48**	**Stable**

6. Conclusions

Fuzzy Logic is a methodology that uses possibility distribution instead the well-known probability-statistical method. The main difference between both is that Fuzzy Logic does not lunch the concept of probability distribution of random parameters. Instead, it uses the concept based on that an interval between two extreme values embraces a possibility to happen. This gives the engineers freedom to use their experience and judgment to evaluate a boundary in which values are acceptable to vary.

Herein, the parameters were chosen to be the water level and the strength parameters C and ϕ, whose boundaries were evaluated from former published researches.

By cross-combining these values, the final stability state for Santa Branca dam, comprising periods before and after the reinforcement, were evaluated and the results are quite satisfactory indicating, thus, a considerable improvement in the stability of the dam.

Acknowledgments

The Authors are grateful to Rio de Janeiro State Research Agency – FAPERJ for financing the second author during this research.

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Symbol List

$\mu(\ )$: Membership degree

$\mu(fsp)$: Membership degree of the factor of safety corresponding to pre-defined curve

$\mu(fsr)$: Membership degree of the resulting factor of safety