Evaluation of stress history and undrained shear strength of three marine clays using semi-empirical methods based on Piezocone Test

Danielle Caroline Ferreira1#, Faiçal Massad1

Abstract

The paper presents a comparative study between semi-empirical methods for the estimation of pre-consolidation pressure and undrained shear strength from Piezocone (CPTu) data. The first method, proposed by Massad, was developed from observing the variation of these parameters with depth; the second method, proposed by Mayne, was developed from simplifications and relationships between the Spherical Cavity Expansion Theory (SCET) and the Critical State Theory; the third method was proposed by Mayne, which considers the variations due to soil type from the CPT Index to estimate the pre-consolidation pressure. The methods were validated based on their applications to the marine clay from Santos Coastal Plain, Brazil, Bothkennar clay from Scotland, and Torp Clay from Sweden. It is intended to verify if the results are consistent with each other, with the stress history of these soils and with the available test results. The application of the Massad’s method led to results close to the available reference values. The results of the Mayne’s method based on SCET showed great variability in behavior comparing to the test data depending on the case study. By the Mayne’s method based on CPT Index values, the calculated pre-consolidation pressures were slightly higher than the values of the available test data. The variations in the results highlighted the importance of validating estimates based on semi-empirical methods through specific tests and the knowledge of geological history contributes to predicting the behavior of clays, since they showed good agreement with the available data from oedometer tests.

Keywords
Piezocone test
Geological history
Marine clays
Consolidation mechanisms
Pre-consolidation pressure
Undrained shear strength

1. Introduction

Due to the recurrence of CPTu test in field investigations, it is common to use its results to estimate geotechnical parameters, such as pre-consolidation pressure ($\sigma''_a$) and undrained shear strength ($S_u$), instead of performing a large number of specific tests, such as Vane Test ($VT$) and oedometer test, which makes the geotechnical investigation more expensive.

The correlation proposed by Kulhawy & Mayne (1990, apud Coutinho & Oliveira, 1993) is often used to determine the $\sigma''_a$, given by:

$$\sigma''_a = \frac{q_t - \sigma_0}{N_{\sigma t}}$$  \hspace{1cm} (1)

In general, $N_{\sigma t}$ is in the order of 3.3 (Mayne et al., 1998) to 3.4 (Demers & Leroueil, 2002), among other values.

For the estimation of $S_u$, Lunne et al. (1985) proposed the second term of Equation 2, based on the Spherical Cavity Expansion Theory (SCET), while Tavenas et al. (1982) proposed its determination in terms of excess pore pressure induced by cone penetration ($\Delta u$), as the third term of Equation 2.

$$S_u = \frac{q_t - \sigma_0}{N_{kl}} = \frac{\Delta u}{N_{\Delta u}}$$  \hspace{1cm} (2)

The most common empirical factor is $N_{\Delta u}$, which varies from 10 to 15 to normally consolidated clays and from 15 to 19 to overconsolidated clays according to Senneset et al. (1989). In practice, its value is usually determined with VT.

This paper aims to evaluate three recent studies of semi-empirical methodologies for estimating $\sigma''_a$ and $S_u$ from CPTu data. The Massad’s method (2009, 2010, 2016) is based on observations of the variations of $\sigma''_a$, $S_u$ and CPTu data with depth. Mayne (2016) proposed the method

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from the relationship between SCET and the Critical State Theory and from simplifications in the determination of some parameters difficult to be obtained directly. Mayne (2017) considered the influence of the particle size to estimate $\sigma'_a$ from CPT Index.

These methods were applied to marine clays from Santos Coastal Plain (Brazil), Bothkennar (Scotland), and Torp area (Sweden). It is intended to verify if the estimates are consistent with each other, with the geological history of the clay deposits and with the available test data.


Massad (2009, 2010, 2016) presented solutions for estimating both $\sigma'_a$ and $S_u$ for SFL clays (from Sediments-Fluvial Lagoon-Bay) in Santos Coastal Plain, based on their variations with depth and the geological history of these sediments.

2.1 Pre-consolidation pressure obtained by Massad’s method (2009, 2010, 2016)

From 20 underground profiles of Santos Coastal Plain with oedometer test data, Massad (2009) noted strong linearity and parallelism in the relationship between $\sigma'_a$ and $\sigma'_v$, with depth for the SFL clay layer, which suggests overconsolidation due to preloading ($\Delta p = $ cte) and allowing to assume the following expression:

$$\sigma'_a = \Delta p + \sigma'_v$$

For other deposits that have the influence of ageing on overconsolidation, the relationship between $\sigma'_a$ and $\sigma'_v$ deviates from parallelism. Therefore, the $r$ factor was inserted in Equation 3 as presented in the relationship between the first and second terms of Equation 4.

$$r = \frac{\sigma'_a}{\sigma'_v + \Delta p} = \left(\frac{t}{t_p}\right) \frac{C'_{ec}/C_c}{1-C'_{ec}/C_c}$$

The determination of the $r$ factor in terms of $C'_e$, $C_c$ and $C'_{ew}$, as presented in Equation 4, was proposed by Massad (2009) as an adaptation to the formula of Mesri & Choi (1979) with the introduction of $\Delta p$ to combine the effects of ageing and preloading.

It is observed from Equation 4 that, when admitting that $r = 1$, the effect of ageing is disregarded (Equation 3); on the other hand, by assuming that there is no preloading ($\Delta p = 0$), then $r = OCR > 1$ and $\sigma'_a$ would vary linearly with depth (Massad, 2009).

From CPTus data performed in Santos Coastal Plain, Massad (2009) observed that the SFL clays presented a practically linear relationship between the cone tip resistance ($q_t$) and the depth ($z$) at a rate “$b$”, so that:

$$q_t = a + b \cdot z$$

By introducing both the relationship between the first and second terms of Equation 4 and the Equation 5 in Equation 1 and matching the dependent terms of the depth, Massad (2009, 2010, 2016) obtained the following formula to determine $N'_{st}$:

$$N'_{st} = \frac{b - \gamma_n}{r' \gamma'}$$

2.2 Undrained shear strength obtained by Massad’s method (2009, 2010, 2016)

Hundreds of VTs performed on SFL clays in Santos city, compiled by Massad (2009, 2010), showed that $S_u$ varies linearly with depth, so that:

$$S_u = c_0 + c_1 \cdot z$$

By relating the Equations 2, 5 and 7 and matching the dependent terms of the depth, Massad (2016) proposed to determine $N_{kt}$ as follows:

$$N_{kt} = \frac{b - \gamma_n}{c_1}$$


The penetration of the CPTu generates a very complex stress state and deformation in the surrounding soil mass. Therefore, simplifying hypotheses are used to interpret the boundary conditions, such as the SCET.

The equations formulated by Vesic (1972, 1977), from the SCET study, are functions of the empirical factors, $N_{st}$ and $N_{ku}$, and the rigidity index ($I_R$) as follows:

$$N_{kt} = \frac{4}{3} \left[\ln(I_R) + 1\right] + \frac{\pi}{2} + 1$$
According to Mayne (2016), $I_R$ can be determined as an exponential function of pore pressure parameter ($B_q$) from its relationship with the empirical factors ($N_{tu} = B_q N_{kt}$) and using the Equations 9 and 10, which was validated through the analysis of CPTu data from 34 soft to firm clays where $B_q$ ranged from 0.45 to 0.75.

\[ N_{tu} = \frac{4}{3} \ln(I_R) \]  

(10)

\[ \sigma'_a = 0.33(q_t - \sigma_{v0})^{m'} \text{ [kPa]} \]  

(14)

Because $m'$ varies with the soil type, Mayne (2017) noted a strong relationship between this exponent and the CPT index ($I_c$), which allowed him to establish the empirical formula presented below:

\[ m' = 1 - \frac{0.28}{1 + \left(\frac{I_c}{2.65}\right)^{25}} \]  

(15)

In general, the values of $I_c$ and the exponent $m'$ vary according to the soil type, as indicated in Table 1.

### 3.1 Pre-consolidation pressure obtained by Mayne’s method (2016 and 2017)

Mayne (2016) searched a relationship between $\sigma'_a$ and $S_u$ to apply the SCET equations, function of $N_{tu}$ or $N_{kt}$, previously presented. This relationship was made through the Critical State Theory, which provided the following equation:

\[ S_u = \left(\frac{M_c}{2}\right) \left(\frac{OCR}{2}\right)^\Lambda \sigma'_v \]  

(11)

where $M = 6\sin\phi'/(3 - \sin\phi')$ that ranges from 0.8 to 0.9 (Jamiolkowski et al., 1985; Larsson & Åhnberg, 2005), but Mayne (2016) assumed $\Lambda = 1$ in a simplified way.

Mayne (2016) used the second term of Equation 2 plus Equations 9 and 11 to get the following expression for estimating the $\sigma'_a$ in terms of cone tip resistance:

\[ \sigma'_a = \frac{q_t - \sigma_{v0}}{M_c \left(1 + \frac{1}{3} \ln I_R \right)} \]  

(12)

To estimate in terms of pore pressure, Mayne (2016) used the third expression of Equation 2 and considered the determination of the empirical factor $N_{tu}$ by Equation 10, so that:

\[ \sigma'_a = \frac{\Delta u}{\frac{1}{3} M_c \ln I_R} \]  

(13)

It can be noted that the denominator of Equation 12 is equivalent to the empirical factor $N_{tu}$ of Equation 1.

In a more recent publication, Mayne (2017) proposed an adaptation of Equation 1 to consider variations due to soil type. The author made a compilation of a data set from a variety of natural soil formations and observed a tendency to divide them into ranges of variation based on their particle size. Therefore, Mayne (2017) introduced the exponent $m'$ that increases with fine contents and decreases with mean grain size, so that:

\[ \sigma'_a = 0.33(q_t - \sigma_{v0})^{m'} \text{ [kPa]} \]  

In general, the values of $I_c$ and the exponent $m'$ vary according to the soil type, as indicated in Table 1.

### 3.2 Undrained shear strength obtained by Mayne’s Method (2016)

Mayne (2016) reformulated the Equations 9 and 10 putting them as an exponential function of $B_q$ (as previously mentioned) and then replaced it with its definition $B_q = \Delta u/(q_t - \sigma_{v0})$, getting a simple equation to determine the $S_u$:

\[ S_u = \frac{q_t - u_2 - \sigma_{v0}}{3.90} \]  

(16)

By rearranging Equation 16 to define it in terms of the empirical factor $N_{tu}$, Mayne (2016) got:

\[ N_{tu} = \frac{3.90}{1 - B_q} \]  

(17)

### 4. Applications to soils with known geological history

Three case studies will be presented in which both information about the geological history of the soils and tests of the most varied types are available. The first refers to a

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>SBT zones</th>
<th>$I_c$</th>
<th>$m'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sands to silty sands</td>
<td>6</td>
<td>1.31-2.05</td>
<td>0.72</td>
</tr>
<tr>
<td>Silty sands to sandy silts</td>
<td>5</td>
<td>2.05-2.60</td>
<td>0.8</td>
</tr>
<tr>
<td>Clayey silts to silty clays</td>
<td>4</td>
<td>2.60-2.95</td>
<td>0.85</td>
</tr>
<tr>
<td>Silty clays to clays</td>
<td>3</td>
<td>2.95-3.60</td>
<td>0.9 ± 0.1</td>
</tr>
<tr>
<td>Organic clays</td>
<td>2</td>
<td>&gt;3.6</td>
<td>0.9 ± 0.1</td>
</tr>
</tbody>
</table>

Table 1. Relationship between $m'$ and soil type (Robertson, 1990; Mayne, 2017).
4.1 SFL clay in Santos (close to Barnabé Island)

In the area close to Barnabé Island, in Santos Coastal Plain, several tests were performed due to the need to build an embankment for a container yard in the Santos Harbor Channel, where the final level of the earth fills should emerge up to an elevation of +3.5 m in relation to sea level.

4.1.1 Geological history and overconsolidation for the SFL Clay in Santos

The genesis of quaternary sediments in Santos Coastal Plain was explained by Suguio & Martin (1978), who indicated that the relative fluctuations in sea level, both in the Pleistocene and in the Holocene, were the main causes of the formation of sedimentary deposits.

At the peak of the last glaciation, near 15,000 years ago, with the great retreat of the sea level at an elevation of -110 m in relation to the current one, there was an intense erosive process, forming deep valleys. The Santos Transgression, about 7,000 years ago, rose the sea level roughly 6 m above the current level. The sea entered the lower areas, originating an extensive system of lagoons, forming the SFL clays, and eroded partially the Pleistocene sediments, originating the SFL sandy deposits (Massad, 2009).

Close to Barnabé Island, the local SFL clays are highly overconsolidated due to dunes that were active in the area until about 50 to 100 years ago, with OCR > 2. These facts imply that $r \approx 1.0$, as shown in Table 2.

There are reports of the existence of dunes about 4 m high on Santo Amaro Island, close to Barnabé Island. By assuming $r = 1$ and $\gamma_s = 19$ kN/m$^3$, the preload due to the dunes is equivalent to $\Delta p \approx 76$ kPa, then it can be said that the equation that represents the geological history of the SFL clay in the area is given by:

$$\sigma'_a = \sigma'_0 + 76 \text{ kPa}$$

as shown by the dotted lines in Figure 1a and Figure 1b.

4.1.2 Soil profile, CPTu and VT for the SFL Clay in Santos

The Figure 2a and Figure 2b presents the CPTu 101 data performed in the area close to Barnabé Island. It is noticed the presence of an upper layer with about 2 to 3 m of a very soft clay (mangrove) followed by sand to the depth of 6 m, where the thick layer of SFL clay begins. The first 6 meters and the isolated points that indicate the occurrence of sand lenses were neglected in the analyses.

From Figure 3, for depths greater than 6 m, $c_1 = 1.85$ kPa/m for the VT points performed close to CPTu 101 hole. However, in general, the VTs performed in the area, compiled by Massad (2009), revealed a trend of $c_1 = 1.47$ kPa/m, value adopted in the analyses.

4.1.3 Geotechnical parameters and considerations for the SFL Clay in Santos

With the underground profile information, CPTu data and the knowledge of the geological history of the SFL clay in the area close to Barnabé Island, it was possible to fill the Table 3 below.

Table 2. Determination of the $r$ factor of the SFL clay close to Barnabé Island.

<table>
<thead>
<tr>
<th>Geotechnical Parameters</th>
<th>Values</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\alpha}$</td>
<td>0.1%</td>
<td>Lambe &amp; Whitman (1979) and Massad (2009)</td>
</tr>
<tr>
<td>$C_{c}/(1+e_0)$</td>
<td>0.43</td>
<td>Massad (2009)</td>
</tr>
<tr>
<td>$C_{\alpha}e_0/C_c$</td>
<td>0.0023</td>
<td>$C_{\alpha}/C_c = C_{\alpha}(1+e_0)$</td>
</tr>
<tr>
<td>$C_e/C_c$</td>
<td>10%</td>
<td>Massad (2009)</td>
</tr>
<tr>
<td>$C_{\alpha}$</td>
<td>3.00x10^{-4}</td>
<td>Massad et al. (2013)</td>
</tr>
<tr>
<td>$H_d$</td>
<td>10 m</td>
<td>CPTu 101</td>
</tr>
<tr>
<td>$t_p$</td>
<td>1.192 years</td>
<td>Terzaghi’s Theory*</td>
</tr>
<tr>
<td>$t$</td>
<td>100 years</td>
<td>Adopted**</td>
</tr>
<tr>
<td>$r$</td>
<td>1.011</td>
<td>Equation 4</td>
</tr>
<tr>
<td>$r$ adopted</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

*Calculated by Terzaghi’s Theory of Consolidation (Terzaghi, 1943); $T \approx 1.128$ was adopted to represent 95% of degree of consolidation (the end of primary consolidation); **$t = 100$ years based on geological history, as the dunes were active until recently.

Figure 1. (a) Pre-consolidation pressure ($\sigma'_a$) and (b) OCR for the SFL Clay in Santos in the context of its geological history and the application of the semi-empirical methods.
practically coincided and came close to oedometer test data performed by Andrade (2009) using Shelby samples extracted at a certain distance from the CPTu 101 borehole, with regular to good qualities. Taking as reference the OCR values indicated by Massad et al. (2013) for the Barnabé Island area, OCR > 2, and for Santo Amaro Island (close to Barnabé Island), OCR = 2.5, it is noticed (Figure 1b) that the oedometer test data and the applications of Massad’s (2009, 2010, 2016) and Mayne’s (2017) methods led to results closer to those expected from the geological history of the local soil. The application of Mayne’s method (2016), however, led to mean OCR of 5, with great dispersion, which does not represent the studied clay. The empirical factor \( N_{\sigma t} \) obtained by Mayne’s (2016) method, both in terms of cone tip resistance and pore pressure, was \( N_{\sigma t} = 1.69 \), thus half of the value obtained by Massad’s (2009, 2010, 2016) method, \( N_{\sigma t} = 3.28 \), which resembled the available reference values (3.3 to 3.4). It should be mentioned that this figure corresponds to the CPTu 101. Working with results of 15 CPTus, in this same area, including the former one, Massad (2016) arrived to \( N_{\sigma t} = 3.9 \) as an average value.

Figure 4a and Figure 4b show the SBT Charts with the CPTu 101 data, performed close to Barnabé Island, and with the CPTu data from the other case studies presented in this paper.

### 4.1.4 Analyses of results for the SFL Clay in Santos

The analyses of results for SFL clay from Barnabé Island area are presented below.

#### 4.1.4.1 Pre-consolidation pressure

By analyzing the results of Figure 1a, the estimates by Massad’s (2009, 2010, 2016) and Mayne’s (2017) methods

<table>
<thead>
<tr>
<th>Geotechnical Parameters</th>
<th>Values</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_n )</td>
<td>14.9 kN/m^2</td>
<td>CPTu 101 data</td>
</tr>
<tr>
<td>( b )</td>
<td>30.98 kPa/m</td>
<td>CPTu 101 data</td>
</tr>
<tr>
<td>( r )</td>
<td>1.00</td>
<td>Table 2</td>
</tr>
<tr>
<td>( c_i )</td>
<td>1.47 kPa/m</td>
<td>Massad (2009) (Figure 3)</td>
</tr>
<tr>
<td>( N_{\sigma t} ) (Massad)</td>
<td>3.28</td>
<td>Equation 6</td>
</tr>
<tr>
<td>( N_{\sigma t} ) (Massad)</td>
<td>10.94</td>
<td>Equation 8</td>
</tr>
<tr>
<td>( I_R )</td>
<td>10.99</td>
<td>Equation 12</td>
</tr>
<tr>
<td>( M_c )</td>
<td>0.94</td>
<td>Equation 17</td>
</tr>
<tr>
<td>( Q_t )</td>
<td>6 to 13</td>
<td>Figure 4ab</td>
</tr>
<tr>
<td>( F_R )</td>
<td>0.4 to 3.0%</td>
<td>Figure 4b</td>
</tr>
<tr>
<td>( I_c )</td>
<td>2.95</td>
<td>Figure 4ab and Table 1</td>
</tr>
<tr>
<td>( m' )</td>
<td>0.982</td>
<td>Equation 15</td>
</tr>
<tr>
<td>( N_{\sigma t} ) (Mayne, 2017)</td>
<td>-</td>
<td>Indeterminable</td>
</tr>
</tbody>
</table>

\( \sigma'_d \) by the Mayne’s method (2016) and it was noted that
variations in $B_q$ (used in the calculation of $I_R$ as proposed by Mayne (2016)) greatly affect the results ($N_{\sigma t}$ values). The range of $B_q$ between 0.45 and 0.75 was the same used by Mayne (2016).

Although the effects of the variations of $B_q$ have been minimized by using an average value for the entire profile, as proposed by Mayne (2016), and restricting it to 0.45, the magnitude of $N_{\sigma t}$ was much lower than the reference values. The relatively low $\phi'$ (used in the calculation of $M_c$ as indicated by Mayne (2016)) of the SFL clay also contributed to reduce $N_{\sigma t}$ as shown in Figure 5a.

### 4.1.4.2 Undrained Shear Strength

The Figure 6 shows the results of applying the methods of Massad (2009, 2010, 2016) and Mayne (2016), the $VT$ data, the correlations $S_u$ vs. $z$ presented by Massad (2009) to the Barnabé Island area and the curve related to $N_{kt} = 15$. The application of Massad’s method (2009, 2010 and 2016) resulted in $S_u$ around 20 kPa higher than the “$VT Ave$” as shown in Figure 6. The Mayne’s method (2016) revealed an even greater difference, with resistance values of about 50 kPa higher than the available data.

For the range of $B_q$ values used by Mayne (2016) ($0.45 < B_q < 0.75$), it can be seen from Figure 5b that $N_{kt}$ varies between 7.1 and 15.6. Senneset et al. (1989) indicated $N_{kt}$ ranging from 10 to 15 for normally consolidated clays and from 15 to 19 for overconsolidated clays. Therefore, the range of the Mayne (2016) dataset is restricted to typical values of normally consolidated clays.

However, as seen above, the SFL clay from Barnabé Island area is overconsolidated, which according to Senneset et al. (1989) would lead to $N_{kt}$ greater than 15, well above the values estimated by Mayne’s (2016) and Massad’s (2009, 2010, 2016) methods, 7.1 and 10.9, respectively. As shown in Figure 6, the curve for $S_u$ calculated with $N_{kt} = 15$ overlapped the “$VT Ave$” curve, getting very close to the $VT$ data, confirming that $N_{kt}$ estimated by the studied methods were lower than expected values.
Finally, an evaluation of the $S_u/\sigma'_a$ relationship was made for the studied methods: by Massad’s method (2009, 2010, 2016), $S_u/\sigma'_a = 0.30$ was obtained; for Mayne’s method (2016), this value was 0.25 in terms of cone tip resistance and 0.30 in terms of pore pressure. Thus, although the $S_u$ and $\sigma'_a$ values have been quite different between the methods, their ratios were close.

As a reference, there is the following empirical correlation:

$$S_u / \sigma'_a = \frac{\sqrt{I_p}}{22}$$

proposed by Mayne & Mitchell (1988), where $I_p$ is the plasticity index of the soil. From the tests performed by Andrade (2009), $I_p = 75\%$ for the SFL clay in the Barnabé Island area, leading to $S_u/\sigma'_a = 0.39$, significantly higher than the figures presented above.

### 4.2 Bothkennar Clay

The study site is in the Bothkennar region, on the edge of the River Forth, situated between Edinburgh and Glasgow, Scotland, UK.

The soft silty clay at Bothkennar attracted the interest of many researchers due to its homogeneity, described by Nash et al. (1992a) as being “remarkably uniform” when compared to other locations in the UK.

#### 4.2.1 Geological history and overconsolidation for Bothkennar Clay

Around 7,000 years ago, the Bothkennar region was going through a process of sediment deposition, which reached a level about +4.5 m in relation to the current sea level (Nash et al., 1992a). Later, with the marine regression and the consequent erosive processes, part of this material was removed, thus the Bothkennar clay suffered an overconsolidation by preloading, which according to Nash et al. (1992a), was equivalent to a load of $\Delta p = 15$ kPa. This observation allowed the authors to assume that the $\sigma'_a$ of this clay could be obtained by the following equation, also represented in a curve in Figure 7a and Figure 7b:

$$\sigma'_a = \sigma'_0 + 15$$

The curve suggested by Nash et al. (1992a) resulted in lower pressures in relation to the oedometer tests data for which Nash et al. (1992a) proposed a second curve (see Figure 7a and Figure 7b), with $OCR = 1.55$, mean value of the oedometer tests, given by:

$$\sigma'_a = 1.55 \cdot \sigma'_0$$

There is a gap between the curves of the two expressions proposed by Nash et al. (1992a). The authors attributed this difference to the possibility that ageing had a greater influence on clay overconsolidation. However, it is evident that there is a contradiction between the premises that gave rise to the two curves: the curve given by Equation 20 only considers the influence of preload while the curve related to Equation 21 assumes that only ageing is responsible for the clay overconsolidation, by considering that $OCR$ is constant.

To combine both overconsolidation mechanisms, ageing + preloading, a third curve is being proposed based on the adjustment of the expression of Equation 20 proposed by Nash et al. (1992a) with the insertion of the $r$ factor given by Equation 4, so that:

$$\sigma'_a = 1.33 \cdot (\sigma'_0 + 15)$$

As shown in Table 4, $r = 1.33$ to the Bothkennar Clay.

#### 4.2.2 Soil Profile, CPTu and VT for Bothkennar Clay

Figure 8a and Figure 8b show the CPTu data performed at the Bothkennar test site by Powell & Lunne (2005). The water level was found at -0.8 m of depth in relation to the ground level and it is important to highlight the existence of a dry crust, up to a depth of 2 to 3 m, which was probably
formed due to variations in sea level according to Nash et al. (1992a). The existence of this crust affected the resistance of the soil; therefore, the first 2.5 m were neglected in the analyses.

Nash et al. (1992a) performed laboratory (triaxial $UU$) and field ($VT$ and pressuremeter) tests to measure $S_u$. The authors also performed an indirect evaluation of this parameter using dilatometer test ($DMT$) data. Figure 9 presents the results.

It is possible that the line of the “Average $VT$” was obtained considering the dry crust. Therefore, to avoid taking parameters distorted by the crust, it was decided to consider the pressuremeter data to estimate the coefficient $c_1$, disregarding the points above 3 m depth; thereby, $c_1 = 2.94 \text{kPa/m}$ was obtained.

### 4.2.3 Geotechnical parameters and considerations for Bothkennar Clay

With the underground profile information, $CPT_u$ data and the knowledge of the geological history of Bothkennar clay, it was possible to fill the Table 5 below.

<table>
<thead>
<tr>
<th>Geotechnical Parameters</th>
<th>Values</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_u/C_c$</td>
<td>0.04</td>
<td>Nash et al. (1992b)</td>
</tr>
<tr>
<td>$C/C_c$</td>
<td>0.10</td>
<td>Nash et al. (1992b)</td>
</tr>
<tr>
<td>$C_s$</td>
<td>$3.17 \times 10^{-7}$ m$^2$/s</td>
<td>Nash et al. (1992a)</td>
</tr>
<tr>
<td>$H_d$</td>
<td>10 m</td>
<td>Nash et al. (1992a)</td>
</tr>
<tr>
<td>$t_\gamma$</td>
<td>10 years</td>
<td>Terzaghi’s Theory*</td>
</tr>
<tr>
<td>$t$</td>
<td>6,000 years</td>
<td>Nash et al. (1992a)</td>
</tr>
<tr>
<td>$r$</td>
<td>1.33</td>
<td>Equation 4</td>
</tr>
</tbody>
</table>

*Calculated by Terzaghi’s Theory of Consolidation (Terzaghi, 1943): $T = 1.128$ was adopted to represent 95% of degree of consolidation (the end of primary consolidation).
4.2.4 Analyses of results for Bothkennar Clay

The analyses of results for Bothkennar Clay are presented below.

4.2.4.1 Pre-consolidation pressure

By the results presented in Figure 7a and Figure 7b, it is evident the great approximation between the values estimated for the $\sigma^\prime_a$ and the OCR by the Mayne’s (2016) and Massad’s (2009, 2010, 2016) methods. When applying the Mayne’s method (2017), the resulting curve indicated a slightly higher overconsolidation, drifting away from the curve of Nash et al. (1992a) adapted with the $r$ factor and the curves proposed by Nash et al. (1992a).

As for this case of Bothkennar clay $m^\prime = 1$, the value of 3.0 referring to the inverse of the factor of Equation 14 ($1/0.33$) can be compared with the values of $N_{\sigma_t}$ obtained by the Mayne’s (2016) and Massad’s (2009, 2010, 2016) methods: 3.56 and 3.30, respectively, highlighting the proximity between them. The Mayne’s (2016) and Massad’s (2009, 2010, 2016) methods were the closest to the values referenced by Mayne et al. (1998) and Demers & Leroueil (2002): 3.3 and 3.4.

4.2.4.2 Undrained shear strength

Figure 10 shows the results of applying the methods of Massad (2009, 2010, 2016) and Mayne (2016), the pressuremeter, triaxial $UU$ and $VT$ data and the indirect evaluation of $S_u$ by DMT data.

The curves of the Massad’s (2009, 2010, 2016) and Mayne’s (2016) methods were very close to each other and had a good agreement with $S_u$ values obtained from the pressuremeter and the “Average VT” data, showing a small deviation for greater depths ($z > 12$ m), possibly because Nash et al. (1992a) did not disregard the points of the VTs obtained at depths below 3 m (dry crust occurrence) when tracing the “Average VT” line. If the value of $c_1 = 2.30$ kPa/m from the VT data was taken as a reference, the curve proposed with the Mayne’s method (2016) would not be affected, only the Massad’s curve (2009, 2010, 2016) would suffer a displacement towards DMT and “Average UU” data.

The $N_{kt}$ values obtained by the Mayne’s (2016) and Massad’s (2009, 2010, 2016) methods, 10.2 and 10, respectively, are almost the same. Senneset et al. (1989) indicated a range of $N_{kt}$ from 15 to 19 for overconsolidated clays, as Bothkennar clay, which suffered overconsolidation due to ageing and preloading. However, it would lead to lower $S_u$ values, moving away from pressuremeter data and “Average VT”, getting closer to other test data (DMT and “Average UU”).

The $S_u/\sigma_a$ relationship obtained was 0.33 by the Massad’s (2009, 2010, 2016) method and 0.35 by the Mayne’s (2016) method, in terms of cone tip resistance, and 0.34, in terms of...
pore pressure. Knowing that for Bothkennar clay $I_p = 40\%$, the correlation of Mayne & Mitchell (1988), Equation 19, gives $S_u/\sigma'_a = 0.29$, quite close to the above figures.

4.3 Torp Clay

Torp Clay is found in the southern part of the municipality of Munkedal, Sweden, in the Torp area, which is located on the west bank of the river Örekilsälven.

In the so-called Section C of the Torp area, $CPT_u$, $VT$ and oedometer tests were performed in points of interest such as: at the bottom of the river channel, in excavated areas and at the top of the slope crest. In this study, it was decided to apply the semi-empirical methods only at a point in an excavated area, denominated point S9, because it is the location with the greatest depth and because it was also the object of analysis by Mayne (2017).

4.3.1 Geological history and overconsolidation for Torp Clay

According to Larsson & Åhnberg (2003), during the last glaciation, the Torp area was covered by ice. About 12,400 years ago, with the retreat of the ice front and the progress of the isostatic uplift of the land, the sea level gradually became shallower, and the deposition of sediments began: postglacial sediments started to overlay the glacial deposits. With the further decline in sea level, the river was formed in the higher areas and the eroded particles were transported by the river and started to deposit far from the river mouth.

Erosive processes, slides in the slopes of the riverbanks and excavations in the area were the main factors responsible for overconsolidating the Torp Clay, involving, above all, preloading and ageing mechanisms.

The Torp Clay consolidated until reaching the maximum preload of $\Delta p \approx 100$ kPa, so that:

$$\sigma'_a = \sigma'_v + 100 \text{[kPa]}$$  \hspace{1cm} (23)

and then suffered a slight overconsolidation due to ageing equivalent to $r = 1.15$, so that:

$$\sigma'_a = 1.15 \cdot (\sigma'_v + 100) \text{[kPa]}$$  \hspace{1cm} (24)

as shown by the dotted lines in Figure 11a and Figure 11b. It is possible to assume that OCR varies from 1.3 to 3.2, thereby it is an overconsolidated clay, confirming the geological history of the area.

4.3.2 Soil Profile, $CPT_u$ and $VT$ for Torp Clay

As described by Larsson & Åhnberg (2003), the underground profile of the area is heterogeneous, composed of a sandy layer at the top, followed by clay with silt/sand lenses and, at greater depths, it returns to granular material. An analysis of the $CPT_u$s at Point S9 (Figure 12a and Figure 12b) and at neighboring points (not shown) allowed to identify that the clay layer, with silt/sand lenses, occurs between elevations +3 and -23 m, which, for Point S9, correspond to depths 11 to 37 m.

Figure 13 presents the results of the $VT$s performed in Point S9 of the Torp area by Larsson & Åhnberg (2003). It is possible to assume $c_f = 1.15$ kPa/m for the Torp Clay layer.

4.3.3 Geotechnical parameters and considerations for Torp Clay

With the underground profile information, $CPT_u$ data and the knowledge of the geological history of Torp Clay, it was possible to fill the Table 6 below.

4.3.4 Analyses of results for Torp Clay

The analyses of results for Torp Clay are presented below.

4.3.4.1 Pre-consolidation pressure

The Figure 11a and Figure 11b show the results of Massad’s (2009, 2010, 2016) and Mayne’s (2016 and 2017) methods in the context of geological history of the Torp area, as mentioned above. There is a good approximation between them. Moreover, comparing the results of these methods with the available data by Larsson & Åhnberg (2003), the estimated
As shown in Table 6, the $N_q$ values obtained by Mayne’s (2016 and 2017) and Massad’s (2009, 2010, 2016) methods were very similar, being slightly lower than the values referenced by Mayne et al. (1998) and Demers & Leroueil (2002): 3.3 and 3.4.

Figure 11. (a) Pre-consolidation pressure ($\sigma' a$) and (b) OCR for Torp Clay in the context of its geological history and the application of the semi-empirical methods.

Pre-consolidation pressures were slightly higher than the curve given by Equation 24, situation in which both preloading and ageing acted. For Point S9, as expected, the occurrence of excavations led the clay to a highly overconsolidated condition, with OCR between 1.5 and 3.0.

Figure 12. CPTu data from Point S9 of the Torp area (a) $q_t$ vs. depth and (b) $u_2$ vs. depth (Larsson & Åhnberg (2003) data).

Figure 13. VT data from Point S9 of the Torp area (Larsson & Åhnberg (2003) data).

As shown in Table 6, the $N_q$ values obtained by Mayne’s (2016 and 2017) and Massad’s (2009, 2010, 2016) methods were very similar, being slightly lower than the values referenced by Mayne et al. (1998) and Demers & Leroueil (2002): 3.3 and 3.4.
Table 6. Geotechnical parameters of Torp Clay to estimate the pre-consolidation pressure and undrained shear strength.

<table>
<thead>
<tr>
<th>Geotechnical Parameters</th>
<th>Values</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_n$</td>
<td>16.4 to 17.7 kN/m$^3$</td>
<td>Larsson &amp; Åhnberg (2003)*</td>
</tr>
<tr>
<td>$b$</td>
<td>41.8 kPa/m</td>
<td>CPTu data</td>
</tr>
<tr>
<td>$r$</td>
<td>1.15</td>
<td>Larsson &amp; Åhnberg (2003)</td>
</tr>
<tr>
<td>$c_i$</td>
<td>1.15 kPa/m</td>
<td>VT data (Figure 12)</td>
</tr>
<tr>
<td>$N_m$</td>
<td>2.75 to 3.50</td>
<td>Equation 6</td>
</tr>
<tr>
<td>(Massad)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_{tu}$</td>
<td>21 to 22</td>
<td>Equation 8</td>
</tr>
<tr>
<td>(Massad)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_q$ average</td>
<td>0.624</td>
<td>$B_q = \Delta u/(q_t-\sigma_u)$</td>
</tr>
<tr>
<td>$I_s$</td>
<td>129.35</td>
<td>$I_s = \exp[2.93. B_q/(1-B_q)]$</td>
</tr>
<tr>
<td>$\varphi'$</td>
<td>30°</td>
<td>Larsson &amp; Åhnberg (2003)</td>
</tr>
<tr>
<td>$M_e$</td>
<td>1.20</td>
<td>$M_e = 6.\sin\varphi'/(3-\sin\varphi')$</td>
</tr>
<tr>
<td>$N_m$</td>
<td>3.14</td>
<td>Denominator of Equation 12</td>
</tr>
<tr>
<td>(Mayne, 2016)</td>
<td></td>
<td>Equation 17</td>
</tr>
<tr>
<td>$N_m$</td>
<td>10.37</td>
<td>Equation 17</td>
</tr>
<tr>
<td>(Mayne, 2016)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q_t$</td>
<td>5 to 8</td>
<td>Figure 4a</td>
</tr>
<tr>
<td>$I_c$</td>
<td>3.275</td>
<td>Figure 4a and Table 1</td>
</tr>
<tr>
<td>$m'$</td>
<td>0.9986</td>
<td>Equation 15</td>
</tr>
<tr>
<td>$m'$ adopted</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$N_m$</td>
<td>3.0</td>
<td>As $m' = 1, N_m = 1/0.33$</td>
</tr>
<tr>
<td>(Mayne, 2017)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*From $\gamma_n$ vs. z graph between elevations +3 and -23 m for Point S9.

4.3.4.2 Undrained shear strength

Figure 14 shows the results of applying the methods of Massad (2009, 2010, 2016) and Mayne (2016) and the VT data performed at the test site.

The curve for the application of the Massad’s method (2009, 2010, 2016) was closer to the VT data when compared to the curve of the Mayne’s method (2016).

There is a large difference between the $N_{tu}$ values obtained by the Mayne’s (2016) and Massad’s (2009, 2010, 2016) methods, 10.4 and 21.5, respectively. The last number is close to the upper limit indicated by Senneset et al. (1989) for overconsolidated clays, as mentioned above.

The $S_u/\sigma_a'$ relationship obtained by the methods was quite different: 0.13 < $S_u/\sigma_a'$ < 0.15 by the Massad’s method (2009, 2010, 2016) and 0.27 < $S_u/\sigma_a'$ < 0.32 by the Mayne’s method (2016), both in terms of pore pressure and in terms of cone tip resistance. To determine $S_u/\sigma_a'$, by the correlation of Mayne & Mitchell (1988), Equation 19, the value of $I_s$ was estimated between 40 and 56%, based on $I_s$ data presented by Larsson & Åhnberg (2003) for the elevations of interest (between +3 and -23 m), resulting $0.29 < S_u/\sigma_a' < 0.34$.

It is interesting to present the studies by Larsson & Åhnberg (2003) regarding the $S_u/\sigma_a'$ relationship. The authors proposed an empirical correlation based on direct shear tests data performed on Torp Clay samples that indicated the trend given, mathematically, by:

$$\frac{S_u}{\sigma_a'} = a^* \cdot OCR^{b^* - 1}$$  \hspace{1cm} (25)

where $a^* = 0.22$ and $b^* = 0.8$.

For the studied clay layer, $OCR$ varies between 1.3 and 3.2, so that, from Equation 25, it follows $0.17 < S_u/\sigma_a' < 0.21$, therefore, greater than the mean value of 0.14 obtained by Massad’s method (2009, 2010, 2016), but far below the mean values of 0.29 and 0.32 of Mayne’s (2016) method and Mayne & Mitchell’s (1988) correlation (Equation 19), respectively. This inconsistency was widely discussed by Larsson & Åhnberg (2005).

5. Conclusions

For all case studies, the curves obtained from $\sigma_a' = r(\sigma_{vo} + \Delta p)$, calculated with knowledge of geological history, considering preloading and ageing mechanisms, had a close approximation with the available oedometer test data.

It was observed that, in general, the application of the Massad’s method (2009, 2010, 2016), both to estimate $\sigma_a'$ and $S_u$, led to results consistent with those obtained through specific tests and with the geological history of the deposits. For all studied marine clays, the application of the Mayne’s
method (2017) led to overconsolidation slightly higher than expected by the same verifications mentioned above. For the Mayne’s method (2016), it was noticed that extreme values of \( B_q \) greatly affected the results, impairing the analyses and it presented better agreement for two clays, in terms of \( \sigma'_a \), and for one clay, in terms of \( S_u \).

The variability of results by different methods on different clays evidences that the use of semi-empirical methods to estimate geotechnical parameters provides a reduction in the number of specific tests required, but do not replace them, because they are essential for validation purposes, considering the knowledge of geological history of the test site.

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Declaration of interest

The authors have no conflicts of interest to declare. All co-authors have observed and affirmed the contents of the paper and there is no financial interest to report.

Authors’ contributions

Danielle Caroline Ferreira: Conceptualization, Data curation, Methodology, Validation, Writing – original draft, review and editing, Visualization. Faiçal Massad: Conceptualization, Data acquisition, Methodology, Supervision. Validation, Writing – review and editing.

List of symbols

- **a**: Cone tip resistance at the surface
- **ave**: Average
- **a***: Soil constant proposed by Larsson & Åhnberg (2003)
- **b**: Cone tip resistance rate of increase with depth
- **b***: Soil constant proposed by Larsson & Åhnberg (2003)
- **B_q**: Pore pressure parameter
- **c_0**: Undrained shear strength at the surface
- **c_1**: Undrained shear strength rate of increase with depth
- **C_c**: Virgin compression index
- **CPT**: Cone Penetration Test
- **CPTu**: Piezocone Test
- **C_{r_e}**: Recompression index
- **C_v**: Vertical coefficient of primary compression
- **C_{ae}**: Vertical coefficient of secondary compression in function of void ratio variation
- **C_{ae}**: Vertical coefficient of secondary compression
- **DMT**: Dilatometer Test
- **e_0**: Initial void ratio
- **F_{r_e}**: Normalized friction ratio
- **H_d**: Drainage height
- **I_c**: CPT index
- **I_p**: Plasticity index
- **I_R**: Rigidity index
- **min**: Minimum
- **max**: Maximum
- **m’**: Exponent relative to soil type
- **M**: Frictional parameter for triaxial compression
- **N_{h1}**: Empirical factor to determine \( S_u \) in terms of \((q_t - \sigma_v^0)\)
- **N_{h0}**: Empirical factor to determine \( S_u \) in terms of \( \Delta u \)
- **N_{m}**: Empirical factor to determine \( \sigma'_a \)
- **OCR**: Over consolidation ratio
- **q_t**: Cone tip resistance
- **Q_t**: Normalized cone resistance
- **r**: Ageing effect consideration factor
- **S9**: Point of study in Torp test site
- **SBT**: Soil Behavioural Type
- **SCET**: Spherical Cavity Expansion Theory
- **SFL**: Sediments-Fluvial Lagoon-Bay
- **S_u**: Undrained Shear strength
- **t**: Time of secondary compression
- **T**: Terzaghi’s Time factor
- **t_p**: Time of primary compression
- **u_0**: Hydrostatic pressure
- **u_2**: Pore pressure measured behind the cone
- **UU**: Unconsolidated undrained triaxial test
- **VT**: Vane Test
- **z**: Depth in relation to the ground level
- **Δp**: Preloading
- **Δu**: Excess pore pressure
- **φ’**: Effective friction angle
- **γ’**: Submerged unit weight
- **γ_n**: Unit weight
- **Λ**: Plastic volumetric strain ratio
- **σ’_a**: Pre-consolidation pressure
- **σ’_{v0}**: Vertical effective pressure
- **σ_{v0}**: Total overburden pressure

References


