Soils and Rocks
An International Journal of Geotechnical and Geoenvironmental Engineering
www.soilsandrocks.com
ISSN 1980-9743
ISSN-e 2675-5475

Limitations of the Danish driving formula for short piles

Silvio Heleno de Abreu Vieira[1#], Francisco R. Lopes[2]

Technical Note

Keywords
Foundations
Piles
Dynamic formulae
Dynamic tests
Load tests

Abstract

Dynamic formulae are a widely used expedient for the control of driven piles to ensure load capacity. These formulae have considerable limitations when used in the prediction of the load capacity on their own, but are very useful in the control of a piling when combined with other tests. This technical note presents an evaluation of the Danish Formula for 54 precast concrete piles, comparing its results with High Strain Dynamic Tests (HSDTs), Static Load Tests (SLTs) and predictions by a semi-empirical static method (Aoki & Velloso, 1975). The data used in the comparison come from three works in the city of Rio de Janeiro, Brazil. All piles were driven with free-fall hammers and in one particular work the piles were relatively short. The predictions of the Danish Formula were evaluated in relation to the pile length/diameter ratio. It was concluded that for short piles - with lengths less than 30 times the diameter - this formula indicates bearing capacities higher than the actual ones. A correction for a safe use of the Danish Formula for short piles is suggested.

1. Introduction

Dynamic formulae are based on elementary laws of Physics, such as those that govern the conservation of energy or the shock between bodies. However, driving a pile is a more complex phenomenon. The pile is not a free body, but an elongated element inserted into the ground, with which it interacts under a hammer blow. An alternative to these formulae is the solution of the Wave Equation, introduced by Smith (1960).

Dynamic formulae basically require the hammer and pile data and provide the set (permanent penetration of the pile per blow). On the other hand, a Wave Equation solution requires, in addition to these data, those related to the driving accessories and the soils (in terms of rigidity, resistance and viscosity), and outputs not only the set, but also the dynamic stresses (stresses along the pile under driving). In their use to estimate pile capacity, the dynamic formulae are fed simply by the measured set, while the Wave Equation solution requires more extensive measurements of the pile response to driving, in what is called the High Strain Dynamic Test (HSDT).

The use of either of the two dynamic methods, however, pose a few questions, such as (e.g., Alonso, 1988):

(i) the energy of the hammer blow is not always sufficient to bring about the maximum resistance of the soil;
(ii) the resistance presented by the soil depends on the time between driving and the measurement of the set or the HSDT, with soil resistance usually increasing with time, hence this phenomenon being known as "set-up" (very rarely, resistance decreases over time, in this case, called "relaxation");
(iii) the energy losses in the accessories and the viscous response of soil are not properly incorporated in most dynamic formulae.

The first two aspects are inherent to any dynamic method, leading to different load capacities obtained (i) with different driving energies and (ii) with set measurements or HSDTs made at different times after driving. As a consequence of aspect (i), it is common practice to refer to load capacity obtained in HSDTs – performed with a given driving equipment – as *mobilized load capacity*, implying that a higher capacity could be obtained with a higher energy.

Despite the above issues, dynamic formulae are very useful in the control of a piling, especially if combined with HSDTs and static load tests (SLTs) − ideally executed right at the beginning of the construction −. The dynamic formulae serve to ensure homogeneity in load capacity, leading to different lengths of piles driven in heterogeneous soils.

[#] Corresponding author. E-mail address: silvioheleno@gmail.com
[1] Vale, Nova Lima, MG, Brasil.
[2] Universidade Federal do Rio de Janeiro, Rio de Janeiro, RJ, Brasil.

Submitted on September 5, 2020; Final Acceptance on March 24, 2021; Discussion open until August 31, 2021.
https://doi.org/10.28927/SR.2021.057320

There are more than one hundred formulae. In the evaluation of Agerschou (1962), the Dutch and the Engineering News Record Formulae presented values with a very large scatter, therefore, were considered unreliable. The Hiley, Janbu and Danish Formulae showed close and reliable values. In the review by Poulos & Davis (1980), the Engineering News Record Formula was considered to be unreliable, with a weak correlation with load test results, while the Janbu and Danish Formulae presented a good correlation. Likins et al. (2012) summarizes the discussions that followed a report by the ASCE "Committee on Pile Driving Formulas and Tests", published in the 1940s after a decade-long study (Greulich, 1941). In these discussion, several very prominent engineers expressed opposing views on the formulae available at the time.

The Danish Formula has been widely used worldwide, both for steel and precast concrete piles. In the evaluation of Danziger & Ferreira (2000), this formula presented a good correlation with results of a Wave Equation solution for steel piles. In another evaluation (Silva et al., 2020), the Danish Formula proved to be suitable for the control of a large piling.

The present technical note aims to evaluate the Danish Formula in the verification of the load capacity of precast concrete piles. Comparisons are made with HSDTs, SLTs and predictions by a semi-empirical static method (based on SPT results). For the latter type of comparison, the Aoki & Velloso (1975) method was chosen for its common use in Brazilian practice. The predictive capacity of this formula is evaluated in particular in relation to the pile length, resulting in the recommendation of a range for its safe use.

2. Construction data used in the study

Data from three works in the city of Rio de Janeiro were used: Metallurgical Laboratory of the Federal University of Rio de Janeiro (UFRJ), located in Fundão Island, Athletes Village for the Pan-American Games (Vila Pan-Americana) and Shopping MAP-Car, the last two located in Barra da Tijuca. Altogether there are 54 precast concrete piles, some with a hollow circular cross-section and others with a full square section. The piles passed through different sediments and had their tips driven into gneissic residual soil. Site investigations were conducted with SPTs.

The present evaluation used pile set measurements, HSDTs and SLTs, as summarized in Table 1 (with more details can be seen in Vieira, 2006). The piles had very different lengths, which allowed an evaluation of this effect.

3. A preliminary evaluation of the Danish Formula

According to Sorensen & Hansen (1957), the pile driving resistance depends on 5 factors: the pile driver efficiency (η); hammer weight (W); hammer drop height (h); set or permanent penetration of the pile per blow (s); pile length (L); pile cross-section area (A) and pile modulus of elasticity (E_p). The driving resistance is given by (Danish Formula):

$$R_{Dan} = \frac{\eta\ W\ h}{s + \frac{1}{2}\sqrt{\frac{2\ \eta\ W\ h\ L}{A\ E_p}}} \tag{1}$$

The second part of the denominator corresponds to the elastic (recoverable) compression of the pile under the energy of the blow. The authors suggest an efficiency factor of the driving system equal to 0.70 for free-fall hammers and 0.90 for diesel hammers.

The set and the cross-section area have their influences on driving resistance easy to perceive in the formula. On the other hand, an increase in its cross-section is known to require greater energy to drive the pile into the ground.

Figure 1 shows, for a pile with a cross-section area of 895 cm^2 (for example, a hollow pile 42 cm in diameter, 10 cm thick wall), how the driving resistance varies with pile length and net energy, considering two final sets: 0 and 3mm/blow (or 0 and 30 mm/10 blows). It can be observed that the load capacity is influenced by pile length and that, for relatively short piles, driving resistance varies very sharply as the set varies.

It can be observed in Figure 1 that short piles with small sets (sets that approach 0) would have exceptionally high resistances, according to the formula; furthermore, as pile length tends to 0, driving resistance tends to infinity. For piles with lengths greater than 10 m (typically 30 diameters in this case), the variation in set has little effect on driving resistance, as does driving energy.

Table 1. Summary of pile data.

Work	Number of Piles	HSDTs	SLTs	Pile length (m)
Metallurgical Lab.	20	7	-	3.70 to 8.40
Vila Pan-Americana	11	10	4	23.00 to 32.20
Shopping MAP-Car	23	7	-	20.50 to 22.20

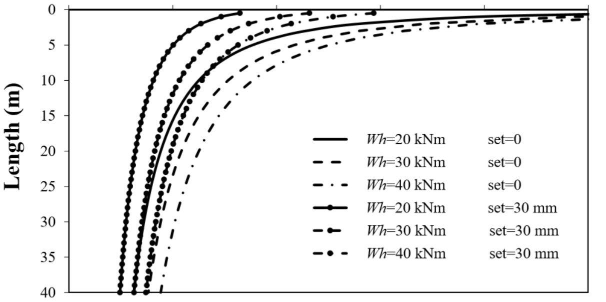

Figure 1. Pile resistance variation with pile length and set by the Danish Formula (Vieira, 2006).

4. Evaluation of the Danish Formula in three foundation works

4.1 Important clarifications and assumptions

In this section, Danish Formula results will be compared to results of HSDTs, SLTs and a static method (Aoki-Velloso method). In the interpretation of a HSDTs it is possible to separate the dynamic resistance and the static resistance (e.g., by the CAPWAP method), the latter corresponding to the static load capacity. However, as HSDTs are often made during the driving process or shortly after, the static resistance usually increases over time, due to set-up, until reaching the load capacity of a SLT or a prediction by static method, Q_{ult}. Thus, it is common practice to use the notation R_u for the static resistance obtained in the HSDT. If there were no set-up, $R_u = Q_{ult}$.

The Danish Formula does not provide, strictly speaking, the static load capacity, Q_{ult}, but the driving resistance, R_{Dan}. To obtain the service load, Sorensen & Hansen (1957) recommend a factor of 2.0 (i.e., $Q_{ser} = R_{Dan} / 2.0$). Since the overall safety factor to be applied to the static load capacity to obtain the service load is 2.0, it can be assumed that the driving resistance given by the formula corresponds to the static load capacity, Q_{ult}.

In the analysis of the data from three foundation works using the Danish Formula, the piles were supposed to have an excess of 2.0 m in length at the end of the driving, that is, above ground level, which is common (later demolished). The hammer efficiency factor, η, was based on the net energies measured in the HSDTs, and were: 0.80 for the Metallurgical Laboratory, 0.70 for the Vila Pan-Americana and 0.60 for the Shopping MAP-Car.

4.2 Comparison between Danish Formula and HSDT results

Figure 2 shows that, for short piles (Metallurgical Laboratory), the Danish Formula indicates load capacities much higher than those measured in HSDTs; the ratio between these values varied between 1.75 and 3.95. On the other hand, for long piles (Shopping MAP-Car and Vila Pan-Americana), values obtained by HSDTs were higher.

Figure 3 shows that, for relatively short piles, the Danish Formula indicates load capacities much higher than those predicted by the Aoki-Velloso static method. The ratio between these values varied between from 1.81 to 3.83. For long piles, load capacities were close, with a ratio ranging from 0.75 to 1.67. Two piles in that figure were not included in this evaluation, as they had a very low load capacity predicted by the static method, probably due a flaw in the SPT.

From Figures 2 and 3, it can be concluded that the Danish Formula greatly overestimates the driving resistance or load capacity of relatively short piles.

Figure 4 shows in more detail the relation between load capacities indicated by the Danish Formula and by

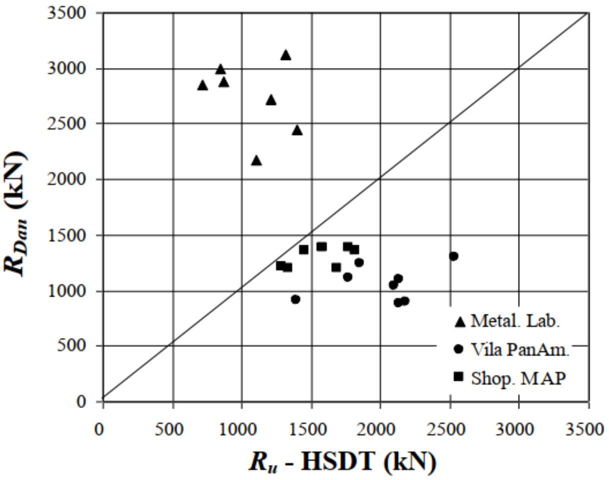

Figure 2. Danish Formula vs. HSDTs – all piles.

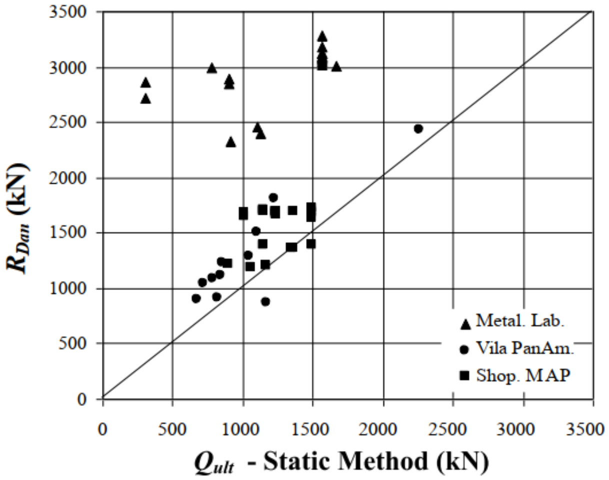

Figure 3. Danish Formula vs. static method (Aoki-Velloso).

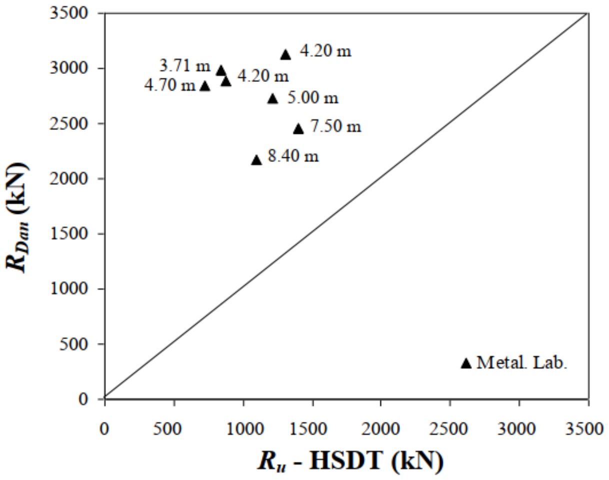

Figure 4. Danish Formula vs. HSDTs - Metallurgical Laboratory.

HSDTs for short piles in the Metallurgical Laboratory. The numbers next to the dots indicate the pile lengths in meters. Note that, as the piles are longer, the results of the dynamic formula approach those of the HSDT.

Figures 5 show how the ratio between the load capacities predicted by the Danish Formula and HSDTs varies with the aspect ratio of the pile (*L/B*). In this figure, a trend curve − with solid line − was fitted, showing how the predictability of the Danish Formula is influenced by pile length. A second line – dashed – was introduced, suggesting that this formula can be used for pile lengths greater than 30 diameters, and that its results need adjustments for lengths below this value.

The ratio between load capacities indicated by the Danish Formula and the HSDT for the Metallurgical Laboratory, with shorter piles, varied between 0.25 to 0.57, while for the Vila Pan-Americana, with longer piles, the ratio varied between 1.50 to 2.45. For MAP-Car, with long piles (but not so much as in the Vila Pan-Americana), the ratio remained between 1.06 and 1.41.

The data in Figure 5 indicates that piles with up to 30 diameters need some adjustment in the application of the Danish Formula − and these piles will be considered here as relatively short −. Based on this figure, it is suggested that the Danish Formula needs the following correction for safe use if $L/B < 30$ (assuming $Q_{ult} = R_{Dan}$ for longer piles):

$$Q_{ult} = 0.033 R_{Dan} \left(L / B \right) \tag{2}$$

4.3 Comparison between dynamic and static methods

Figure 6 shows the results of 4 Static Load Tests (SLTs) compared to those of the Danish Formula. These static tests were performed on long piles in the Vila Pan-Americana, where results of the Danish Formula were lower than those obtained in the HSDTs. The failure loads in the SLTs are higher than those obtained by the Danish Formula, a possible explanation being the occurrence of a significant set-up after driving.

Figure 7 presents a comparison between static load capacities obtained in HSDTs and given by the Aoki-Velloso static method. In the Metallurgical Lab., where piles were shorter, HSDTs static load capacities are close to those provided by the static method. At the other two sites, where long piles passed through very soft clay layers, HSDTs results are higher than those of the Aoki-Velloso method, most likely because the latter does not consider any shaft friction in clays with $N_{SPT} = 0$. In fact, a skin friction of about 10 kN/m^2 develops in soft clays after consolidation which follows pile driving (e.g., Décourt & Quaresma, 1978).

Figure 8 presents a comparison between static load capacities obtained in HSDTs and SLTs. These tests were carried out on long piles at Vila Pan-Americana. Two HSDTs results were close to those obtained in SLTs, while another was considerably lower.

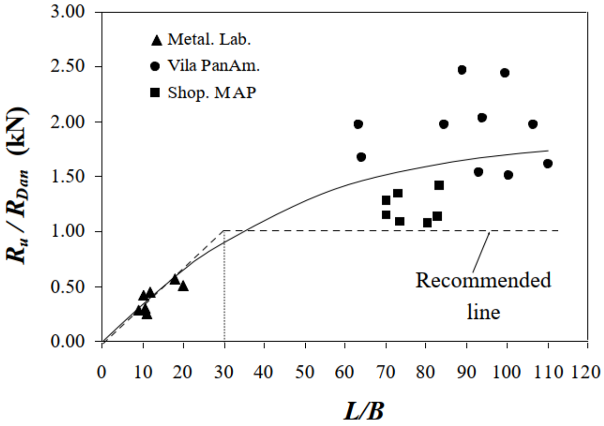

Figure 5. Predictive capacity of the Danish Formula as a function of pile aspect ratio (*L/B*).

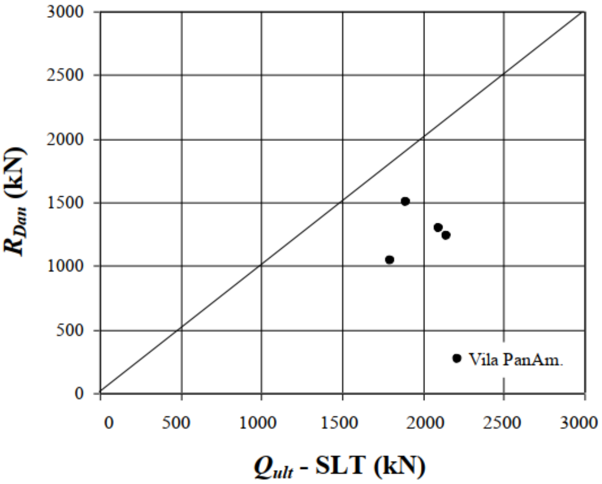

Figure 6. Danish Formula vs. SLTs - Vila Pan-Americana.

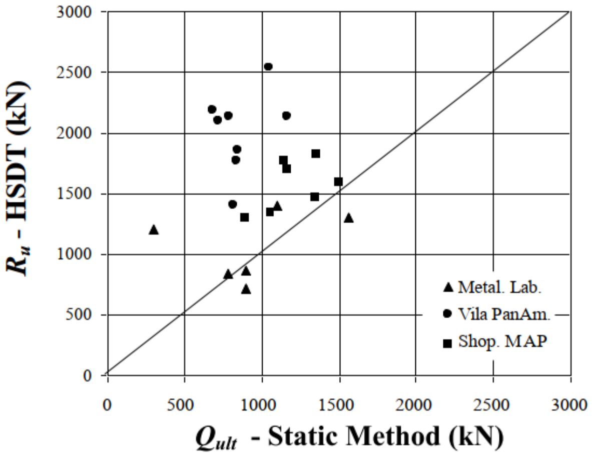

Figure 7. Static method (Aoki-Velloso) vs. HSDTs.

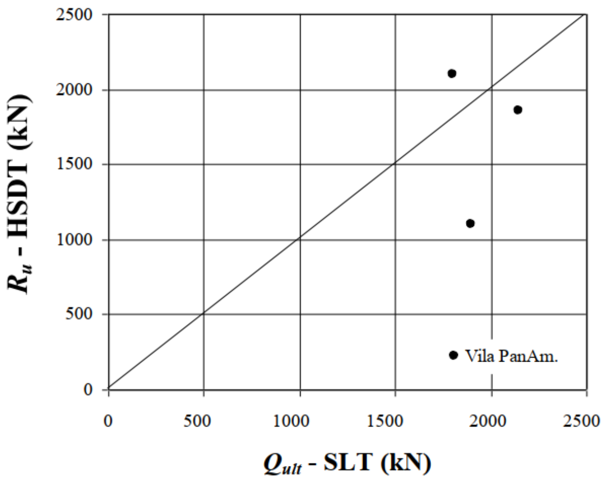

Figure 8. HSDTs vs. SLTs.

5. Concluding remarks

The development of quality control techniques based on measurements of pile response to driving should be encouraged, either by the simple set measurements or by the acquisition of more complete data in a dynamic test HSDTs. In any dynamic method, special attention should be given to the question of soil recovery after driving – set-up –, a phenomenon capable of considerably altering the load capacity of driven piles, especially in fine grained soils. The assessment of set-up can be made by carrying out HSDTs or even set measurements on two or three occasions after driving.

The comparison of the pile load capacities indicated by the Danish Formula with those of other methods, in particular HSDTs, showed that this formula overestimates the load capacity of relatively short piles (length less than 30 times the diameter). For longer piles, the results of the Danish Formula were consistent with those of dynamic and static tests (HSDTs and SLTs). The results of the present paper for this particular formula must be confirmed with a larger data base.

In view of the natural heterogeneity of the subsoil, the control of a piling through set measurements and dynamic formulae is quite efficient to ensure homogeneity in terms of load capacity. However, the use of these formulae should be restricted to the piling control process and not as a predictive tool. The best practice suggestion is: (i) prediction of pile depths by static methods (based on SPT, CPT, etc.) in the design stage, (ii) confirmation of pile depths and capacities during actual construction by HSDTs and SLTs, performed right at the beginning of the works, and (iii) adjustment of the selected dynamic formula (using measured pile response to driving – set – and capacities) to control the homogeneity of the pilling.

Acknowledgements

Thanks are due to Prof. Bernadete Ragoni Danziger, Rio de Janeiro State University, for her kind review of the manuscript.

Declaration of interest

There is no conflict of interests in the material presented.

Author’s contributions

Silvio Heleno de Abreu Vieira: Conceptualization, Methodology, Formal analysis. Francisco R. Lopes: Discussion of results, Writing - Reviewing and Editing.

List of symbols

A = pile cross-section area
B = pile cross dimension (diameter if circular pile)
E_p = modulus of elasticity of the pile material
h = hammer drop height
η = efficiency factor of the driving system
L = pile length
Q_{ult} = pile (static) load capacity
Q_{ser} = pile service load
R_{Dan} = pile driving resistance by the Danish Formula
R_u = static load capacity or static resistance obtained in HSDT
s = pile set (permanent penetration per blow)
W = weight of hammer
HSDT = High Strain Dynamic Test
SLT = Static Load Test
CAPWAP = Case Pile Wave Analysis Program (Pile Dynamics, Inc.)

References

Agerschou, H.A. (1962). Analysis of the engineering pile formula. *Journal of the Soil Mechanics and Foundations Division*, 88(5), 1-11. http://dx.doi.org/10.1061/JSFEAQ.0000450.

Alonso, U.R. (1988). *Previsão e controle das fundações* (1. ed.). São Paulo: Edgar Blucher.

Aoki, N., & Velloso, D.A. (1975). An approximate method to estimate the bearing capacity of piles. In *Proceedings of the 5th Panamerican Conference on Soil Mechanics and Foundation Engineering* (pp. 367-376), Buenos Aires.

Danziger, B.R., & Ferreira, J.S. (2000). Back-analyses of steel pile driving records for quality assurance. In *Proceedings of the International Conference on the Application of Stress-Wave Theory to Piles* (pp. 657-663), São Paulo.

Décourt, L., & Quaresma, A.R. (1978). Bearing capacity of piles from SPT values. In *Proceedings of the 6th Brazilian Conference on Soil Mechanics and Foundation Engineering* (pp. 45-53). Rio de Janeiro.

Greulich, G.G. (1941). Progress report of the committee on the bearing value of pile foundations. *Proceedings of the American Society of Civil Engineers*, 67(7), 1391-1396.

Likins, G.E., Fellenius, B.H., & Holtz, R.D. (2012). Pile driving formulas: past and present. In *Proceedings of the GeoCongress 2012* (pp. 737-753), Oakland, California. http://dx.doi.org/10.1061/9780784412084.0051.

Poulos, H.G., & Davis, E.H. (1980). *Pile foundation analysis and design.* New York: John Willey.

Silva, E.R., Danziger, B.R., & Pacheco, M.P. (2020). Comparação entre critérios de controle de estacas cravadas. *Geotecnia*, 149, 3-16. http://dx.doi.org/10.24849/j.geot.2020.149.01.

Smith, E.A.L. (1960). Pile-driving analysis by the wave equation. *Journal of the Soil Mechanics and Foundations Division*, 86(4), 35-61. http://dx.doi.org/10.1061/JSFEAQ.0000281.

Sorensen, T., & Hansen, J.B. (1957). Pile driving formulae, an investigation based on dimensional considerations and a statistical analysis. In *Proceedings of the 4th International Conference on Soil Mechanics and Foundation Engineering* (pp. 61-65), London.

Vieira, S.H.A. (2006). *Controle da cravação de estacas pré- moldadas: avaliação de diagramas de cravação e fórmulas dinâmicas* [MSc dissertation]. Federal University of Rio de Janeiro.