Analogy to the chaos theory applied to the study of rockfalls

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Article

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Abstract

Chaos Theory is a mathematical theory devoted to study dynamic systems presenting very peculiar characteristics – sensitivity to initial conditions, positive or close to zero Lyapunov exponents, statistics governed by gaussian or non-gaussian distributions, among others - which make them, in the long run, unpredictable in time and space. This article aims at applying Chaos Theory to rockfall phenomenon. More precisely, the fall of unstable rock blocks was simulated through the RocFall 6.0 program by four preliminary case studies, having different rock slope geometry, different heights of the fall and blocks with different size and shapes. Moreover, the trajectories and reaches of gneissic rock blocks in a section of a phacoidal augen gneiss slope located in Morro do Cantagalo, in the city of Rio de Janeiro, were also simulated from the perspective of Chaos Theory. More precisely, the results suggest that the statistics of the number of fallen blocks at each end point of the trajectories located downstream of the respective slopes can be described by distributions derived from Chaos Theory. In addition, weakly or strongly chaotic behavior seems to be very specially associated with the concavity or convexity of the slopes.

1. Contextualization

Rio de Janeiro is inserted in an environment of rugged relief, with heterogeneous and discontinuous rock masses, and humid tropical climate, marked by manifestations of different types of mass movements. These phenomena, combined with the disorderly urban growth of cities and the absence of planning linked to infrastructure, lead to serious socioeconomic consequences (Menezes Filho, 1993; Ignacio, 2019).

Rockfalls have been considered an extremely important phenomenon due to its high sensitivity to the initial conditions of the unstable blocks, usually involving high kinetic energies and occurring abruptly, which turns out to be difficult to forecast. In order to prevent this type of movement, several rockfall prediction methods (Back analysis of events, *in situ* tests, laboratory tests, Fahrböschung principle, Minimum shadow angle and numerical methods, for example) have been developed and improved over the years. Based on observations and records of real cases, most of them aim at identifying its main characteristics, enabling to subsidize the selection and dimensioning of coexistence and mitigation measures (Evans & Hungr, 1993; Rocha, 2009; Gálvez, 2012; Vijayakumar et al., 2012; Spadari et al., 2012; Tavares, 2015).

However, a common characteristic of those methods is their very restricted application for global analysis of the movement, making the delimitation of potential areas to

be eventually affected and parameter estimation seriously impaired. This is mainly due to the non-linearity found in the block falling process and, consequently, to the difficulty of understanding the movement mechanism and its particularities (Ignacio, 2019; Tavares, 2015).

The peculiarities of chaotic systems make them difficult to analyze, for the rockfall phenomena manifestly depend on the launching initial conditions and the impact points along the slope. In addition, several other parameters influence the final response, such as the geometric shape of the slope and the block, the dimensionality of the system and the restitution coefficient of the materials involved in the successive shocks. Such restrictions naturally lead to a probabilistic-statistical approach of investigation, in which the search for regularities in the phenomenon, in particular the final position of the blocks downstream of the slope and their associated probabilistic distributions, can bring a new understanding of the process of instability (Rocha, 2009; Freitas, 2013; Tavares, 2015).

The fall process and the successive shocks of the blocks against the slopes can be viewed as an exchange of information within the unstable system itself, thus changing its entropy over time. In this sense, it is known that the Boltzmann-Gibbs entropy is the one that most adequately describes the evolution of strongly chaotic dynamic systems, having distinctive characteristics, among others, short-range

spatial and temporal correlations (or even absence of memory), ergodicity, markedly positive Lyapunov exponent, strong "mixing" in the phase space and compliance with the Central Limit Theorem. The probability distribution that maximizes this entropy is the Gaussian distribution (Equation 1 - where a, b and c are fitting parameters) (Beck & Schlögl, 1993; Tsallis, 1988, 2009).

Systems that present some of the above-mentioned characteristics are the following:

a. The classic Boltzmann gas, an idealized one where the particles move freely inside a stationary recipient, without interacting with one another – except for very brief collisions, where energy and momentum are exchanged with each other – is a paradigm of strongly chaotic system. It presents, for the velocity distribution of its particles, a Maxwell-Boltzmann (Gaussian) distribution (Sears, 1963);

b. Non–linear iterated maps are dynamical systems whose iterative processes are measured in discrete intervals (years, generations etc). One of these famous maps is the 'logistic model of populational growth', also known as 'logistic map'. It shows Gaussian distributions in the regions characterized by positive Lyapunov exponent (i.é, strongly chaotic regions) (Peak & Frame, 1994);

c. Some strongly chaotic systems appear in certain classes of billiard problems. A billiard is a two-dimensional planar region in which a particle moves subjected to a constant velocity along a straight line trajectory between successive bounces from the boundary of the domain. In a special kind of billiard proposed by the Russian mathematician Yakov Sinai (called 'Sinai billiards'), characterized by having convex domains (arcs of circles facing inwards to the interior domain), it is impossible to foresee the exact trajectories of nearby particles after the impact onto the boundaries. The origin of chaos is intuitively clear in this case, for the curvature of the boundary has a dispersing effect on two parallel trajectories, leading to a rapid separation of nearby trajectories after a few bounces. Furthermore, the velocity distribution of the particles turned out to be Gaussian (Sinai, 1970).

On the other hand, weakly chaotic systems - also called 'complex systems'- are described by 'generalized non-additive entropies', among which those proposed by Tsallis, such as S_q, S_δ and $S_{q,\delta}$. This time, dynamic systems are characterized by long-range spatial and temporal correlations (long-range memory), broken ergodicity, Lyapunov exponent close to zero, weak "mixing" in the phase space, some type of energy dissipation, and non-compliance to the Central Limit Theorem (Landsberg, 1999; Tsallis, 1988, 2009).

Systems that show some of the above-refered characteristics are the following:

a. Certain evolutionary dynamic systems spontaneously arrange themselves, after a long period of time, in a state of 'self-organized criticality', that is, a state of weak chaos characterized by the development of self-similar (fractal) spatial and temporal patterns. That is the case, for example, of the time evolution of elasto-plastic models of geologic materials analyzed under dynamic relaxation. Besides generating fractal fracture patterns, the equivalent plastic deformation time series show their respective Lyapunov exponents very close to zero (Menezes Filho, 2003);

b. Non-linear iterated maps at the primary edge of chaos show non-gaussian statistics, that is, Extended q-Exponentials with the exponent δ = 1 in Equation 2 (Tsallis, 2009);

c. The 'Stadium Billiard', discovered by the Russian mathematician Leonid Bunimovich, is a two-dimensional planar billiard having concave domains (arcs of circle facing outward to the interior domain) linked by two parallel straight lines. This non-dispersing billiard shows, for certain particle orbits, a power law divergence of trajectories, with zero Lyapunov exponent (Bunimovich, 1979).

Therefore, weakly (strongly) chaotic systems show very specific characteristics: statistics governed by non-Gaussian (Gaussian) distributions, interacting (non-interacting or interacting for very brief moments) particles and a power-law (exponential) separation of trajectories, among others.

The reader will see further that, applied to the rockfall phenomena, those cited concepts lead to a very rich spectrum of questions: the identification of strong or weak chaotic phenomena through the probability distributions of the number of fallen blocks arrived at certain points downstream of the slope and their gaussianity or not, as well; the geometry of the rock slopes – concave or convex – and a possible analogy with Sinai and Bunimovich billiards; the rapid (exponential) or slow (power-law) divergence of trajectories in convex or concave slopes, due to their dispersing (diverging) or non-dispersing (converging) properties.

It should be pointed out that the application of generalized entropies to natural hazard phenomena – e.g. earthquakes - is quite recent. The researchers have been focusing on the statistics of foreshocks and aftershocks time signals, aiming at predicting these very intrincated phenomenon (Kalimeri et al., 2008; Papadimitriou et al., 2008; Eftaxias, 2010; Papadakis et al., 2015; Vallianatos, 2016).

However, the Authors of this paper are, to the best of their knowledge, unaware of any previous application of Chaos Theory and probability distributions derived from generalized entropies to the rockfall phenomenon in the technical literature.

The optimization of those parametric nonadditive entropies furnishes non-Gaussian probability distributions, among which the Extended q-Exponential distribution (Equation 2 - where a', b', c', q and δ are fitting parameters, q is the entropic parameter and δ is an adjustment parameter) adopted in this research. It should be noted that the parameters

q ($-\infty < q \leq 3$) and δ, when approaching unity, transform the above generalized distribution into Gaussian (Tsallis, 2009; Menezes Filho, 2003; Tsallis & Cirto, 2013).

$$p(x)_g = a\left[e^{\left[-\left(\frac{x-c}{b}\right)^2\right]}\right] \tag{1}$$

$$p(x)_{q,s} = a'\left[1 - (1-q)\left(\frac{x'-c'}{b'}\right)^{\frac{2}{\tilde{a}}}\right]^{\frac{1}{1-q}} = a'\, e_q^{-\left[\left(\frac{x-c'}{b'}\right)^2\right]^{\frac{1}{\tilde{a}}}} \tag{2}$$

Here $e_q^{-x} \equiv [1 - (1-q)x]^{\frac{1}{1-q}}$ ($q \in R$) is the generalized exponential function, whose inverse is the generalized logarithmic function, defined by $\ln_q x \equiv \left(\frac{x^{1-q}-1}{1-q}\right)$ ($q \in R$). The above probabilistic distributions derived from the optimization (maximization) of generalized entropic forms, make it possible to describe random-dependent phenomena, enabling to solve several practical problems in highly and weakly chaotic nonlinear systems, such as rockfalls along a slope.

Thus, the present article aims at contributing to a better understanding of the fall behavior, through an interdisciplinary analysis, associating Chaos Theory with the fall movement of rock blocks.

2. Materials and methods

To compose the study, four sections of gneissic (augen) rock slopes (C1, C2, C3 and C4) and the case of study were initially modeled with distinct concave and convex geometries, as shown in Figures 1, 2, 3, 4, and 5, respectively, in the RocFall 6.0 program. For each of these sections, typical blocks were modeled, having the same irregular geometric shape and different sizes, hereafter refered to as 'Small Irregular Block' (BIP), 'Medium Irregular Block' (BIM) and 'Large Irregular Block' (BIG).

For a given geometry of the slopes and the blocks, 100,000 simulations of the falls of these BIP, BIM and BIG blocks were performed in each of the proposed sections. A high number of simulations is mandatory in order to get robust statistics of the final block location points downstream of the slopes, since the number of blocks per each final position is the most relevant distribution for understanding the phenomenon.

In all cases, the Rigid Body Method was selected in the program, in which the unstable blocks were supposed not to be deformable. Moreover, use was made of Monte Carlo statistical-probabilistic sampling of the origin of launching points, generating results that are more realistic.

The histograms of the number of blocks per each final location points downstream of the slopes were adjusted to Gaussian and Extended q-Exponential distributions, according to their strongly or weakly chaotic behavior, derived, as refered, from the Boltzmann-Gibbs and the $S_{q,\delta}$ entropies, respectively. Afterwards, the process carried out in the preliminary study

was also applied to the case of the Cantagalo Massif – RJ cut, in one of the sections mapped by GEO-RIO in 2009 during the field research work for the implementation of impact flexible barriers that took place in 2011.
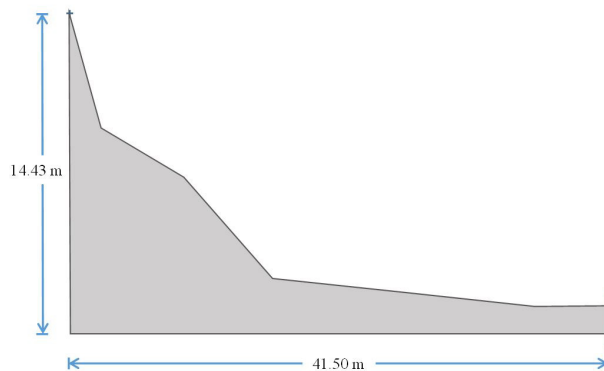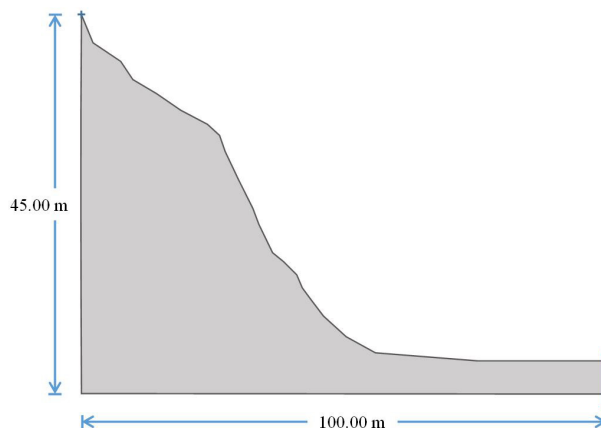

Figure 1. Case C1 geometry (not to scale).

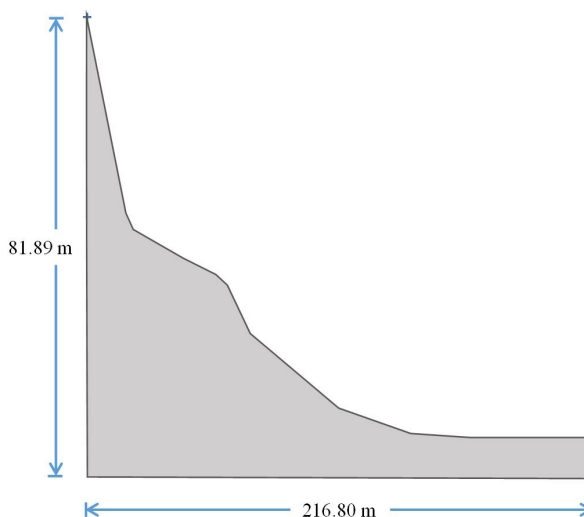

Figure 2. Case C2 geometry (not to scale).

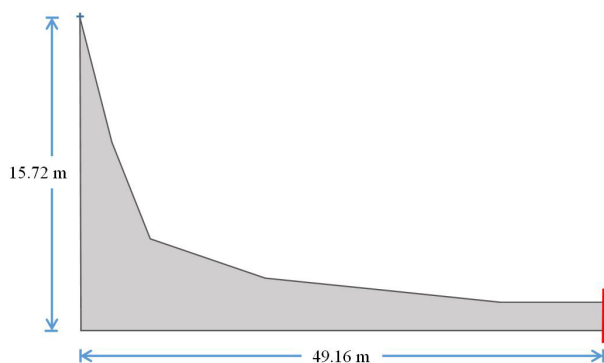**Figure 3.** Case C3 geometry (not to scale).

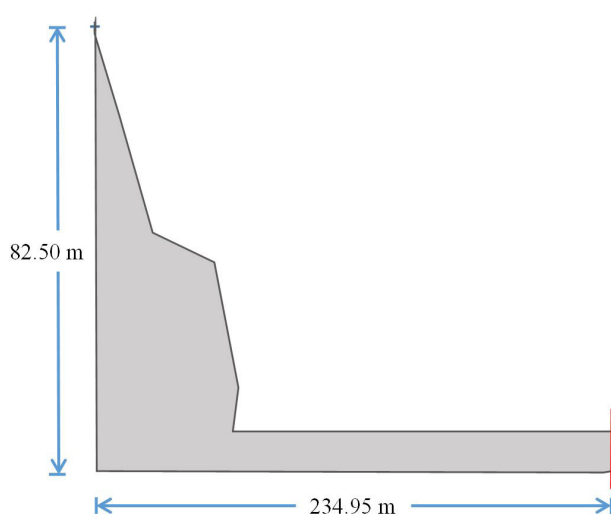**Figure 4.** Case C4 geometry (not to scale).

Figure 5. Case of study geometry (not to scale).

The geotechnical parameters of the phacoidal (augen) gneissic slopes and their respective rock blocks (normal and tangential restitution coefficients, dynamic or kinetic friction and rolling friction) were selected from Menezes Filho (1993), Rocha (2009) and Pelizoni (2014), as well as technical studies provided by GEO-RIO in 2009 at Cantagalo Massif. Table 1 shows the parameters required by the RocFall 6.0 program for the geotechnical modeling of the sections of the rock slopes.

The shape of the BIP, BIM and BIG rock blocks and the characteristics of each one of them are shown in Table 2. It should be noted that 'approximated size' refers to the edge of the larger size of the irregular block, whose area served as basis for calculating the mass of the blocks according to their specific weight.

All the geometries selected for the slope sections have an elongated base, perfectly articulated to their base level, therefore accommodating all trajectories developed by the fallen rock blocks. In addition, their lauching points are fixed to be the highest in all sections. It was also chosen to investigate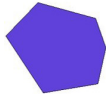 the quality of the adherence of the probability distributions to the histogram points in the region of their tail. So, graphs with the vertical axis on a logarithmic decimal scale were made for each slope section and specific block size.

In addition, for each block size, the relation Ln_q $(P(x)/a')$ versus $((x-c')/b')^{(2/\delta)}$ was also plotted, in order to test the functional universality of the Extended q-Exponential - and its Gaussian counterpart - in describing the statistics of the unstable blocks per each of their final positions downstream of the slopes. Figure 6 shows the flowchart of the methodology adopted.

Table 1. RocFall 6.0 modeling parameters – C1, C2, C3, C4 and case study.

Rock material	Normal restitution coefficient	Tangential restitution coefficient	Dynamic friction	Rolling friction
Augen gneiss	0.35	0.85	0.5	0.15

Table 2. RocFall 6.0 modeling parameters – BIP, BIM and BIG

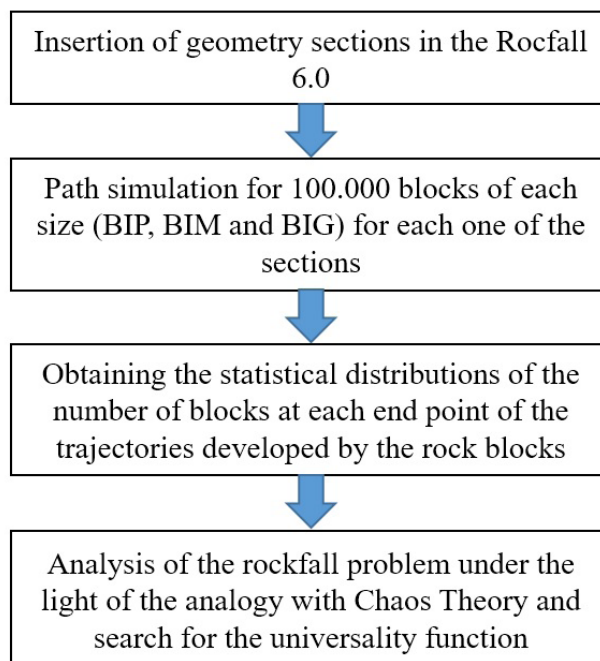| Block type	Representative format	Approx. size (m)	Mass (kg)	Specific weight (kN/m³)
BIP		0.5	337.50	27.00
BIM		1.00	2,700.00	27.00
BIG		1.40	7,408.80	27.00

**Figure 6.** Flowchart of the methodology adopted.

3. Results

3.1 Preliminary study (Cases C1, C2, C3 and C4)

Figures 7, 8, 9, and 10 show the statistics of the number of fallen blocks per each final position downstream of the slopes C1, C2, C3 and C4, plotted in natural and semi-log scales (dot points). As previously explained in the text, the semi-log plot allows a better visualization of the tail of the distributions, a place of occurrence of rare events. Besides, it is also shown the fitting of Gaussian (blue continuous line) and Extended q-exponential (red continuous line) probability distributions. For the each case, the correlation coefficients (R) are also presented.

3.2 Case study

The C1, C2, C3 and C4 geometries previously presented made it possible to apply the same methodology to the case study of Cantagalo Massif – RJ, in which there is a real mapping of the slope, confering greater veracity to the data and analyzes found in the preliminary study.

The Cantagalo Massif is located in the Southern Zone of the City of Rio de Janeiro, specifically between the neighborhoods of Copacabana and Ipanema. On the face of the hill around Professor Gastão Bahiana Street, close to Barata Ribeiro Street and Mayor Sá Freire Alvim Tunnel,
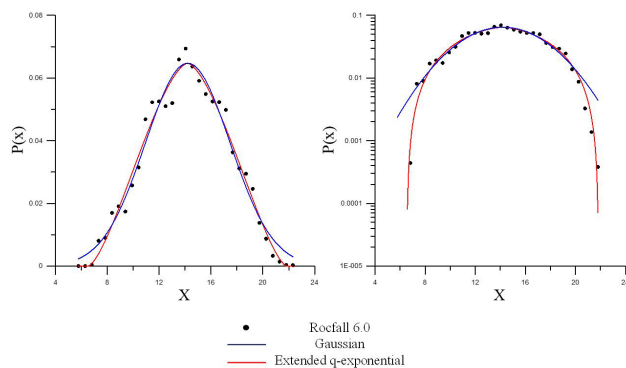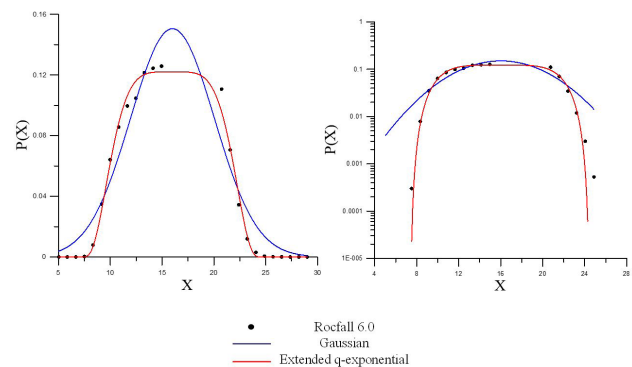

Figure 7. Case C1 – BIG (Gaussian R = 0.9843; Extended q-exponential R = 0.9883).
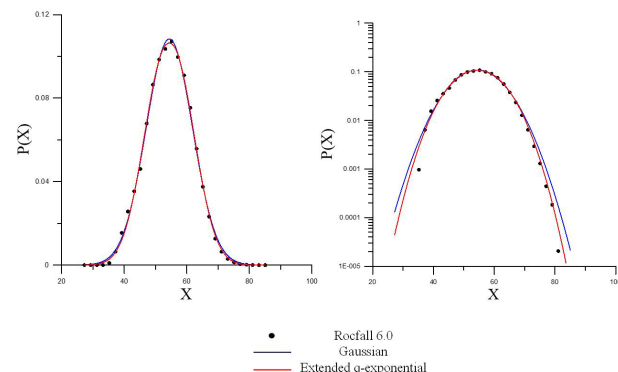
Figure 8. Case C2 – BIP (Gaussian R = 0.9988; Extended q-exponential R = 0.9991).
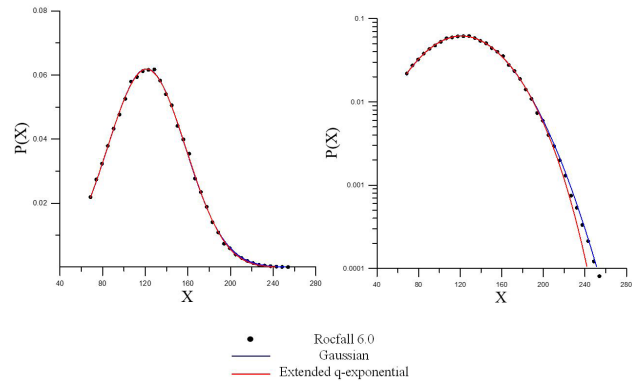
Figure 9. Case C3 – BIP (Gaussian R = 0,9994; Extended q-exponential R = 0.9995).

Figure 10. Case C4 – BIP (Gaussian R = 0.9599; Extended q-exponential R = 0.9954).

there is a rocky escarpment with approximately 175 meters high, in an area of approximately 8,000 m^2 composed mostly of phacoidal augen gneiss.

Due to the little vegetation that covers the escarpment, the face of the hill is constantly exposed to the action of physical and chemical weathering, mainly due to the thermal variation during the day - expansion and contraction of minerals with different thermal expansion coefficients. This has been leading to a gradual formation of (micro) fissures - fractures inside the rock volume, constantly growing over the years. For this reason, it is quite common to have, as a result, irregular blocks and thin, discontinuous and partially embedded chips along the entire escarpment, which may occasionally come off.

The inspections carried out by GEO-RIO identified that the rock mass has few fractures of tectonic origin, which could possibly detach more voluminous portions of the slope. These very superficial and low persistent fractures are mostly the result of thermal exfoliation isolating small to medium blocks.

Historically, the GEO-RIO database indicates that the geological accidents in the last 60 years on this slope refer to the detachment of small splinters and irregular blocks in the rock mass, with the capacity to cause damage from small

to medium size in the nearby buildings. Although the massif does not provide previous signs of movement, the existence of a situation of geological-geotechnical risk on the slope cannot be ruled out, as well as future interventions, mainly due to the proximity of local buildings to the southeastern slope of Cantagalo Massif.

Considering the geological-geotechnical risk, it appears that the possibility of the detachment of blocks is considerable and has been presented over the years, but so far without serious consequences. The profile was mapped by GEO-RIO in 2009 in order to study and enable the solution of coexistence of flexible barriers of impact on the site (GEO-RIO, 2009).

As in the preliminary study, Figure 11 shows Gaussian and Extended q-exponential probability distributions with the respective correlation coefficients (R) as an example for the case study.
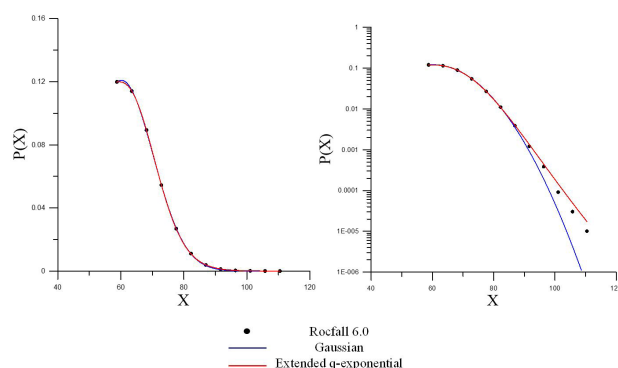
Figure 11. Case of study – BIM (Gaussian R = 0.9999; Extended q-Exponential R = 0.9999).

4. Discussions

The histograms of the number of unstable blocks located at each point downstream of the slopes show, after 100,000 simulations, consistent statistical data, sufficient to confirm the chaotic nature of the phenomenon in question. That is, the results suggest, as indicated in Table 3, that the Gaussian and the Extended q-exponential probability distributions, derived from the Boltzmann-Gibbs and $S_{q,\delta}$ entropies, respectively, describe strongly and weakly chaotic behavior.

Table 3. Compilation of the results obtained

Slope profile	Slope surface	Adjustment probability distribution	Chaotic behavior
Case C1	Concave	Extended q-Exponential	Weak
Case C2	Convex	Gaussian	Strong
Case C3	Convex	Gaussian	Strong
Case C4	Concave	Extended q-Exponential	Weak
Case study	Convex	Gaussian	Strong

Furthermore, in all slope profiles, it was possible to fit Extended q-exponential and Gaussian probability distributions to the experimental data of different block sizes (BIP, BIM and BIG) for the same slope material (phacoidal augen gneiss). So, it was initially found in each case that the size of the blocks (BIP, BIM and BIG) does not significantly influence the probability distributions obtained in the final analysis of the phenomenon. However, it is necessary to expand these studies, focusing on different block geometries, since, in the present research, the same geometry was used for all block sizes.
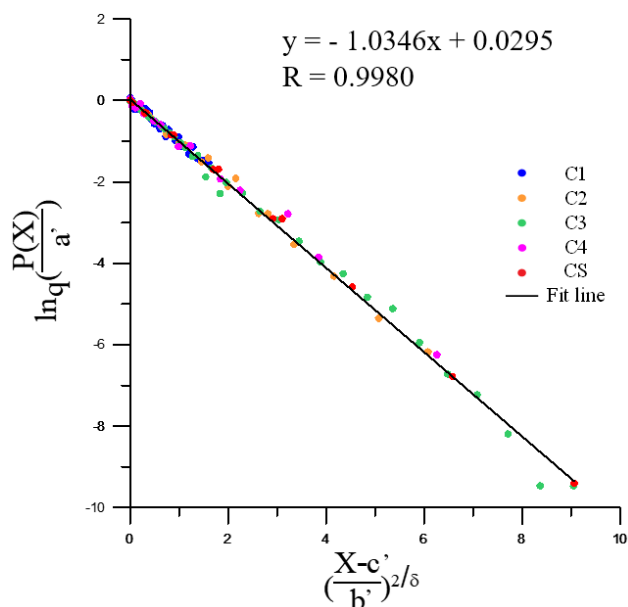
The experimental results clearly suggest that the slope profiles with markedly convex surfaces, such as C2, C3 and Case Study (Figures 2, 3, and 5) display a strongly chaotic behavior, having statistics characterized by Gaussian distributions (Figures 8, 9 and 11). It is intuitively clear that this may be due to the convex shape of the profiles involved, allowing the ejection of blocks in different trajectories, which promotes a wide dispersion along the slope, a very similar behavior encountered in Sinai billiard.

Therefore, it might be argued that the divergence between different block trajectories might follow an exponential law, typical of strongly chaotic phenomena. Accordingly, the two distributions provide very close results (Figures 8, 9 and 11), even in their tail regions, where both show strong adherence to experimental data.

On the other hand, the profiles of slopes with concave or globally concave surfaces, such as the C1 and C4 cases (Figures 1 and 4), present a weakly chaotic behavior. This time, the Extended q-exponential distribution provides much better results than the Gaussian one (Figures 7 and 10). Thus, it is a phenomenon ruled by non-Gaussian distributions, particularly in the region of the tails, which show strong adherence to experimental data. This seems to be mainly related to the concave shape of those profiles which, unlike the convex ones, tend to converge trajectories. In addition, the simulations show that the blocks remain in contact with the slope for a longer time, favoring the development of long-range memory due to the exchange of information within the system, while facilitating the dissipation of energy.

Through the previous analyzes, and with the data obtained in the preliminary study and in the case study, it is possible to elaborate the graph of the Figure 12, in which the argument $[(x-c')/b']^{2/\delta}$ and the generalized logarithmic function lnq $[p(x)/a']$ are plotted. More specifically, just as a common exponential function appears as a straight line, when plotted on a graph whose vertical axis is its inverse logarithmic function, in the same way a generalized exponential function will appear straight on a graph in which, on the vertical axis, is its generalized logarithmic inverse function.

Thus, in addition to providing a better visualization of the results obtained, it illustrates the tendency of the experimental data to collapse around the same and unique line (general equation: y = - 1.0346x + 0.0295; correlation coefficient $R = 0.9980$) (Figure 12), a clear manifestation of a

Figure 12. Functional universality graph.

type of functional universality that occurs in the phenomenon of rockfalls.

Therefore, the experimental results seem to suggest that, at least in the milestones and limitations involved in this research, regardless of the slope geometry, the height of the fall and the size of the block, the phenomenon's statistics seems to be adequately described by the Extended q-exponential distribution (and its Gaussian particular case).

5. Conclusions

The results obtained through numerical modeling showed that the profile of Cantagalo Massif (convex surface) provided probability distributions of trajectory-reach invariably Gaussian (or Extended q-exponentials in which q and δ tend to 1), whereas, in concave or globally concave profiles which gradually articulate with the base level, the Extended q-exponential probability distribution provides better results than the Gaussian one.

This suggests that convex slope profiles present a highly chaotic behavior, allowing the ejection of blocks in different trajectories and promoting a wide diffusion along the slope, in which the various trajectories might present an expressive (exponential) divergence between them.

On the other hand, the concave profiles show a weakly chaotic behavior, mainly related to the fact that in these profiles, unlike the convex ones, there is a tendency for the trajectories to converge. In addition, the blocks remain in contact with the slope for a longer time, which favors the development of a long-range memory due to the exchange of information within the system, while facilitating the dissipation of energy.

Furthermore, the results suggest that mass movements of rockfall type can be described by the Chaos Theory, in

view of the unique functional universality found. More precisely, the Extended q-exponential probability distribution (and the Gaussian, for cases where q and δ tend to 1) is able to statistically describe the phenomenon, regardless of the geometric shape of the slopes, the height of the fall and the size of the blocks. This functional universality is typical of chaotic phenomena, which denote its great power of unification and systematization of acquired knowledge.

It is noteworthy that the correct definition of the probabilistic distributions that govern the phenomenon of rockfalls with respect to their trajectory and reach has a decisive influence on the establishment of safety zones for buildings and civil works around these potentially dangerous areas, enabling, therefore, for a better urban planning for the growth of cities, especially those established in mountaineous regions, as well as assisting current methods.

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Declaration of interest

Absence of conflicting interests.

Author's contributions

Fernanda Valinho Ignacio: writing – original draft, investigation, formal analysis, visualization. Armando Prestes De Menezes Filho: conceptualization, methodology, supervision, writing – review & editing, validation. Ana Cristina Castro Fontenla Sieira: methodology, supervision, writing – review & editing, validation.

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List of Symbols and Abbreviations

a Random variable a of the Gaussian probability distribution
a' Random variable a'of the Extended q-Exponential probability distribution
b Random variable b of the Gaussian probability distribution
b' Random variable b'of the Extended q-Exponential probability distribution
c Random variable c of the Gaussian probability distribution
c' Random variable c'of the Extended q-Exponential probability distribution
q Entropic parameter of the degree of non-additivity
R Correlation coefficient
S_q Tsallis' Generalized entropy
$S_{q,\delta}$ Generalization of Tsallis' Generalized Entropy
x Variable of probability distributions
y Fit line equation
δ Entropic adjustment parameter